Kow et al. 2018. *Int. J. Vehicle Structures & Systems*, 10(2), 133-136
ISSN: 0975-3060 (Print), 0975-3540 (Online)
doi: 10.4273/ijvss.10.2.11

International Journal of Vehicle Structures & Systems
Available online at www.maftree.org/eja

Sliding Mode Controller with Nonlinear Sliding Surface for Two-Link Planar Manipulator

Yu-Yang Kow[a], Jee-Hou Ho[a] and Tong-Yuen Chai[b]

[a]*Dept. of Mech., Materials and Manuf. Engg., The University of Nottingham Malaysia Campus, Malaysia*
[b]*Dept. of Mechatronics and Biomedical Engg., Universiti Tunku Abdul Rahman, Malaysia*
Corresponding Author, Email: chaity@utar.edu.my

ABSTRACT:

The objective of this paper is to propose a method to control a two-link planar manipulator using a sliding mode controller with higher order sliding surface. Earlier approaches of varying sliding surface are time dependent and this may not yield optimal performance as the rotating surface does not depend on the dynamic states of the system. In this paper, the proposed control law alters the sliding surface based on the error states to drive the trajectory approaching the sliding phase quicker. A new sliding manifold is established and tuned until error decreases to zero. Stability is ensured through Lyapunov theorem and the trajectory is driven towards a designed sliding surface. The performance of the proposed controller is evaluated and compared against the conventional sliding mode controller as well as the previous approach of rotating sliding surface controller. Results showed that the proposed controller improved the respond speed and shortened the reaching phase. Although the chattering phenomenon remains, it enhanced the flexibility to adapt to the variation of system settings (e.g. torque limit).

KEYWORDS:

Sliding mode control; Nonlinear sliding surface design; Robot manipulator

CITATION:

Y.Y. Kow, J.H. Ho and T.Y. Chai. 2018. Sliding Mode Controller with Nonlinear Sliding Surface for Two-Link Planar Manipulator, *Int. J. Vehicle Structures & Systems*, 10(2), 133-136. doi:10.4273/ijvss.10.2.11.

1. Introduction

During the last few decades, the development of sliding mode control (SMC) has advanced rapidly in the field of automation. It has been recognized as a robust and reliable solution for many nonlinear dynamical systems. It follows the basic principle of variable structure control system (VSS) by which the control algorithm is a discontinuous function of system states through proper switching logic [1]. Due to the presence of nonlinearity in those highly complicated machines, linear control methods have their limitation in achieving accurate motion control. Thus, as a nonlinear discontinuous switching control system, SMC possess an advantage in controlling these highly complicated motions [2]. The primary motivation of developing SMC is that it eliminates the effects of the disturbance and uncertainty parameters during the sliding phase [3-4]. In this phase, control signal switches rapidly to neutralize the disturbance effect. Although, SMC has high capability in overcoming the disturbance effect during the sliding phase, the system motion may be distorted during the reaching phase. Therefore, many variants of SMC are proposed to improve its performance through shortening the reaching phase by altering the sliding surface [5-6]. Although the reaching phase is sharply reduced by moving linear sliding surface, it trades off the dwelling time to the sensitivity of disturbance [7].

Nonlinear sliding surfaces such as higher order sliding surface and PID sliding surface, are suitable candidates to replace the linear sliding surface for enhancing the system performance [8-10]. However, they impose complexity to determine a stable control algorithm and merely one parameter is allowed to be tuned for some higher order SMCs [11]. SMC has some disadvantages too, noticeably the chattering effect as a result of high frequency oscillations of control outputs, partly due to hysteresis and signal delaying. It might degrade the precision electronic device because of the high vibration frequency. The chattering issue could be overcome by changing the control law of SMC or frequently tuning the gain of SMC. For example, super-twisting SMC and fuzzy SMC are proposed to adjust the control gain so as to reduce the chattering effect [12-14]. However, it is required to predefine the initial states of the control input to ensure the system performance fulfilling certain criteria such as settle time and stability.

This project aims to propose a sliding surface design as an attempt to improve the system performance. The structure of sliding surface is tuned by the difference between the actual value and desired value. Due to less variances of system model, the flexibility of the control setting is enhanced in the proposed SMC. Furthermore, the stability of the proposed SMC (named as SMC-H, see Section 2) is ensured through the Lyapunov stability theorem. As a case study, the controller will be applied in the control of two link planar robotic manipulator. Its performance will be compared with the conventional SMC (SMC-C) and time-varying SMC (SMC-T) [15] in terms of settling time, reaching phase and torque output.

2. Theory

SMC has good disturbance rejection capability because the discontinuous state function is activated as the trajectory converges to the sliding surface. Sliding surface is a designed path to guide the trajectory towards the desired state variable. As the sliding surface is near to the initial conditions, the duration of reaching phase will be reduced. Thus, sliding surface directly influences to the system performance. The sliding surface Eqn. for second order system, S is

$$S(x,t) = \dot{e}(t) + ce(t) \tag{1}$$

Where x is the state variable, c is the slope of the linear function and it is a positive value, $e(t)$ the feedback error and $\dot{e}(t)$ is the rate of change of feedback error. Control law is another important design process of SMC. It contains the sign function and relies on the feedback loop to force the trajectory slides along the sliding surface. However, to design a suitable control law, Lyapunov stability theorem is required. The control law of SMC is given by,

$$k\,\mathrm{sign}(S) = \begin{cases} k & \text{if } S(x) > 0 \\ -k & \text{if } S(x) < 0 \end{cases} \tag{2}$$

where k is the gain of control system. Lyapunov stability theorem is adopted to assure the direction of trajectory driven by control law. The stability condition is given as,

$$\dot{V} = S\dot{S}(x,t) \le -n|s(x,t)| \tag{3}$$

To ensure the trajectory will converge towards the sliding surface in a finite time (S = 0 when T∞), n should be a positive real number. The control input is determined by the Lyapunov function [15],

$$u = \frac{-f + \ddot{x}_d - c\dot{e}}{a} - k\,\mathrm{sign}(S) \tag{4}$$

Where f(t) is the dynamic Eqn. of the system, u(t) is the control input while a(x, t) is the coeff. of control input.

3. Proposed SMC with nonlinear sliding surface (SMC-H)

Since convectional SMC (SMC-C) has a static linear sliding surface, it indicates that the trajectory could be possibly distorted by disturbance for a long period. Although the parabolic shape sliding surface shorten the reaching phase of time-varying SMC (SMC-T) [15], its sliding surface will lose its curve attribute as $k_s(t)$ approaches to zero. Moreover, SMC-T is less flexible in adjusting the system performance because it is required to assure the curve shape is retained before it reaches the desired slope. Although, the allowable maximum velocity is increased by raising the control gain, the consequence is that there is a high chattering effect in the control signal. Thus, new sliding surface function (SMC-H) should address those limitations. The proposed SMC is similar to SMC-T which has a curved sliding surface but it has even acceleration and deceleration motion to promote the trajectory speed to higher speed region. Besides that, the surface also depends on the initial condition, to shorten the duration to reach the sliding phase. Moreover, the control strategy incorporates the error feedback loop to calculate the appropriate control signal to drive the system. Therefore, it has higher flexibility to define the optimal setting under equipment limitations. The proposed sliding surface equation $S_p(x,t)$ is given as:

$$\begin{aligned} S_p(x,t) &= \dot{e}(t) + (Ae^2(t) + B(e)e(t)) \text{ for } e(t) < 0 \\ S_p(x,t) &= \dot{e}(t) - (Ae^2(t) + B(e)e(t)) \text{ for } e(t) > 0 \end{aligned} \tag{5}$$

Where B is the bending coefficient and A is the acceleration coefficient. The bending efficient, B is the primary parameter to regulate the reaching phase. To minimize the reaching phase, the sliding surface always been set around the trajectory at early state. Besides that, the sliding surface and bending coefficient are continuously tuned by the feedback error to retain the invariance effect. However, the consequence of early sliding phase is that the chattering phenomenon might sustain for the whole operation. This issue could be mitigated by increasing the acceleration coefficient, A. The acceleration coefficient is a scale value to promote the acceleration rate.

During the sliding phase, partly discontinuous control input induces the rapid change of speed. Hence, higher velocity change rate could mitigate the chattering effect. Equation for bending coefficient, $B(e)$:

$$\begin{aligned} B(e) &= Ap^*\exp(D(e(t) + (p^* + H)) \text{ for } e(t) < 0 \\ B(e) &= Ap^*\exp(-D(e(t) + (p^* + H)) \text{ for } e(t) > 0 \end{aligned} \tag{6}$$

Where p^* is gap parameter, D defines a scaling parameter while H is a shifting parameter. The value of p^* directly influences the disturbance effect on the trajectory. p^* denotes the distance between the switching line and initial trajectory. Shorter distance indicates less time for reaching sliding phase and easily activates the discontinuous function to neutralize the disturbance. Shifting parameter is another vital factor in adjusting the disturbance sensitivity. Increasing H value indicates that the sliding surface deviate away from the initial condition. It could accelerate the tuning rate without increasing the torque output but the consequence is higher disturbance sensitivity. Due to those parameters of the bending coefficient are invariable to the system model, variable combinations of the control parameter are set to optimize the system performance under different dynamic limitations. Therefore, the flexibility of the system has been enhanced by this sliding surface.

4. Simulation results and analysis

A case study is presented to compare the performance of SMC-C, SMC-T and the proposed SMC-H. A two-link planar robotic manipulator, shown in Fig. 1, is studied here and the joint variables are θ_1 and θ_2. Its dynamic model is common and could be referred to some standard references. The dynamic model and control system are developed in Matlab Simulink. To have a realistic study, the gravitational effect and disturbance are included in the analysis. The control inputs u_1 and u_2, subscripts 1 and 2 denote links 1 and 2 respectively, are given by Eqn. (7).

$$\begin{aligned} u_1 &= -\frac{2}{3}\sin\theta_2\dot{\theta}_2(2\dot{\theta}_1+\dot{\theta}_2)-\left(\frac{2}{3}+\cos\theta_2\right)\sin\theta_2\dot{\theta}_1^2+2g\cos\theta_2-g\cos(\theta_1+\theta_2)\cos\theta+\left(\frac{16}{9}-\cos^2\theta_2\right)(-2A_1\dot{e}_1-B_1\dot{e}_1-e_1\dot{B}_1-k_1\ \mathrm{sign}(s_1)) \\ u_2 &= \left(\frac{2}{3}+\cos\theta_2\right)\sin\theta_2\dot{\theta}_2(2\dot{\theta}_1+\dot{\theta}_2)+2\left(\frac{5}{3}+\cos\theta_2\right)\sin\theta_2\dot{\theta}_1^2+\frac{8}{3}g\cos\theta_2-2g\cos\theta_2+\left(\frac{16}{9}-\cos^2\theta_2\right)(-2A_1\dot{e}_2e_2-B_2\dot{e}_2-e_2\dot{B}_2-k_2\ \mathrm{sign}(s_2)) \end{aligned} \quad (7)$$

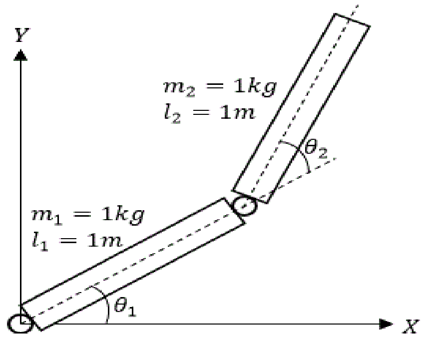

Fig. 1: Two-link plane manipulator system

Disturbance equation is given by:

$$D_1 = D_2 = 2.5\sin(3.5\pi t) \quad (8)$$

The initial conditions are set as θ_1 = 0 rad, $\dot{\theta}_1 = -\pi/9$ rad/s, θ_2 = 0 rad and $\dot{\theta}_2$ = 0 rad/s while the desired states are $\theta_1(t_f) = \pi/3\,\mathrm{rad}$ and $\theta_2(t_f) = 4\pi/9\,\mathrm{rad}$ for step input testing [15]. Three SMCs are sharing the same value of control gains $k_1 = k_2 = 3$ and the slope parameters $c_1 = c_2 = 5$. Tables 1 and 2 give the control parameters for the SMC-T and SMC-H. The trajectories for 1st and 2nd link are presented in Fig. 2 and 5 respectively.

Table 1: Control parameters of SMC-T

First link parameter set				Second link parameter set			
k_s^+	k_s^-	a_1	a_2	k_s^+	k_s^-	a_1	a_2
0	-220	22	0	0	-110	8.6	-0.035

Table 2: Control parameters of the proposed SMC-H

First link parameter set				Second link parameter set			
A_1	p^*_1	D_1	H_1	A_2	p^*_2	D_2	H_2
9	1.0602	0.3	0.09	6	1.4	0.025	0.09

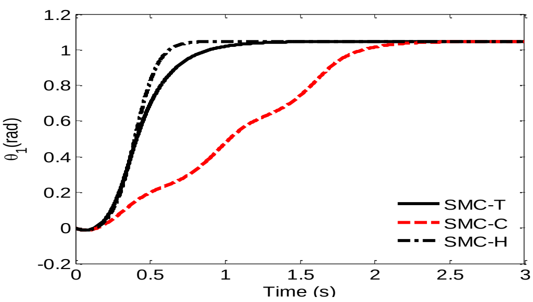

Fig. 2: System trajectory vs. Time at 1st link

Compared with the second link, first link plane is the most effective in shortening the settling time and reaching phase. Fig. 3(a) shows that the SMC-H spent the shortest time to reach the steady state which improved more than 50% respond speed compared to SMC-C or 20% faster than SMC-T. According to Fig. 3(b), the chattering effect of SMC-H started after 0.17 s which was the quickest to reach the sliding surface. It is because SMC-H has the shortest distance between the initial condition and the sliding surface. Thus, SMC-H has the least disturbance effect. Moreover, SMC-H could boost the trajectory in higher speed region without increasing the control gain and slope parameter. In Fig. 3(c), the SMC-H has the highest trajectory speed compared to other two SMCs. It indicates that SMC-H could improve the respond speed without sacrificing the system performance. Although the nonlinear SMCs require more torque to accelerate the dynamic model, SMC-H has higher efficiency on energy management. SMC-H has higher acceleration rate compared to SMC-T, but the required torque of SMC-H has merely increased. Thus, SMC-H has higher flexibility in adapting the variable of system setting.

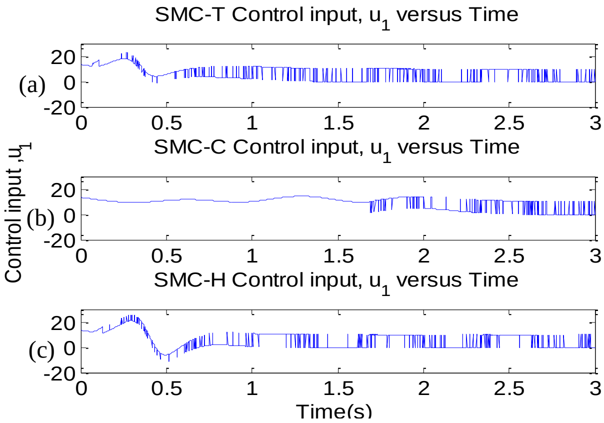

Fig. 3: Control outputs for each SMC at 1st link

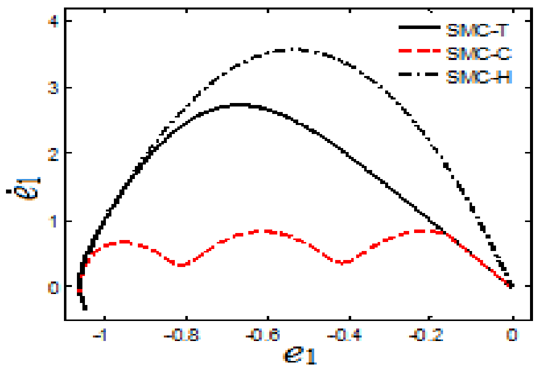

Fig. 4: Rate of change of error $\dot{e}_1(t)$ vs. Error $e_1(t)$

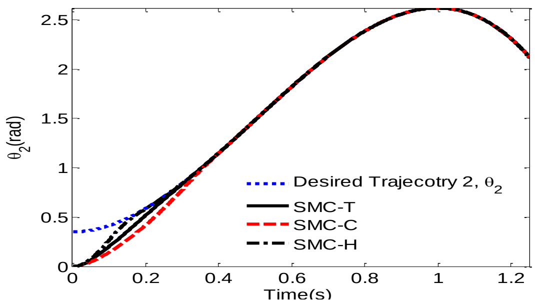

Fig. 5: Trajectory tracking performances for each SMC at 2nd link

To obtain the tracking capability, all SMCs have been tested under the specific trajectory path. The trajectory path of first link will start from 20° to 80° (see Fig. 2) while for second link (Fig. 5), it starts at 20° to 150° and both link have the constant accelerations. While the gain and slope parameter for each SMC are similar (k_1 = 30, k_2 = 40, c_1 = 30, c_2 = 40). The error plot is shown in Fig. 4. Fig. 5 shows the tracking results. The second link plane showed significant improvement compared to the first link. Although the large control gain greatly increases the required torque amount, SMC-H still performed efficiently on tracking capability. Figs. 6(a) and 6(b) show that the SMCs required the same amount of torque to accelerate the manipulator and no overshoot issue, but SMC-H required least time to align back on the trajectory path (see the first 0.2 s in Fig. 6(a)). Hence, it has the best trajectory tracking ability.

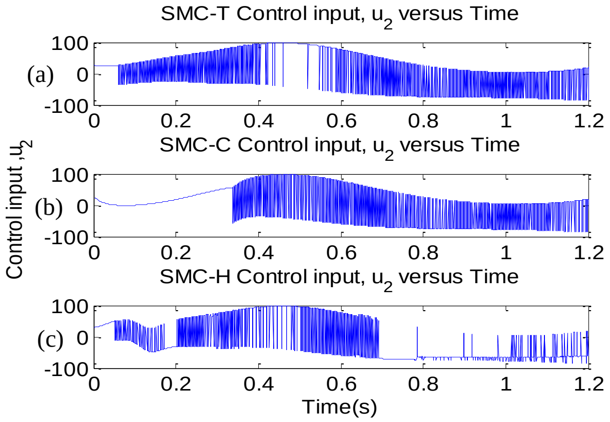

Fig. 6: Control inputs for trajectory tracking test

5. Conclusion

In this paper, a higher-order sliding surface and its controller (SMC-H) has been successfully developed and studied. Results showed that SMC-H has the best settling time in step input test. Besides that, the SMC-H trajectory has sustained the least disturbance effect as it was the fastest to reach the sliding phase. Furthermore, SMC-H has been efficient on energy management in trajectory tracking ability. Under similar torque input, SMC-H was able to accelerate to higher speed region and spent least time to align the trajectory path. Although, the system performance has been improved by SMC-H, the chattering effect remains. This could be a focus for future work to mitigate the chattering effect.

REFERENCES:

[1] T.C. Manjunath. 2007. Design of moving sliding surfaces in a variable structure plant & chattering phenomena, *Int. J. Mech. Aerospace, Ind. Mechatron. Manufacturing Engg.*, 1(9), 752-758.

[2] R. Fellag and M. Hamerlain. 2015. Discrete first and second order sliding mode controllers for a pneumatic artificial muscles robot manipulator, *Proc. 2015 4th Int. Conf. Electr. Engg. Boumerd k s, Alger.*, 1-6. https://doi.org/10.1109/INTEE.2015.7416754.

[3] J.S.V.U.J. Guldner. 2009. *Sliding Mode Control in Electro-Mechanical Systems*, Second Edition.

[4] H. Yadegari, H. Chao, and Z. Yukai. 2016. Finite time sliding mode controller for a rigid satellite in presence of actuator failure, *Proc. 3rd Int. Conf. Inf. Sci. Control Engg.*, 1327-1331. https://doi.org/10.1109/ICISCE.2016.283.

[5] M.Z. Shah. 2011. Sliding mode based lateral control for uavs using piecewise linear sliding surface, *Commun. Comput. Control Appl. Int. Conf.*.

[6] T. Mizoshiri and Y. Mori. 2016. Sliding mode control with a linear sliding surface that varies along a smooth trajectory, *Proc. SICE Int. Symp. Con. Syst.*, 1(c), 31-36.

[7] S. Choi, C. Cheong and D. Park. Moving switching surfaces for robust control of second - order variable structure systems, *Int. J. Control*, 58(1), 229-245. https://doi.org/10.1080/00207179308922999.

[8] Tavakoli, A.R. Seifi and A. Reza. 2016. Adaptive self-tuning PID fuzzy sliding mode control for mitigating power system oscillations, *Neurocomputing*, 218, 146-153. https://doi.org/10.1016/j.neucom.2016.08.061.

[9] M.U. Salamci and G.S. Tombul. 2006. Sliding mode control design with time varying sliding surfaces for a class of nonlinear systems, *Proc. IEEE Int. Conf. Control Appl.*, 996-1001.

[10] N.M. Dehkordi, N. Sadati and M. Hamzeh. A back stepping high-order sliding mode voltage control strategy for an islanded microgrid with harmonic/interharmonic loads, *Control Engg. Pract.*, 58, 150-160.

[11] S. Mondal and C. Mahanta. 2011. Nonlinear sliding surface based second order sliding mode controller for uncertain linear systems, *Commun.. Nonlinear Sci. Numer. Simul.*, 16(9), 3760-3769. https://doi.org/10.1016/j.cnsns.2010.12.020.

[12] Y. Shtessel, M. Taleb and F. Plestan. 2012. A novel adaptive-gain supertwisting sliding mode controller: Methodology and application, *Automatica*, 48(5), 759-769. https://doi.org/10.1016/j.automatica.2012.02.024.

[13] S.A. Tchenderli-Baham, F. Hamerlain and N. Saadia. 2015. Adaptive sliding mode controller applied on an under actuated wheeled mobile robot, *Proc. 15th Int. Conf. Control, Automation and Systems*, 716-719. https://doi.org/10.1109/ICCAS.2015.7365013.

[14] M. Morsy, M. Moteleb and H.T. Dorrah. 2008. Design and implementation of fuzzy sliding mode controller for switched reluctance motor, *Proc. IEEE Int. Conf. Ind. Technol.*, II, 19-21.

[15] S. Tokat, I. Eksin, M. Güzelkaya and M.T. Söylemez. 2003. Design of a sliding mode controller with a nonlinear time-varying sliding surface, *Trans. Inst. Meas. Control*, 25(2), 145-162. https://doi.org/10.1191/0142331203tm079oa.