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Determination of Suitable Hyper Elastic Material Model through Non-Linear Regression Analysis

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ABSTRACT:

The material behaviour of elastomers can be simulated through Strain Energy Density (SED) function which can be defined by the following hyper plastic material models: (i) Neo-Hookean, (ii) Mooney-Rivlin, (iii) Yeoh and (iv) Ogden. The stress-strain relations of the above-mentioned SED functions for uni-axial tension, planar (pure shear) tension and equi-biaxial tension are validated with Treloar's data. Different combinations of Treloar's data are used to determine the co-efficient of SED functions of the above said models. These co-efficient values are determined using the software like ANSYS, MATLAB and POLYMATH and the validation of the results is carried out based on sum of squared error (SSE) which is calculated between the experimental values and predicted values. From the result, it is found that SSE less than 5 and closer to 0 can be taken as good prediction for selection of material model and co-efficient of material models. The engineering stress-strain behaviour of synthetic rubber (NBR) is obtained experimentally from uni-axial tension test and the co-efficient of SED functions are determined.

KEYWORDS:

Hyper elasticity; Stress-strain function; Material model; Mathematical simulation; Regression fit

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1. Introduction

Rubber has various applications in the engineering field, such as pneumatic tires, suspension systems, conveyors, and athletic shoes etc. Non-linear regression has become a standard tool for engineers to predict the mechanical behaviour of structures made using rubber materials. Rubber materials are generally regarded as incompressible, isotropic and hyper elastic and their mechanical properties can be conveniently described in terms of a strain energy density (SED) function. SED is a mathematical constitutive relation between the engineering stress and strain. It is very necessary to have a comprehensive understanding before using these SEDs, because making use of these models without enough understanding may introduce errors. Furthermore, determining the co-efficient of these SED functions is also more difficult than for linear elastic materials.

The co-efficient of SED functions that are based on phenomenological theory should be determined using some special tests like uni-axial tension (UT), planar tension (PT) and equi-biaxial tension (ET) are popular practices to determine the co-efficient of these functions; the samples are always cut from a single sheet to ensure the consistency of the data [2]. The strain-stress relations obtained from these tests are used to determine the co-efficient of the SED functions using a curve-fitting process. In this article, first the validation of hyper elastic material model for determining the co-efficient of SED functions and in the second section the co-efficient of SED functions for NBR are determined.

2. Hyper elastic material model

A hyper elastic material is still an elastic material, which means, that it returns to its original shape after the forces have been removed. Hyper elastic material is also called as Cauchy-elastic, which means that the stress is determined by the current state of deformation and not the path or history of deformation. Rubber typically undergoes large strains at small loads (low modulus of elasticity). The specific form of the strain energy controls the elastic material properties of the model. There are many kinds of functions. They all try to follow the stress-stretch curve for different loading cases. At the same time, they shall be as uncomplicated as possible. The simplest ones are built as a polynomial. They are usually written in the following form

$$W = \sum_{i,j,k=0}^{\infty} C_{ijk} (I_1-3)^i (I_2-3)^j (I_3-1)^k \quad (1)$$

In the case of incompressibility (1) is reduced to

$$W = \sum_{i,j,k=0}^{\infty} C_{ijk} (I_1-3)^i (I_2-3)^j \quad (2)$$

The different types of hyper elastic material models are available in the literature but familiar models such as Neo-Hookean, Mooney-Rivlin, Yeoh and Ogden forms, are briefly reviewed.

The Neo-Hookean material is an extension of Hooke's law for the case of large deformations. It is useable for certain plastics and rubber-like substances. A Neo-Hookean function uses only the first term. It will be a polynomial of the first order with one constant to be determined

$$W = C_{10}\,(I_1-3) \quad (3)$$

Another model derived from Eqn. (1) is the well-known Mooney-Rivlin function [4].

$$W = C_{10}\,(I_1-3) + C_{01}\,(I_2-3) \quad (4)$$

This strain energy function shows similar characteristics as the Neo-Hooke model and the main difference is determination of two unknown co-efficient C_{10} and C_{01}.

In 1993, Yeoh [5] proposed a phenomenological model in the form of third order polynomial based only on first strain invariant I_1. The Yeoh model [7] is also called the reduced polynomial model and for compressible rubber SED can be defined as follow

$$W = C_{10}\,(I_1-3) + C_{20}\,(I_1-3)^2 + C_{30}\,(I_1-3)^3 \quad (5)$$

Ogden developed a model called Ogden model [6] and difference is that Ogden uses principal stretch ratios λ_i instead of strain invariants I_i. In addition to the integer exponent in the ordinary polynomial Ogden uses real numbers. The benefit of this method is better adjustment possibilities to experimental curves.

$$W(\lambda_1, \lambda_2, \lambda_3) = \sum_{p=1}^{N} \mu_p / \lambda_p (\lambda_1^{\alpha_p} + \lambda_2^{\alpha_p} + \lambda_3^{\alpha_p} - 3)$$

3. Validation of SED function determination

For the validation purpose, the Treloar's experimental data [1], also used as reference in Li and Wei [3], are taken for study (Literature – Data set "A"). The co-efficient of SED functions for incompressible rubber-like materials (i.e.) Neo-Hookean, Mooney-Rivlin, Yeoh and Ogden forms are determined by ANSYS (Data set "B" with normalised error and Data set "C" with absolute error), MATLAB (Data set "D") and POLYMATH (Data set "E"). Four combinations (UT + PT + ET, UT + ET, UT + PT and UT) of the UT, PT and ET data were used to fit the co-efficient of the SED function. The validity of the model is verified using a measure called sum of squared errors (SSE). The SSE is defined as:

$$SSE = \sum_{i=1}^{n} (S_{Pi} - S_{Ei})$$

Where n – No. of data points, S_{pi} - Engineering stress value predicted using fitted model for the given experimental strain value, S_{Ei} - Experimental stress value for given experimental strain value.

The SED function of the material model is validated with the published Treloar's experimental data (1943) with the help of ANSYSv12 curve fitting module of hyper elasticity, a developed MATLAB program based on fminsearch optimization algorithm and POLYMATH 6.0. In ANSYS, the co-efficient can be determined with normalized and absolute error criteria. Whereas in POLYMATH 6.0, linear and non-linear regression solver can be used with mrqmin optimization algorithm to obtain co-efficient of the material model. The obtained co-efficients using software are given as initial guess value in the developed MATLAB program to determine the SED function of the material model. The obtained parameters of the different material model are presented in Tables 1 to 5. It is found that SSE less than and closer to 0 can be taken as good prediction for selection of material and co-efficient of material model. In validation, from these Tables and Fig. 1, it is found Treloar's experimental data can be fitted best with second order Ogden model. When UT is alone considered, reference value is better as it gives SSE = 0.87. And all other cases polymath prediction is better.

Table 1: Co-efficient of the Neo-Hookean model obtained for Treloar's experimental data with SSE

Data		UT	UT+PT	UT+ET	UT+PT+ET
A	C_{10}	0.182	0.176	0.194	0.186
	SSE	44.96	42.97	53.4	318.85
B	C_{10}	0.355	0.338	0.381	0.359
	SSE	24.72	519.45	694.18	2260
C	C_{10}	0.558	0.525	0.546	0.519
	SSE	188.06	1830	1920	5680
D	C_{10}	0.4219	0.2057	0.19	0.1371
	SSE	56.395	74.53	70.09	70.3
E	C_{10}	0.2881	0.1322	0.1373	0.086
	SSE	14.85	20.36	16.75	21.66

Table 2: Co-efficient of the Mooney-Rivlin model obtained for Treloar's experimental data with SSE

Data		UT + ET	UT + PT + ET
A	C_{10}	2.29E-01	2.12E-01
	C_{01}	4.21E-04	1.27E-03
	SSE	114.01	499.67
B	C_{10}	1.86E-01	1.71E-01
	C_{01}	3.54E-04	4.14E-03
	SSE	43.84	244.4
C	C_{10}	2.77E-01	2.62E-01
	C_{01}	-2.06E-03	-1.49E-03
	SSE	238.5	944.2
D	C_{10}	0.122	0.073
	C_{01}	1.129	1.162
	SSE	2.63E+02	6.73E+03
E	C_{10}	9.00E-02	3.60E-02
	C_{01}	3.98E-03	3.97E-03
	SSE	42.27	95

Table 3: Co-efficient of the Yeoh model obtained for Treloar's experimental data with SSE

Data		UT	UT + PT	UT + ET	UT+PT+ET
A	C_{10}	1.61E-01	1.68E-01	1.82E-01	1.78E-01
	C_{20}	-1.30E-03	-1.10E-03	-1.20E-03	-1.20E-03
	C_{30}	3.90E-05	3.60E-05	3.70E-05	3.60E-05
	SSE	61.8	475.6	466.7	1.20E+03
B	C_{10}	1.50E-01	1.50E-01	1.70E-01	1.60E-01
	C_{20}	-9.00E-04	-5.00E-04	-7.00E-04	-5.00E-04
	C_{30}	3.30E-05	2.80E-05	2.90E-05	2.60E-05
	SSE	64.53	452.88	453.4	1.02E+03
C	C_{10}	1.60E-01	1.70E-01	1.80E-01	1.79E-01
	C_{20}	-1.60E-03	-1.60E-03	-1.40E-03	-1.10E-03
	C_{30}	4.10E-05	4.06E-05	3.66E-05	3.55E-05
	SSE	60.4	478.4	467.3	645.7
D	C_{10}	1.35	1.35	1.4	1.29
	C_{20}	1.2	1.2	1.2	1.21
	C_{30}	-0.24	-0.22	-0.23	-0.013
	SSE	1.70E+04	3.70E+04	3.30E+04	1.77E+07
E	C_{10}	1.80E-01	1.83E-01	1.19E-01	1.16E-01
	C_{20}	-2.00E-03	-2.30E-03	1.40E-03	-1.50E-03
	C_{30}	4.18E-05	2.67E-05	2.30E-05	1.91E-05
	SSE	56.2	470.5	330.6	10.46

Table 4: Co-efficient of the first order Ogden model obtained for Treloar's experimental data with SSE

Data		UT	UT+PT	UT+ET	UT+PT+ET
	μ	2.02E-01	2.40E-01	3.04E-01	3.04E-01
A	α	2.462	2.328	2.22	2.187
	SSE	15.33	93.79	160.28	471.79
	μ	1.93E-01	2.14E-01	2.71E-01	2.62E-01
B	α	2.48	2.39	2.29	2.28
	SSE	15.98	91.74	102.9	492.5
	μ	1.30E-02	3.30E-02	1.07E-01	9.40E-02
C	α	3.925	3.487	2.88	2.952
	SSE	3.67	200	203.6	905.6
	μ	1.12	1.073	1.059	1.119
D	α	1.86	1.54	1.536	1.31
	SSE	67.2	1.80E+02	1.59E+02	3.41E+02
	μ	1.40E-02	1.50E-02	4.40E-02	3.00E-02
E	α	3.919	3.52	2.992	2.98
	SSE	2.81	4.29	6.41	7.42

Table 5: Co-efficient of the second order Ogden model obtained for Treloar's experimental data with SSE

Data		UT	UT + PT	UT + ET	UT + PT + ET
	μ_1	9.8E-5	2.4E-1	2.48E-2	2.33E-2
	A_1	6.24	2.34	3.58	3.64
A	μ_2	4.1E-1	-4.64	-4.06	-3.28
	A_2	1.65	1.8E-6	-1.6E-1	-2.4E-7
	SSE	0.87	47.76	82.76	1.075E3
	μ_1	1.21E-5	1.17E-8	5.6E-11	2.49E-7
	A_1	7.19	10.49	13.07	9.02
B	μ_2	3.3E-1	2.15E-1	3.6E-1	3.3E-1
	A_2	1.88	2.27	2.05	2
	SSE	2.01	1.178	1.73E12	790.99
	μ_1	1.17E-8	1.57E-8	4.92E-6	5E-10
	A_1	10.49	10.36	7.57	12.03
C	μ_2	2.15E-1	2.4E-1	3.86E-1	0.342
	A_2	2.27	2.2	1.88	2.06
	SSE	2.28	173.65	184.99	842.72
	μ_1	0.92	0.96	0.97	1.01
	A_1	1.56	1.20	1.23	1.02
D	μ_2	1.02	1.03	1.01	0.99
	A_2	1.61	1.24	1.22	1.03
	SSE	94.883	2.79E2	2.76E2	5.59E5
	μ_1	0.28	0.17	8 E-4	0.93
	A_1	1.50	1.09	4.93	0.26
E	μ_2	6.2E-3	7.1E-3	25.4	6.2E-3
	A_2	4.28	3.87	0.02	3.73
	SSE	2.10	2.24	3.11	4.793

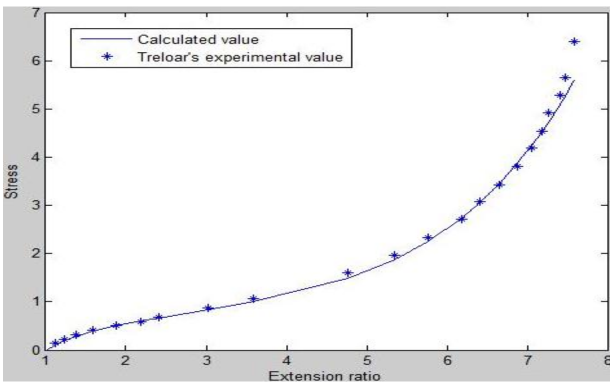
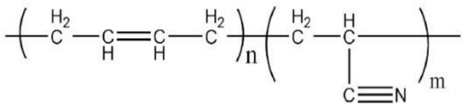


Fig. 1: Engineering stress vs. Extension ratio curve

4. Tensile strength of NBR

Nitrile rubber also known as Buna-N, Perbunan or NBR, is a synthetic rubber copolymer of acrylonitrile (ACN) and butadiene. Nitrile butadiene rubber (NBR) is a family of unsaturated copolymers of 2-propenenitrile and various butadiene monomers (1, 2-butadiene and 1, 3-butadiene). The polymer's composition of nitrile is as follows,



In the production of NBR, acrylonitrile ($CH_2 = CHCN$) and butadiene ($CH_2 = CH - CH = CH_2$) are emulsified in water and then polymerized (their single-unit molecules linked into large, multiple-unit molecules) through the action of free-radical initiators. The amount of acrylonitrile present in the final copolymer varies from 15 to 50 percent [8]. With increasing acrylonitrile content the rubber shows higher strength, greater resistance to swelling by hydrocarbon oils and lower permeability to gases. At the same time, however, the rubber becomes less flexible at lower temperatures, owing to the higher glass transition temperature of poly-acrylonitrile (i.e., the temperature below which the molecules are locked into a rigid, glassy state). Nitrile rubber is mostly used where high oil resistance is required, as in automotive seals, gaskets or other items subject to contact with hot oils. The rolls for spreading ink in printing and hoses for oil products are other obvious uses. NBR is also employed in textiles, where its application to woven and nonwoven fabrics improves the finish and waterproofing properties.

Table 6: Formulation of compounds

Constituent/ingredients	Composition in phr
Nitrile butadiene rubber (100%)	100
Processing oil (naphthenic oil)	2
Zinc oxide	5
Stearic acid	1
TDQ / Pilflex 13	2
Anti-ozonant (PF WAX)	2
Primary accelerator, Monobenzothiazole disulfide (MBTS)	1
Secondary accelerator, Tetra methyl Thiuram Monosulfide (TMTS)	1
DTDM	0.6

The compounding of NBR and other ingredients was done on a laboratory sized open two roll mill (160mm, 320mm) at the room temperature and at a speed ratio of 1:1.4 as per the ASTM D3182 according to the formulation listed in Table 6. Processing aids and rubber were first blended. Then curatives were added orderly. The samples were then cured at 160ºC in an electrically heated hydraulic press for their respective cure times t_{90} determined by oscillatory disk remoter measurements. Figs. 2 & 3 show the dimensional detail of specimen, actual cut specimen and holding arrangement of specimen while testing the specimen to get actual engineering stress-strain curve [9]. Using this curve the coefficients are determined by adopting non-linear regression software. To verify these coefficients non-linear FE analysis is conducted including both material and geometrical non-linearity. Finally, the numerical stress-strain curve is compared with experimental stress-strain curve.

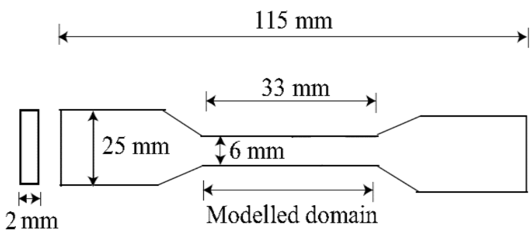


Fig. 2: Specimen dimension of NBR for tensile test

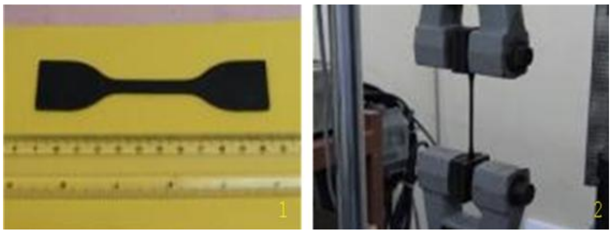


Fig. 3: Specimen of NBR for tensile test

Table 7: Co-efficients of the Neo-Hookean, Mooney-Rivlin and 1st order Ogden models obtained for NBR with SSE

Data	Neo-Hookean		Mooney-Rivlin		1st order Ogden	
B	C_{10}	0.6464	C_{10}	0.433	μ	0.389
			C_{01}	-0.2148	α	2.585
	SSE	43.661	SSE	0.754	SSE	0.091
C	C_{10}	0.755	C_{10}	0.548	μ	0.35
			C_{01}	-0.475	A	2.687
	SSE	84.861	SSE	-0.155	SSE	0.054
D	C_{10}	0.4282	C_{10}	0.115	μ	1.174
			C_{01}	1.148	α	1.766
	SSE	2.78	SSE	26.58	SSE	5.5
E	C_{10}	0.377	C_{10}	0.547	μ	0.22
			C_{01}	-0.472	α	2.528
	SSE	1.3005	SSE	0.1557	SSE	19.39

Table 8: Co-efficients of the Yeoh and 2nd order Ogden models obtained for NBR with SSE

Data	Yeoh		2nd order Ogden	
B	C_{10}	0.271	μ_1	0.12
	C_{20}	0.008	A_1	3.41
	C_{30}	-1.79	μ_2	1.00E+05
			A_2	7.21
	SSE	3.90E+05	SSE	3.00E+17
C	C_{10}	0.239	μ_1	0.4
	C_{20}	0.015	A_1	2.5
	C_{30}	-4.30E-04	μ_2	1.00E+05
			A_2	-1.97
	SSE	1.9	SSE	2.10E+11
D	C_{10}	1.42	μ_1	0.95
	C_{20}	1.27	A_1	1.3
	C_{30}	-0.53	μ_2	0.9
			A_2	1.4
	SSE	3.50E+03	SSE	42
E	C_{10}	0.93	μ_1	1.00E-07
	C_{20}	-0.14	A_1	9.5
	C_{30}	0.018	μ_2	0.9
			A_2	1.48
	SSE	9.55	SSE	11

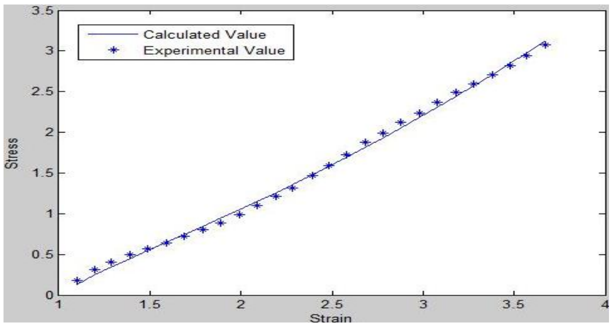


Fig. 4: Engineering stress vs. Engineering strain curve

From the Tables 7 and 8 and Fig. 4, it can be noted that the SSE value obtained for first order Ogden model from ANSYS prediction is less than 5 and closer to 0. Hence for the NBR compound taken for study first order Ogden model is best.

5. Conclusion

The following conclusions are derived based on the validation of Li and Wei (2015) and the test carried out on NBR taken for study:

- For better prediction of material model and its co-efficient, SSE can be taken as measure of degree of fitness and for better prediction, SSE should be less than 5 and closer to 0.
- For the NBR taken for study, first order Ogden model is the best material model.

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