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Rail Vehicle Modelling and Simulation using Lagrangian Method

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ABSTRACT:

Formulation of vehicle dynamics problem is dealt either with Newton's method or Lagrange's method. This paper provides a broad understanding of Lagrange's method applied to railway vehicle system. The Lagrange's method of analytical dynamics provides a complete set of equations through differentiations of a function called Lagrangian function which includes kinetic and potential energy with respect to independent generalised coordinates assigned to the system. This paper also discusses rigid body rotational dynamics along with the concept of generalised coordinates (constrained and un-constrained) and generalised forces in detail.

KEYWORDS:

Lagrangian function; Euler's angle; Newton's method; Generalized forces; Generalized coordinates; Body fixed axes

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NOMENCLATURE:

y_i, z_i	Translational coordinates in lateral & vertical directions respectively (i =1 for car body, 2&3 for front & rear bolster, 4&5 for front & rear bogie frame and 6, 7, 8 & 9 for four wheel axles)
ψ_i, ϕ_i, θ_i	Euler yaw, pitch and roll rotations
ω_i	Angular velocity of rigid bodies
$k_{CB}^{z,y}$	Vertical and lateral stiffness between car body and bolster respectively (½ part)
$k_{BBF}^{z,y}$	Vertical (¼ part) and lateral stiffness between bolster and bogie frame
F_{Zj}, F_{Yj}	Vertical and lateral force at the wheel-rail contact point in vertical and lateral direction *(j = 1,….8)*
t_W	Lateral distance (bogie frame c.g. – vertical suspension between bogie frame and wheel axle)
t_C	Lateral distance (car body c.g. – side bearings)
t_B	Lateral distance (bolster c.g. – vertical suspension between bolster and bogie frame)
l_A	Longitudinal distance (wheel axle c.g. – vertical suspension between bogie frame & wheel axle)
x_{12}	Long. distance (car body c.g. – bolster c.g.)
z_{12}	Vertical distance (car body c.g. – bolster c.g.)
z_{24}	Vertical distance (bolster c.g. – bogie frame c.g.)
x_{46}	Long. distance (bogie frame c.g. – wheel axle c.g.)
z_{46}	Vertical dist. (bogie frame c.g. –wheel axle c.g.)
z_{STi}	Vertical distance of different rigid bodies c.g. from $X_I Y_I$ plane in normal static position, i = 1 (car body), 2 (bolster), 4 (bogie frame), 6 (wheel axle)
r	Radius of wheel axle set to the track center line

1. Introduction

In general there are two approaches to solution of any vibratory systems - Newton's method and Lagrange's Method. Newton's method is usually applied to a simpler system modelled with lesser degree of freedoms. It deals with restoring and disturbing forces which a vector quantity. For the analysis magnitudes and directions of the forces acting on the system must be analysed. Constraint forces must also be determined. Sometimes constraint forces are not determined directly and required to be considered as added unknown variables in the equations of motion. Moreover, the particle accelerations also present kinematical difficulties in analysis. An alternate approach is that of analytical dynamics, as represented by Lagrange's equations. This method provides a complete set of equations of motion through differentiations of a scalar function i.e. Lagrangian function. The function accounts kinetic and potential energies; however ideal constraint forces are not involved. Thus, sequential methods for obtaining the equations of motion are possible and wide range of problems can be analysed.

Lagrange's Method also includes spring strain energy and Rayleigh's dissipation energy. Generalized forces are determined in Lagrange's method; however they are determined using the principle of virtual work using the differentiating expressions of position vectors with respect to each independent generalized coordinates in turn to get the equations of motions as follows [1-6],

$$\frac{d}{dt}\left[\frac{\partial L}{\partial \dot{q}_i}\right] - \frac{\partial L}{\partial q_i} + \frac{\partial E_P}{\partial q_i} + \frac{\partial E_D}{\partial \dot{q}_i} = Q_i \tag{1}$$

Lagrangian L is defined as $(T - V_g)$ where T is the kinetic energy and V_g is the potential energy, E_P is the strain energy stored due to springs, E_D is the dissipation energy of the system and Q_i are the generalized forces corresponding to y_i the generalized coordinates. The term energy is a scalar quantity which is convenient and easier to deal with. Lagrange's method involves calculation of transformation matrix of body fixed rotations of different rigid bodies, angular velocities, position vectors, linear velocities and finally rotational and translational kinetic energy [7].

For a complex system having many degrees of freedom e.g. rail vehicle system these above-mentioned terms are still much easier to formulate as compared to using Newton's method. A number of rigid body dynamics modelling cases considers very large rotations. The appropriate examples involve aircraft [8] and spacecraft [9-10]. The study of large rotation dynamics is of prime interest in so many cases with application of mechanism and machine theory [7, 11] and molecular dynamics [12]. Most models of rigid body dynamics problems consider body fixed Euler rotation of axis [13]. Euler formulations provide independent equations of motion, but which contain singularities [8]. Laning-Bortz-Stuelpnagel [14] and Rodriguez [15] have also formulated other methods. The consideration of singularities in all these methods developed an alternative four parameter modelling system [14], considering Euler parameters [16]. Such formulations replace the three Euler angles with four parameters and an algebraic constraint. This avoids the Euler angle singularities but leads nominally to a system level model in differential-algebraic form. In order to escape from singular equations of motion and differential-algebraic systems, few researchers reformulated Euler parameter based models, to analyse in three dimensional rigid body dynamics problems. Chang et al [9], Nikravesh et al [17-20], and Vadali [22] presented equivalent replacement formulations based on Lagrange's equations.

Since Lagrange's method consider a solution as a path in configuration space, and since the Euler parameters are accounted as generalized coordinates, this approach starts with a differential system of second-order for the rotational dynamics of a single rigid body developed with a single algebraic constraint. Nikravesh and co-workers began from this starting point and proceed to find a closed form solution for the Lagrange multiplier associated with the algebraic constraint, resulting in an unconstrained formulation of order eight. They do not include a potential energy function in the Lagrangian approach. Similar results are obtained by Vadali [22]. Proceeding in a different manner, Chang et al introduce as quasi-velocity variables the rigid body angular velocities in the body fixed frame, and project the original order eight Lagrange equations onto an order seven subspace. In the process they eliminate the unknown Lagrange multiplier. As an alternative to Lagrange's equations, a Hamiltonian formulation of rigid body dynamics with Euler parameters has been proposed by Morton [23]. However his final formulation is of order eight, and includes a superfluous momentum variable as well as a ''generally arbitrary'' unspecified scalar parameter. It appears that no previous work has attempted to revise or improve upon the Morton formulation. The usefulness of formulations based on Hamilton's canonical equations is well recognized [7]. They offer an explicit state space description of system dynamics problems which is convenient for numerical integration and well suited for coupling to automatic control system models.

2. Generalized coordinate, constrained equation and degree of freedom

If the starting position or static equilibrium position of a system is known, it is needed to select a set of geometrical parameters whose value uniquely defines a new position relative to the initial position. It is possible to draw a diagram in its current position by knowing the fixed dimensions and the position parameters. Geometrical quantities that meet this specification are called generalized coordinates [7]. The minimum number of generalized coordinates required to specify the position of the system are the degrees of freedom of that system. Generalized coordinates do not form a unique set of parameters that may be equally suitable for describing the motion. There is a situation where generalized coordinates equals the number of degrees of freedom. Those generalized coordinates are indeed unconstrained and are known as independent generalized coordinates [7, 13]. Constrained generalized coordinates (P) are related to degrees of freedom (Q) with constrained equations (R) using, P-Q = R. The concept is illustrated by considering the example of bolster body of railway vehicle as shown in Fig. 1. Consider the bolster of railway vehicle AB of length L. Coordinates of A and B are (x_A, y_A) and (x_B, y_B) respectively. AB makes angle θ with the horizontal. Now x_A, y_A, x_B and y_B are constrained generalized coordinates. The bolster can be configured with minimum three degrees of freedom which may be any three out of four x_A, y_A, x_B and y_B coordinates or may be x_A, y_A and θ. The one constrained equation is Pythagorean Theorem.

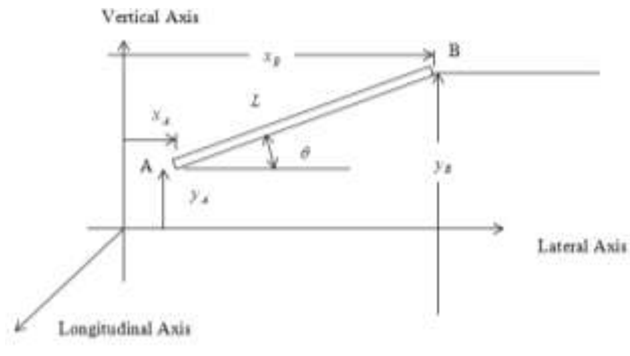

Fig. 1: DoF/Generalised coordinates for bolster

3. Euler's Axes Transformation & Angle

For an arbitrary vector "r", a linear relationship between its components in the primed and un-primed coordinate systems (see Fig. 2) is obtained [13] as r' = Cr and

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} c_{x'x} & c_{x'y} & c_{x'z} \\ c_{y'x} & c_{y'y} & c_{y'z} \\ c_{z'x} & c_{z'y} & c_{z'z} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad (2)$$

$r = x\mathrm{i} + y\mathrm{j} + z\mathrm{k} = x'\mathrm{i}' + y'\mathrm{j}' + z'\mathrm{k}'$ (3)

Where C is rotation matrix which describes the relative orientation of coordinate systems [13]. Generally, these elements of a rotation matrix are different. The first column of the C^T matrix is the components of the unit vector i in primed frame. Similarly, the second and third columns of *C* represent j and k respectively. Hence, the sum of the squares of the elements of a single column must be unity, or the scalar product of a unit vector with itself must equal one. The scalar product of any two different columns is zero, as the unit vectors are mutually orthogonal. Altogether, there are three independent equations for single columns, and three independent equations for pairs of columns, giving a total of six independent constraining equations. The number of direction cosines (nine) minus the number of independent constraining equations (six) yields three, the number of rotational degrees of freedom [13]. By interchanging primed and un-primed subscripts, the transposed rotation Matrix C^T is the rotation matrix for the transformation from the primed frame to the un-primed frame. Hence, the sequence of transformations *C* and C^T, in either order, will return a coordinate frame to its original orientation as $C^TC = CC^T = U$, where *U* is a 3×3 unit matrix, that is, with ones on the main diagonal and zeros elsewhere as $C^T = C^{-1}$. Rotation matrices whose transpose and inverse are equal are classed as orthogonal matrices. The determinant of any rotation matrix is equal to +1. In general, a rotation of axes given by C_a followed by a second rotation C_b is equivalent to a single rotation as $C = C_bC_a$.

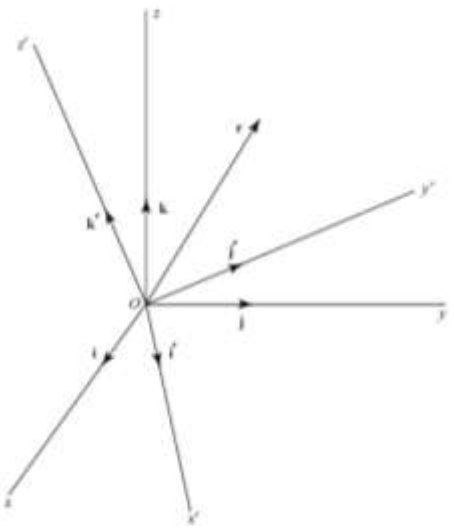

Fig. 2: Euler's orientation of axes

As the order of matrix multiplications is important, indicating that the order of the corresponding finite rotations is also important. The rotation of a rigid body from some reference position to an arbitrary final position is represented by three rotations in a given sequence about specified body axes. The resulting angles of rotation are called Euler angles shown in Fig. 3 and represent an axis-of-rotation order *z*-*y*-*x*, where each successive rotation is about the latest position of the given body axis. The *xyz* body axes and the *XYZ* inertial axes coincide initially. Then three rotations are made in the following order: (1) ψ about the *Z*-axis, resulting in the primed axis system; (2) ϕ about the *y'* axis (line of nodes) resulting in the double-primed system; (3) θ about the *x"* axis, resulting in the final *xyz* body-fixed frame. A rotation matrix *T*, defined in terms of direction cosines. Euler's body fixed rotation, axes systems and inertial frame of reference is illustrated in Fig. 4 for the railway vehicle's car body [23].

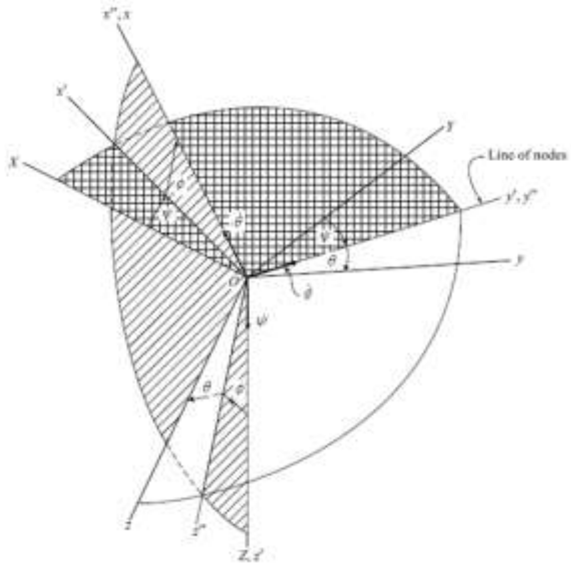

Fig. 3: Euler's angle and Euler's angular velocity

Three rotational degrees of freedom ψ_1(yaw), ϕ_1(pitch) and θ_1(roll) are assigned to car body. Initially X_i, Y_i and Z_i (mass centre of car body) coincide with inertial axes XYZ. The orientation of the body fixed axes X_1, Y_1 and Z_1 system of car body mass centre is defined by the three Euler rotationsψ_1, ϕ_1 and θ_1. The first rotation (ψ)is given about Z1 axis transforming X_1, Y_1 and Z_1 axes to $x_1^{'}, y_1^{'}, z_1^{'}$ [23] and the transformation matrix is written as:

$$\begin{bmatrix} x_1^{'} \\ y_1^{'} \\ z_1^{'} \end{bmatrix} = \begin{bmatrix} \cos\psi_1 & \sin\psi_1 & 0 \\ -\sin\psi_1 & \cos\psi_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \end{bmatrix} \quad (4)$$

The second rotation is ϕ_1 (pitch) is given about $y_1^{'}$ axis resulting in $x_1^{''}$, $y_1^{''}$, $z_1^{''}$ axes system is given by,

$$[\varphi_1] = \begin{bmatrix} \cos\varphi_1 & 0 & -\sin\varphi_1 \\ 0 & 1 & 0 \\ \sin\varphi_1 & 0 & \cos\varphi_1 \end{bmatrix} \quad (5)$$

The third rotation is θ_1 (roll) is about $x_1^{''}$, resulting in $X_1^{'}, Y_1^{'}, Z_1^{'}$ as follows,

$$[\theta_1] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_1 & \sin\theta_1 \\ 0 & -\sin\theta_1 & \cos\theta_1 \end{bmatrix} \quad (6)$$

$$[T_1] = [\theta][\phi][\psi] \quad (7)$$

$$[T_1] = \begin{bmatrix} \cos\psi_1\cos\varphi_1 & \sin\psi_1\cos\varphi_1 & -\sin\varphi_1 \\ -\sin\psi_1\cos\theta_1 + \cos\psi_1\sin\varphi_1\sin\theta_1 & \cos\psi_1\cos\theta_1 & \cos\varphi_1\sin\theta_1 \\ \sin\psi_1\sin\theta_1 + \cos\psi_1\sin\varphi_1\cos\theta_1 & -\cos\psi_1\sin\theta_1 & \cos\varphi_1\cos\theta_1 + \sin\psi_1\sin\varphi_1\cos\theta_1 \end{bmatrix} \quad (8)$$

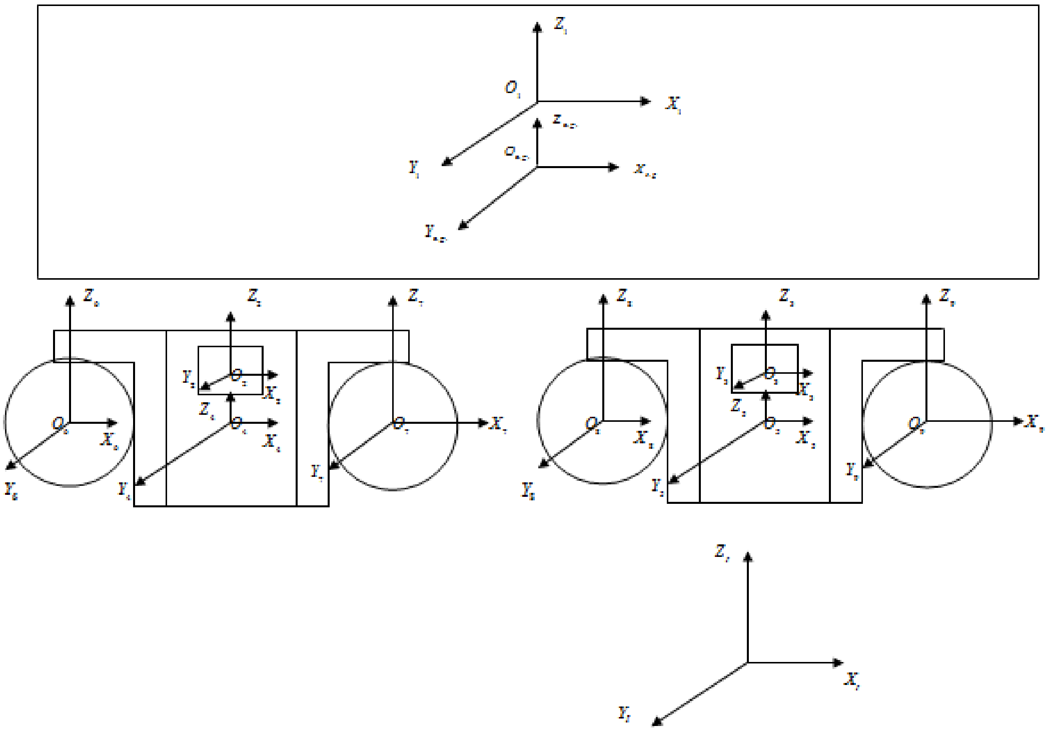

Fig. 4: Inertial axis frame & rigid body mass centre axis system for rail vehicle

The unit vector i, j, k of the body fixed coordinate system will be related to unit vectors I, J, K in the inertial frame using,

$$\begin{bmatrix} i \\ j \\ k \end{bmatrix} = [T_1] \begin{bmatrix} I \\ J \\ K \end{bmatrix} \tag{9}$$

Expressions of the transformation matrix of rotations for other rigid bodies can be written in the same way.

4. Angular and linear velocity

Incremental changes in the Euler angles are indicated by the corresponding angular velocity vectors; namely, $\dot{\psi}$ about the Z-axis, $\dot{\phi}$ about the y-axis (line of nodes), and $\dot{\theta}$ about the x-axis. The absolute angular velocity of the xyz body-axis frame is given by,

$$\omega = \dot{\psi} + \dot{\phi} + \dot{\theta} \tag{10}$$

In terms of body-axis components,

$$\omega = \omega_x \mathrm{i} + \omega_y \mathrm{j} + \omega_z \mathrm{k} \tag{11}$$

For the car body mass of rail vehicle, ω_x is the roll rate, ω_y is the pitch rate, and ω_z is the yaw rate. The velocity vector for the car body can be written as [23]:

$$\dot{\rho}_{1'} = \dot{x}_1 I + \dot{y}_1 J + \dot{z}_1 K \tag{12}$$

This expression can also be written in matrix form as:

$$\dot{\rho}_{1'} = [\dot{x}_1 \quad \dot{y}_1 \quad \dot{z}_1]^T \tag{13}$$

Expressions of angular and linear velocity for other rigid bodies can be written in the same way.

5. Kinetic energy

The kinetic energy of car body mass is sum of rotational kinetic energy and translational kinetic energy [23],

$$[K_1] = [K_1]_{ROT} + [K_1]_{TRANS} \tag{14}$$

The rotational kinetic energy can be written as:

$$[K_1]_{ROT} = ½\, \omega_1^T [I_C] \omega_1 \tag{15}$$

Where $\omega_1 = [S_1]\Omega_1$ and $\omega_1^T = \Omega_1^T [S_1]^T$. $[S_1]$ is sweeping matrix, corresponds to transformation matrix of rotation of car body [T_1] and is expressed as,

$$[S_1] = \begin{bmatrix} 1 & 0 & -\sin\varphi_1 \\ 0 & \cos\theta_1 & \cos\varphi_1 \sin\theta_1 \\ 0 & -\sin\theta_1 & \cos\varphi_1 \cos\theta_1 \end{bmatrix} \tag{16}$$

Now next to car body if front bolster is assigned only roll rotational DoF (θ_2) angular velocity of front bolster will be expressed as [23],

$$[\omega_2] = [T_2][S_1] \begin{bmatrix} \dot{\theta}_1 \\ \dot{\phi}_1 \\ \dot{\psi}_1 \end{bmatrix} + [S_2^{'}] \begin{bmatrix} \dot{\theta}_2 \\ 0 \\ 0 \end{bmatrix} \tag{17}$$

$$[T_2] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_2 & \sin\theta_2 \\ 0 & -\sin\theta_2 & \cos\theta_2 \end{bmatrix} \tag{18}$$

$$[S_2^{'}] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{19}$$

$$[\omega_2] = \begin{bmatrix} 1 & 0 & -\sin\varphi_1 \\ 0 & \cos(\theta_1+\theta_2) & \cos\varphi_1 \sin(\theta_1+\theta_2) \\ 0 & -\sin(\theta_1+\theta_2) & \cos\varphi_1 \cos(\theta_1+\theta_2) \end{bmatrix} \begin{bmatrix} \dot{\theta}_1+\dot{\theta}_2 \\ \dot{\varphi}_1 \\ \dot{\psi}_1 \end{bmatrix} \tag{20}$$

$$\omega_2 = [S_2]\,\Omega_2 \tag{21}$$

$$[K_2]_{ROT} = ½\, \omega_2^T [I_B] \omega_2 \tag{22}$$

If the car body is assigned two translational degrees of freedom i.e. y_1 and z_1. $\rho_{1'}$ denotes the position vector from the origin O_I of the inertial system to the centroid $O_{1'}$ of the car body mass using,

$$\rho_{1'} = y_1 J + z_1 K \tag{23}$$

The velocity vector for the car body can be written as:

$$\dot{\rho}_{1'} = \dot{y}_1 J + \dot{z}_1 K \tag{24}$$

This expression can also be written in matrix form as:

$$\dot{\rho}_{1'} = [0 \quad \dot{y}_1 \quad \dot{z}_1]^T \tag{25}$$

$$[K_1]_{TRANS} = ½\, \dot{\rho}_{1'}^T [M_C] \dot{\rho}_{1'} \tag{26}$$

Expressions of rotational kinetic energy for other rigid bodies can be written in the same way.

6. Potential energy

The potential energy function of the car body can be written as [23]:

$$V_1 = M_C \{ d^T h_1 - d^T h_{ST1} \} \tag{27}$$

Where $d^T = [0 \quad 0 \quad g]$, $h_1 = [x_1 \quad y_1 \quad z_1]$ and $h_{ST1} = [0 \quad 0 \quad z_{ST1}]$. z_{ST1} is the vertical distance from $X_I Y_I$ plane to the normal static position of O_1 in Z_I direction as follows,

$$V_1 = M_C \{ [0\; 0\; g] \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} + [0\; 0\; g] \begin{bmatrix} 0 \\ 0 \\ z_{ST1} \end{bmatrix} \} \tag{28}$$

$$V_1 = M_C g \,[z_1 - z_{ST1}] \tag{29}$$

The potential energy function of the front bolster is determined by applying the transformation matrix of rotation of car body on to bolster axis [23] using,

$$V_2 = M_{Bf} \{ d^T h_2 - d^T h_{ST2} \} \tag{30}$$

$$h_2 = \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} + [T_1]^T \begin{bmatrix} 0 \\ y_2 \\ z_2 \end{bmatrix} + [T_1]^T \begin{bmatrix} x_{12} \\ 0 \\ -z_{12} \end{bmatrix} \tag{31}$$

$$h_2 = \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} + [T_1]^T \begin{bmatrix} x_{12} \\ y_2 \\ z_2 - z_{12} \end{bmatrix} \tag{32}$$

$$h_{ST2} = [0 \quad 0 \quad z_{ST2}] \tag{33}$$

z_{ST2} is the vertical distance from $X_I Y_I$ plane to the normal static position of O_2 in Z_I direction. The potential energy of front bolster can be written as:

$$V_2 = M_{Bf}\, g \{ z_1 - \sin\phi_1 x_{12} + \cos\phi_1 \sin\theta_1 y_2 + \cos\phi_1 \cos\theta_1 (z_2 - z_{12}) - z_{ST2} \} \tag{34}$$

Expressions of potential energy for other rigid bodies can be written in the same way. Spring potential energy and Rayleigh Dissipation energy terms are determined on the basis of degree of freedoms assigned to each rigid body, location of suspension in between them and geometrical parameters (Figs. 5-7). Spring potential energy and Rayleigh dissipation energy terms between car body and bolster is (Figs. 5 - 6),

$$\begin{aligned} E_{sp}^{1-2} + E_d^{1-2} = &+ ½\, k_{CB}^z (z_1 - t_C\theta_1 + t_C\theta_2 - z_2 - x_{12}\varphi_1)^2 + ½\, k_{CB}^z (z_1 + t_C\theta_1 \\ &- t_C\theta_2 - z_2 - x_{12}\varphi_1)^2 + ½ (2k_{CB}^y)(y_1 - y_2 + x_{12}\psi_1)^2 \\ &+ ½\, c_{CB}^z (\dot{z}_1 - t_C\dot{\theta}_1 + t_C\dot{\theta}_2 - \dot{z}_2 - x_{12}\dot{\varphi}_1)^2 + ½\, c_{CB}^z (\dot{z}_1 + t_C\dot{\theta}_1 \\ &- t_C\dot{\theta}_2 - \dot{z}_2 - x_{12}\dot{\varphi}_1)^2 + ½ (2c_{CB}^y)(\dot{y}_1 - \dot{y}_2 + x_{12}\dot{\psi}_1)^2 \end{aligned} \tag{35}$$

Spring potential energy and Rayleigh dissipation energy terms between bolster and bogie frame (5-DoF bogie frame is assigned i.e. y_4, $\dot{z}_4$, $\dot{\theta}_4$, ϕ_4, ψ_4) is,

$$\begin{aligned} E_{sp}^{2-4} + E_d^{2-4} = &½ (2k_{BBF}^z)(z_2 + t_B\theta_2 - z_4 - t_B\theta_4)^2 + ½\, k_{BBF}^y (y_2 - y_4)^2 \\ &+ ½\, k_{BBF}^{\psi} (\psi_4)^2 + ½\, c_{BBF}^z (\dot{z}_2 + t_B\dot{\theta}_2 - \dot{z}_4 - t_B\dot{\theta}_4)^2 \\ &+ ½\, c_{BBF}^z (\dot{z}_2 - t_B\dot{\theta}_2 - \dot{z}_4 + t_B\dot{\theta}_4)^2 + ½\, c_{BBF}^y (\dot{y}_2 - \dot{y}_4)^2 \end{aligned} \tag{36}$$

Expressions of spring potential energy and Rayleigh dissipation energy terms for other connections can be written in the same way.

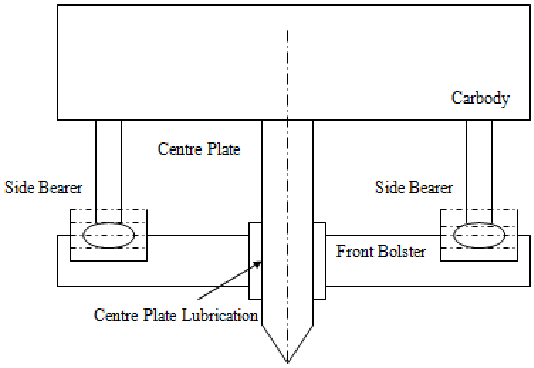

Fig. 5: Vertical suspension between car body and front bolster: Actual configuration

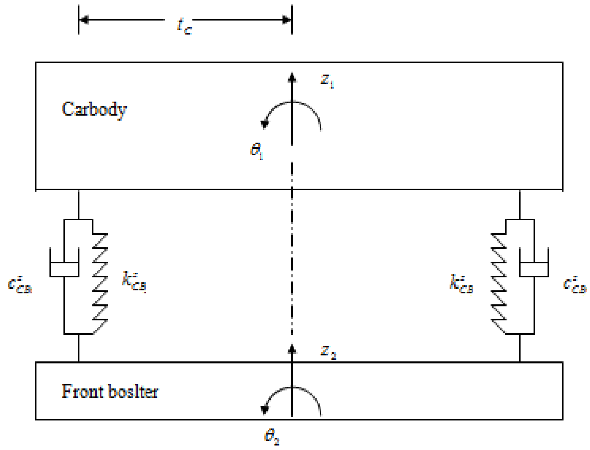

Fig. 6: Vertical suspension between car body and front bolster (end view): Equivalent configuration

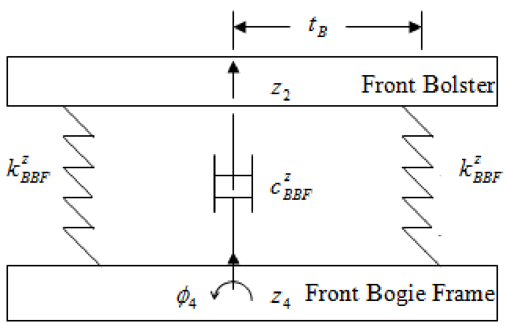

Fig. 7: Vertical suspension between bolster and bogie frame

7. Generalized forces and moments

While determining the generalized forces in railway vehicle system creep forces and moments at the wheel rail contact point, wind drag forces, static forces and moments from them need to be considered [5, 23]. The directions of vertical & lateral normal forces and yaw moment at wheel-rail contact point are shown in Fig. 8. The position vector from O_I (origin of inertial axes system) to O_6 (c.g. of front bogie front wheel axle set) can be written as:

$$\rho_6 = \rho_{1'} + \rho_{1'2} + \rho_{22'} + \rho_{2'4} + \rho_{44'} + \rho_{46} \tag{37}$$

The position vector from O_I to front bogie front wheel axle set left wheel rail contact point L_1 is given by [23]:

$$\rho_{IL1} = \rho_6 + \rho_{66'} + \rho_{6'L1} \tag{38}$$

$$\begin{aligned}\rho_{IL1} = {} & [x_1 \;\; y_1 \;\; z_1]\begin{bmatrix} I \\ J \\ K \end{bmatrix} + [x_{12} \;\; 0 \;\; -z_{12}]\begin{bmatrix} i_1 \\ j_1 \\ k_1 \end{bmatrix} \\ & + [0 \;\; y_2 \;\; (z_2 + z_{24})]\begin{bmatrix} i_2 \\ j_2 \\ k_2 \end{bmatrix} + [x_{46} \;\; y_4 \;\; (z_4 - z_{46})]\begin{bmatrix} i_4 \\ j_4 \\ k_4 \end{bmatrix} \\ & + [0 \;\; a + y_6 \;\; (z_6 - r)]\begin{bmatrix} i_6 \\ j_6 \\ k_6 \end{bmatrix}\end{aligned} \tag{39}$$

$$\begin{bmatrix} i_6 \\ j_6 \\ k_6 \end{bmatrix} = [T_6][T_4][T_2][T_1] \begin{bmatrix} I \\ J \\ K \end{bmatrix} \tag{40}$$

For the case of lateral displacement of car body, an imaginary small displacement of δy_1 is assumed to be given to the car body and work done at the eight wheel rail contact points by the forces shown in Fig. 8 is calculated. The corresponding generalized force is obtained by dividing the work done by δy_1 . Similar methodology has been adopted for the remaining generalized coordinates [5]. For the front bogie front wheel axle set, the partial derivative of position vector ρ_{IL1} (Fig. 7) with respect to lateral displacement y_1 of car body is given by,

$$\frac{\partial \rho_{IL1}}{\partial y_1} = J \tag{41}$$

The displacement of the wheel rail contact point is,

$$\left[\frac{\partial \rho_{IL1}}{\partial y_1}\right]\delta y_1 = J.\delta y_1 \tag{42}$$

The work done by the forces at this wheel rail contact point is given by,

$$\begin{aligned}(\delta W)_{L1} = {} & [F_{X1}I + F_{Y1}J + F_{Z1}K]\left[\frac{\partial \rho_{IL1}}{\partial y_1}\right] \\ & \times \delta y_1 = [F_{X1}I + F_{Y1}J + F_{Z1}K]\; J.\delta y_1\end{aligned} \tag{43}$$

$$(\delta W)_{L1} = F_{Y1}.\delta y_1 \tag{44}$$

The component of generalized force at this wheel rail contact point corresponding to generalized coordinate y_1 is given by the Eqns. (45)-(46).

$$Q_{y_1} = \frac{(\delta W)_{L1}}{\delta y_1} = \frac{F_{Y1}.\delta y_1}{\delta y_1} \tag{45}$$

$$(Q_{y_1})_{L1} = F_{Y1} \tag{46}$$

Similar treatment was done for other wheel rail contact points. The generalized force for lateral motion of car body i.e. y_1 is given by,

$$Q_{y_1} = F_{Y1} + F_{Y2} + F_{Y3} + F_{Y4} + F_{Y5} + F_{Y6} + F_{Y7} + F_{Y8} \tag{47}$$

The generalized forces for lateral motion of front bolster i.e. y_2 is given by,

$$\begin{aligned}Q_{y_2} = {} & (F_{Y1} + F_{Y2} + F_{Y3} + F_{Y4}) \\ & + (F_{Z1} + F_{Z2} + F_{Z3} + F_{Z4})(\theta_1 + \theta_2)\end{aligned} \tag{48}$$

Expressions of generalised forces for the other rigid bodies can be written in the same way.

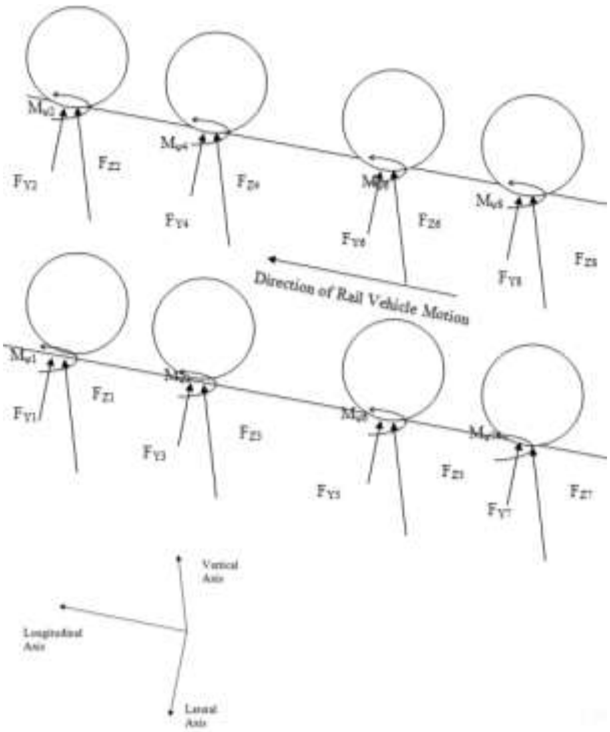

Fig. 8: Vertical and lateral normal forces and yaw moment at eight wheel-rail contact point

8. Conclusions

Newtonian's method is vectorial in nature. When the equations of motion of any system are formulated using Newtonian's method, the acceleration is required to be absolute, therefore it is required to be measured relative to inertial frame. Sometimes the motion of a particle is known relative to a rotating and accelerating frame, and it is desired to find its absolute velocity and acceleration. In general, these formulations are complex except moving frame is not rotating [7, 13]. Lagrangian method on the other hand is scalar in approach. It main deals with calculation of energies and their differentiation with respect to each independent generalised coordinates on the basis of degrees of freedom assigned to the system [7, 13]. When applied to vehicle system both linear and angular degrees of freedom are assigned. In this context body fixed Euler's rotation is accounted and transformation matrix of rotation can be determined with simplicity. For particular railway vehicle system the approach of determining the mathematical expression of position vector, linear or rotational kinetic energy, potential energy and generalised forces is from inertial axis to car body mass centroidal axis then to bolster, bogie frame, wheel axle and finally wheel-rail contact points [23]. The equations of motion once obtained can be utilised to investigate the several performance indices of the vehicle [1-6, 24-29]. Lagrangian approach is used to investigate the performance of rail-road vehicle, marine ship, aircraft, and all - terrain vehicles. The advantage with approach is the complex systems having many degrees of freedom can be solved comparatively with more simplicity [7]. Both linear and non-linear systems can be formulated using this approach. These systems are analysed using different approach once the system is formulated.

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