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Numerical Analysis of a Semi-Infinite Solid with Temperature Dependent Thermal Conductivity using Truly Meshfree Method

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ABSTRACT:

This article presents Meshless Local Petrov-Galerkin (MLPG) method to obtain the numerical solution of linear and non-linear heat conduction in a semi-infinite solid object with specific heat flux. Moving least square approximants are used to approximate the unknown function of temperature T(x) with $T^h(x)$. These approximants are constructed by using a linear basis, a weight function and a set of non-constant coefficients. Essential boundary condition is imposed by the penalty function method. A predictor-corrector scheme based on direct substitution iteration has been applied to address the non-linearity and two-level θ method for temporal discretization. The accuracy of MLPG method is verified by comparing the results for the simplified versions of the present model with the exact solutions. Once the accuracy of MLPG method is established, the method is further extended to investigate the effects of temperature-dependent properties.

KEYWORDS:

Transient; Temperature dependent; Semi-infinite solid; Penalty method; Heat flux; Meshless local Petrov-Galerkin

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ACRONYMS AND NOMENCLATURE:

A	Across-sectional area of fin, m^2
c	Reference heat capacity, J/kg°C
k_0	Reference thermal conductivity, W/m°C
$k(T)$	Temperature- dependent thermal conductivity, $k_0\,(1+\beta T)$
L	Length of object, m
n	Direction cosines pointing outward to the boundary
P	Perimeter of an object, m
$\bar{q}$	Heat flux, W/m^2
T_i	Initial temperature, °C
$\bar{T}$	Specified temperature on essential boundary,°C
$T^h(x)$	Moving Least Squares (MLS) approximant
v	Test function for MLPG method
Δt	Time stepping, sec
α	Penalty parameter
β	Coefficient of thermal conductivity of the material, / °C
Γ	Boundary of global domain
Γ_1	Essential boundary
Γ_2	Natural boundary
Γ_3	Convective boundary
δ	Width of the object, m
$\mathcal{E}$	Emissivity of the semi-infinite object surface
ρ	Material density, kg/m^3
σ	Stephen- Boltzmann constant, (5.67×10^{-8} W/m^2-K^4)
$\Phi_I(\mathbf{x})$	MLS shape function

1. Introduction

A semi-infinite solid is an idealized body that has a single plane surface and extends to infinity in all but one direction. If a sudden change is imposed to this surface, transient 1D conduction occurs in the solid. The semi-infinite solid provides a useful idealization for many practical problems. It may be used to determine the heat transfer near the surface of earth over the period of time or to approximate transient response of a finite solid, such as a thick slab. No object is semi-infinite although many approach this limit as the earth. Furthermore, it can be seen that every object is essentially semi-infinite with respect to surface processes that occur over a sufficiently small time scale. A number of engineering and science problems can be modelled as linear and nonlinear heat transfer in semi-infinite medium. These problems have been addressed by various analytical methods including Hankel and Fourier transformation, Separation of variables, Duhamel's theorem, Greens function, Laplace transformation, integral method and similarity transformation [1-4]. But, real engineering problems, in general, are nonlinear in nature.

A heat conduction problem becomes nonlinear either due to nonlinearity of the differential equation or boundary conditions or both. Since there is no general theory available for solution of nonlinear partial differential equations, the analysis of such problems becomes difficult and each problem should be treated individually. Bianco et al [5] have identified the numerical solution for the transient heat conduction in semi-infinite solids irradiated by a moving heat source. Thermal conductivity is taken as temperature dependent. They have used COMSOL Multiphysics 3.3 code for investigation. Chang [6] have studied the behaviour of the semi-infinite solids under transient conditions by taking temperature dependent thermal properties. Singh et al [7] have conducted the analysis of unsteady-state heat transfer in semi-infinite solid with temperature-dependent thermal conductivity using Element Free Galerkin (EFG) method.

Yu et al [8] have identified the approximate solution of the nonlinear heat conduction equation in a semi-infinite domain using Taylor polynomials of different degrees. They have taken thermal conductivity, material density and heat capacity as the function of temperature. Chambré [9] has studied the time-dependent heat conduction in a semi-infinite medium subject to a boundary condition which can involve the temperature in a nonlinear manner. A simple iterative solution method has been proposed. Badran and Abd-el-malek [10] have solved the nonlinear and transient heat conduction problem in a semi-infinite body using transformation group theoretic approach. Chung and Yeh [11] have used integral method to solve nonlinear transient heat conduction equation in a semi-infinite solid. Ceretani et al [12] have identified the explicit solutions for a non-classical heat conduction problem for a semi-infinite strip with a non-uniform heat source using separation of variables method.

To the best of authors' knowledge, meshless local Petrov-Galerkin (MLPG) method has not been used to study the behaviour of semi-infinite objects. The MLPG method was developed by Atluri and Zhu [13-14]. Unlike FEM and most other meshfree methods, MLPG method operates on local weak form and performs integration over overlapping simple local domains. This has removed the need of mesh at any stage of analysis. Hence, it is truly a meshfree method. The method was further elaborated and developed by a few researchers [15-22]. They concluded that MLPG has a very high rate of convergence, it does not need any post processing technique and does not exhibit any volumetric locking. MLPG method works on Petrov-Galerkin formulation i.e. trial and test functions are selected from different spaces. This provides a large number of possible combinations to formulate MLPG method.

In the present work, the MLPG method has been employed to obtain the numerical solution of nonlinear heat conduction in semi-infinite solids with temperature dependent thermal conductivity. The meshless formulation has been given for a model problem of non-linear heat transfer. The results obtained by MLPG method are compared with those obtained by EFG method, established finite element (ANSYS 8.0) and analytical methods respectively. Moving least square (MLS) approximants are used to approximate the unknown function of temperature T(x) with T^h(x).These approximants are constructed by using a linear basis, a fourth order spline weight function and a set of non-constant coefficients. The MLPG method does not possess Kroneker delta function property as FEM; hence the essential boundary condition (EBC) is imposed by the penalty function method (PM). A predictor-corrector scheme based on direct substitution iteration has been applied to address the non-linearity and two-level θ method for temporal discretization.

2. MLPG method

The MLPG method operates on Petrov-Galerkin formulation i.e. it picks up test and trial functions from different function spaces. The original formulation [13-14] has subsequently evolved in various versions either by changing the meshfree approximation scheme or by selecting a new test function. Hence, the proposed method provides a rational basis for constructing meshfree methods with a greater degree of flexibility. The discretization of the governing equation by the MLPG method requires MLS approximants which are made up of two components of a weight associated with each node, a monomial basis and a set of non-constant coefficients. The unknown function T(x) is approximated by moving least- square approximants T^h (x). In 1D, for linear basis T^h (x) can be written as,

$$T^h(\mathbf{x}) = \sum_{j=1}^{m} p_j(\mathbf{x}) a_j(\mathbf{x}) \equiv \mathbf{p}^T(\mathbf{x})\mathbf{a}(\mathbf{x}) \tag{1}$$

Where $p^T(x) = (p_1\ (x),\ p_2\ (x),\ p_m\ (x))$ is a complete monomial basis and m is the number of terms in the basis. For example, in 1-D space the basis can be,

Linear basis: $\mathbf{p}^T(\mathbf{x}) = \{1,\ x\},\quad m = 2$

Quadratic basis: $\mathbf{p}^T(\mathbf{x}) = \{1,\ x, x^2\},\quad m = 3$

The unknown coefficients $a_j(x)$ at any given point are determined by minimizing the functional J,

$$J = \sum_{I=1}^{n} w(\mathbf{x} - \mathbf{x}_I)[\mathbf{p}^{\mathbf{T}}(\mathbf{x})\mathbf{a}(\mathbf{x}) - T_I]^2 \tag{2}$$

Where n is the number of nodes in the neighbourhood of x for which the weight function $w(x - x_I) \neq 0$, and T_I is the nodal parameter of T at x= xI. The stationarity of *J* in Eqn. (2) with respect to a_j (x) leads to the following set of linear equations:

$$\mathbf{a}(\mathbf{x}) = \mathbf{A}^{-1}(\mathbf{x})\mathbf{B}(\mathbf{x})T \tag{3}$$

Where $\mathbf{A}(\mathbf{x}) = \sum_{I=1}^{n} w(\mathbf{x} - \mathbf{x}_I)\mathbf{p}(\mathbf{x}_I)\mathbf{p}^{\mathbf{T}}(\mathbf{x}_I)$ (4)

$$\mathbf{B}(\mathbf{x}) = [w(\mathbf{x} - \mathbf{x}_I)\mathbf{p}(\mathbf{x}_I), \ldots\ldots, w(\mathbf{x} - \mathbf{x}_n)\mathbf{p}(\mathbf{x}_n)] \tag{5}$$

$$T = [T_1, T_2, \ldots T_n] \tag{6}$$

By substituting Eqn. (3) in Eqn. (1), the MLS approximants can be defined as,

$$T^h(\mathbf{x}) = \sum_{i=1}^{n} \Phi_{I(x)} T_I = \mathbf{\Phi}(\mathbf{x})T \tag{7}$$

Where meshless shape function $\Phi_I(\mathbf{x})$ is defined as,

$$\Phi_I(\mathbf{x}) = \sum_{j=1}^{m} p_j(\mathbf{x})\left(\mathbf{A}^{-1}(\mathbf{x})\mathbf{B}(\mathbf{x})\right)_{jI} \tag{8}$$

The derivatives of shape function is given by,

$$\Phi_{ix} = (p^T A^{-1} B_I)_{,x} = p^T_{,x} A^{-1} B_I + p^T (A^{-1})_{,x} B_I + p^T A^{-1} (B_I)_{,x} \tag{9}$$

The weight function w(x- x_i) is non-zero over a small neighbourhood of x_I called the domain of influence of node I. The choice of weight function w (x- x_i) affects the resulting approximation Th(x_I), therefore the selection of appropriate weight function is essential. In this article the fourth order spline weight function is used. It is represented by,

$$w(\mathbf{x}-\mathbf{x}_i) = \begin{cases} 1-6d^2+8d^3-3d^4 & \text{if } 0 \le d \le 1 \\ 0 & \text{if } d > 1 \end{cases} \tag{10}$$

3. Discrete equations

One-dimensional governing equation for transient heat transfer in semi-infinite solids with temperature-dependent thermal conductivity is given by,

$$\frac{\partial}{\partial x}[k(T)\frac{\partial T}{\partial x}] = \rho c \frac{\partial T}{\partial t} \tag{11}$$

Where $k(T) = k_0(1+\beta T)$

Initial and boundary conditions:

$$T(x,0) = T_i \qquad \text{on } \Omega \tag{12}$$

$$\left[-k(T)\frac{\partial T}{\partial x}\right]_{(0,t)} = \bar{q} \tag{13}$$

Where $q = k(\frac{\partial T}{\partial n})$ and $\bar{T}$ are the specified temperature on essential boundary, $\bar{q}$ is the given heat flux at the natural boundary and **n** is the outward unit normal to the boundary. MLPG method is based on local weak form. Weighted residual formulation for Eqn. (11) in local domain can be expressed as,

$$\int_{\Omega_Q} v\{\frac{\partial}{\partial x}[k(T)\frac{\partial T}{\partial x}] - \rho c \frac{\partial T}{\partial t}\} d\Omega = 0 \tag{14}$$

Where, v is the test function. MLS weight function (MLPG 1) from [16] is employed in this study. Using divergence theorem, Eqn. (14) yields the desired weak form given by,

$$\left[vk(T)\frac{\partial T}{\partial n}\right]_{\Gamma_Q} - \int_{\Omega_Q}\left[k(T)\frac{\partial T}{\partial x}\frac{\partial v}{\partial x} + v\rho c\frac{\partial T}{\partial t}\right] d\Omega = 0 \tag{15}$$

Where Γ_Q is the boundary of the local domain, Ω_Q . In case of 1D problem, boundary integrals turn to be a point value on boundaries. Taking advantage of MLPG method's flexibility, the test function is selected such that it vanishes at the boundary of the local domain. Hence, boundary integral remains non-zero only when local domain intersects the global boundary. The EBCs are imposed by PM, developed by Zhu and Atluri [23].

Therefore, Eqn. (15) can be written as

$$\left[\bar{q}v\right]_{\Gamma_{1Q}} - \int_{\Omega_Q}\left[k(T)\frac{\partial T}{\partial x}\frac{\partial v}{\partial x} + v\rho c\frac{\partial T}{\partial t}\right] d\Omega - \alpha\left[\left(T-\bar{T}\right)v\right]_{\Gamma_{1Q}} = 0 \tag{16}$$

Where $\Gamma_{1Q} = \Gamma_1 \cap \Gamma_Q$ and α is the penalty function parameter = 1 x 10^{10} and $\bar{q} = k(\frac{\partial T}{\partial n})$

The unknown function, T, at any instant of time t, is approximated by MLS scheme (Lancaster and Salkauskas [24]) as follows

$$T^h(\mathbf{x}) = \sum_{i=1}^{n_s} \Phi_i T_i = \mathbf{\Phi T} \tag{17}$$

Where $\mathbf{\Phi}$ is the vector of the mesh free shape functions Φ_i , T represents the vector of nodal parameters T_i at time t and n_s is the number of nodes in the support domain at point x. EBC is imposed by the method of direct interpolation. Substituting the approximation (17) in Eqn. (16) and performing integration over all local domains corresponding to all field nodes, the semi-discrete system can be obtained as follows:

$$\mathbf{C\dot{T}} + \mathbf{KT} = \mathbf{F} \tag{18}$$

$$K_{ij} = \int_{\Omega_Q}\left(\frac{\partial v_i}{\partial x}k(T)\frac{\partial \Phi_j}{\partial x}\right)d\Omega - \left[v_i\frac{\partial \Phi}{\partial n}\right]_{\Gamma_{1Q}} + \left[\alpha v_i \Phi_j\right]_{\Gamma_{1Q}} \tag{19}$$

$$C_{ij} = \int_{\Omega_Q} \rho c v_i \Phi_j d\Omega \tag{20}$$

$$F_i = \left[v_i\bar{q}\right]_{\Gamma_{1Q}} + \left[\alpha v_i \bar{T}\right]_{\Gamma_{1Q}} \tag{21}$$

Spatial discretization of governing PD Eqn. (11) results in a system of semi-discrete ordinary differential equations. Two- level θ method for temporal discretization has been used. It can vary between explicit and implicit strategies and results in the algebraic system

$$[\mathbf{C} + \theta\Delta t\mathbf{K}]\mathbf{T}^{n+1} = [\mathbf{C} + (\theta-1)\Delta t\mathbf{K}]\mathbf{T}^n + \Delta t\mathbf{F} \tag{22}$$

Where is the time step and n denotes the time level (*i.e.* $t_n = n\Delta t$ if uniform time step is employed). According to Morgan [25], nonlinear systems can be very complicated, if not impossible, to solve explicitly. The majority of nonlinear analysis of systems of ODEs focuses on whether or not the systems have stable equilibria. The equilibrium characterizes as stable or unstable based on the behavior of solutions whose initial conditions are in the neighborhood of the equilibrium. If solutions near a critical point of a system stay close to the critical point as time approaches infinity, the critical point is assumed to be stable. An iterative predictor-corrector scheme [26], based on direct substitution iteration is used to handle nonlinearity in this work. This scheme proceeds in two steps. It calculates a rough approximation of the desired quantity in the first step and refines approximation in the next by any other means. It combines the advantages associated with explicit and implicit time schemes. Hence, it provides the stable solution to solve complex nonlinear problems.

Predictor:

$$\left[\mathbf{C}(\mathbf{X}^n)+\theta\Delta t\mathbf{A}(\mathbf{X}^n)\right]\mathbf{X}_*^{n+1}= \left[\mathbf{C}(\mathbf{X}^n)+(1-\theta)\Delta t\mathbf{A}(\mathbf{X}^n)\right]\mathbf{X}^n+\Delta t\mathbf{B}(\mathbf{X}^n) \qquad (23)$$

Corrector:

$$\left[\mathbf{C}(\mathbf{X}_p^{\bar{n}})+\theta\Delta t\mathbf{A}(\mathbf{X}_p^{\bar{n}})\right]\mathbf{X}_{p+1}^{n+1}= \left[\mathbf{C}(\mathbf{X}_p^{\bar{n}})+(1-\theta)\Delta t\mathbf{A}(\mathbf{X}_p^{\bar{n}})\right]\mathbf{X}^n+\Delta t\mathbf{B}(\mathbf{X}_p^{\bar{n}}) \qquad (24)$$

Where p = 0, 1, 2, 3…up to convergence and

$$\mathbf{X}_p^{\bar{n}}=w\mathbf{X}_p^{n+1}+(1-w)\mathbf{X}^n \qquad 0\le w\le 1$$
$$\mathbf{X}_0^{n+1}=\mathbf{X}_*^{n+1} \qquad (25)$$

4. Results and discussions

Numerical solution has been obtained for transient heat conduction in semi-infinite solids with specific heat flux. A model has been solved by using constant and variable thermal conductivities of the material. The thermal conductivity of the material is assumed to vary linearly with temperature. Consider a sample problem of 1D semi-infinite solid object as mentioned in the Fig. 1. The analytical solution for the mentioned boundary conditions can be obtained by following correlations [27] for constant heat flux,

$$T(x,t)=\frac{2\bar{q}(\alpha t/\pi)^{1/2}}{k}\exp\left(\frac{-x^2}{4\alpha t}\right)-\frac{\bar{q}x}{k}erfc\left(\frac{x}{2\sqrt{\alpha t}}\right)+T_i$$

Where erf is the error function. erfc is the complementary error function, erfc w = 1- erf w.

Table 1: Data for semi-infinite solid problem

Parameter	Value	Parameter	Value
L	1.00 m	*c*	400 J/kg ^{0}C
W	0.10 m	k_0	400 W/m ^{0}C
a	1 x 10^5 W/m^2	T_i	0^0C
ρ	9000 kg/m^3	Δ*t*	1 sec

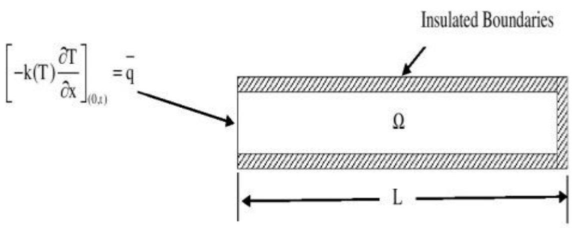


Fig. 1: Semi-infinite model for transient heat transfer

Case I: Constant thermal conductivity

The thermal conductivity of the material is assumed to be constant i.e β=0 and hence k= k0. Number of nodes is taken as 51. Extant of quadrature and support domains are taken as 1.66 and 2.5 respectively. Table 2 shows a comparison of MLPG results and the results obtained by FEM, EFG and analytical methods respectively. The error in EFG, FEM and MLPG results has also been evaluated and presented in Table 2. The maximum error in EFG, FEM and MLPG has been found to be 24.27%, 47.30% and 24.39% respectively. From the results presented in Table 2, it can be observed that EFG and MLPG results are more accurate than FEM results and furthermore MLPG results are in good agreement with the established meshfree EFG method.

Table 2: Comparison of MLPG results with EFG, FEM and analytical method results at x= 0.00 m for constant thermal conductivity

	Analytical	EFG [7]		FEM [7]		MLPG	
T, sec	T, ^{0}C	T, ^{0}C	% Error	T, ^{0}C	% Error	T,^{0}C	% Absolute Error
	0.00	0.00	0.00	0.00	0.00	0.00	0.00
10	9.41	7.12	24.27	4.96	47.30	11.70	24.39
20	13.30	11.67	12.27	9.04	32.04	15.05	13.15
30	16.29	15.03	7.73	12.48	23.41	17.64	8.29
40	18.81	17.76	5.57	15.43	17.95	19.83	5.43
50	21.03	20.11	4.38	18.03	14.28	21.77	3.49
60	23.04	22.21	3.60	20.34	11.71	23.51	2.06
70	24.88	24.13	3.03	22.44	9.84	25.12	0.95
80	26.60	25.90	2.63	24.36	8.44	26.62	0.07
90	28.22	27.56	2.33	26.14	7.37	28.03	0.67
100	29.74	29.12	2.09	27.80	6.53	29.36	1.29

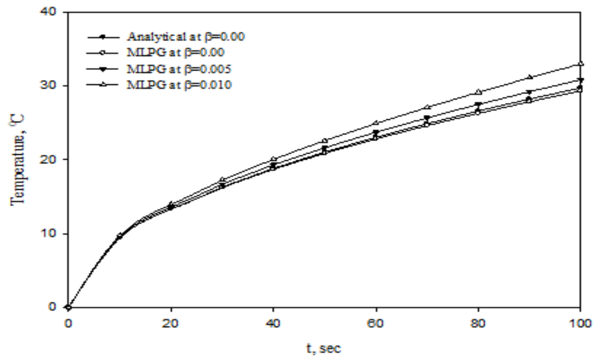
Case II: Variable thermal conductivity

In case of variable thermal conductivity, it is assumed that thermal conductivity of the material is varying linearly with temperature i.e. k= k_0 (1+βT). Number of iterations is taken as less than 10. Table 3 shows a comparison of MLPG results with obtained by FEM, EFG and analytical methods respectively. The error in EFG, FEM and MLPG results has also been evaluated and presented in Table 3. The error in EFG, FEM and MLPG results has also been evaluated and presented in Table 3. The maximum error in EFG, FEM and MLPG has been found to be 25.83%, 48.36% and 22.15% respectively. It is evidential that MLPG results are more accurate than EFG and FEM results.

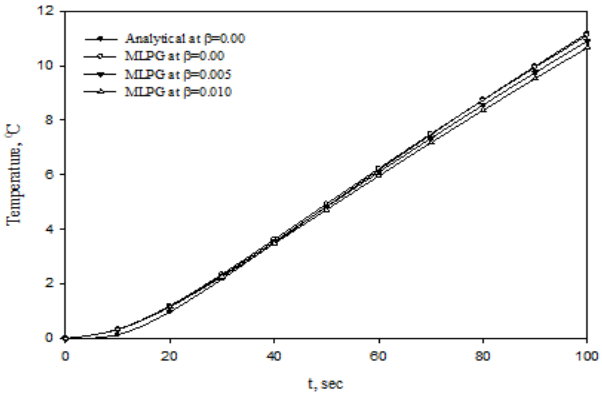
Table 3: Comparison of MLPG results with EFG, FEM and analytical method results at x= 0.00 m for variable thermal conductivity

	Analytical	EFG [7]		FEM [7]		MLPG	
T, sec	T, ^{0}C	T, ^{0}C	% Error	T, ^{0}C	% Error	T,^{0}C	% Absolute Error
0	0.00	0.00	0.00	0.00	0.00	0.00	0.00
10	9.60	7.12	25.83	4.96	48.36	11.73	22.15
20	13.56	11.65	14.08	9.04	33.37	15.09	11.29
30	16.60	14.99	9.67	12.47	24.88	17.70	6.63
40	19.15	17.70	7.58	15.42	19.49	19.90	3.92
50	21.40	20.03	6.41	18.01	15.87	21.85	2.09
60	23.43	22.10	5.67	20.31	13.33	23.62	0.78
70	25.30	23.99	5.17	22.39	11.49	25.23	0.26
80	27.03	25.74	4.80	24.30	10.11	26.74	1.07
90	28.66	27.37	4.52	26.07	9.05	28.16	1.74
100	30.20	28.90	4.30	27.72	8.22	29.51	2.30

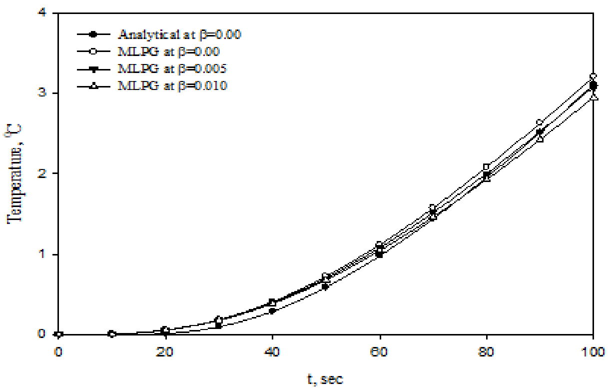
Temperature-time history of a semi-infinite solid at three consecutive nodes is presented in Fig. 2. Thermal conductivity varies linearly with temperature, number of nodes in the computational domain are taken as 151, time stepping of 5 sec, extant of quadrature domain as 1.32 and that of support domain as 2.5 respectively. The heat conduction is investigated for three different values of β. It is found that temperature increases with time due to low initial temperature and constant heat flux on the boundary. Constant temperature profile is close to that of analytical solution but as the value of β increases the nonlinear temperature profile parts away from the other profiles and reaches to the highest temperature. This is because the variable thermal conductivity is a linear function of the difference between the temperatures of surface and the sink. At high values of β, the thermal conductivity of the material decreases and hence the heat transfer.



At X=0.00m



At X=0.10m



At x=0.20m

Fig. 2: Temperature gradient at x=0.00 m, 0.10 m and 0.20 m

5. Concluding remark

In this paper, MLPG method has been successfully implemented to solve nonlinear heat transfer in semi-infinite solids. A model problem has been solved by taking constant and temperature-dependent thermal conductivities with specific heat flux. The thermal conductivity of the material is assumed to vary linearly with temperature. For the linearization of nonlinear system of equations, predictor-corrector scheme has been used successfully and two level theta method for temporal discretization. The MLPG results have been obtained for linear and nonlinear system of equations and are compared with those obtained by FEM, analytical and EFG methods respectively. The MLPG results are found to be in good agreement with all the established and analytical methods. In all the circumstances MLPG method has proved its worth as a truly meshless method and demonstrated its potential to solve nonlinear heat conduction problems of semi-infinite solids.

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