Measurement of Productive Efficiency - A Stochastic frontier production function approach to Indian Industries

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The efficiency term describes the maximum outputs attainable from utilizing the available inputs. A production is efficient if it cannot improve any of its inputs or outputs without worsening some of its other inputs or outputs. Efficiency can be increased by minimizing inputs while holding output constant or by maximizing output while holding inputs constant or a combination of both may increase efficiency (Alias Radam et al,2010). Productive efficiency (also known as technical efficiency) is defined as a situation in which the most production is achieved from the resources available to the producer It occurs when the economy is utilizing all of its resources efficiently, producing most output from least input.

Productive efficiency can be determined by estimating the best-practice production frontier and individual industries gives the measure of inefficiency. In view of the growing high production costs , productive efficiency and profitability will become increasingly important determinants of the future of Indian industries . In addition to developing and adopting new production technology , the industries can maintain their economic viability by improving efficiency of existing operation with a given level of technology.In other word the industries total costs can be reduced and the industries total output can be increased by making better use of available inputs and technology.

This study examined the industry level efficiency so as to identify the sources where improvement can be made. The study will provide vital information to help individual industries in using their resources more efficiently and to assist the industries in becoming more competitive and maintaining its long term survival. The determination of frontier technology and knowledge of productive efficiency and its relationship with firm size can provide important insights into future Indian industries. Further more,the relationship between efficiency levels and various industry- specific factors can provide useful policy –relevant information. A comparison of industry's frontier or " best practice" function and its average practice function will produce useful information about possible future structural adjustments for the industries.

Methodology

Net Value Added (NVA) was taken as output. Labour input (L) consisted of both workers directly involved in production and persons other than workers like supervisors, technicians, managers, clerks and similar type of employees. The invested capital (K)was taken into account as capital. Wages included renumeration paid to workers.The basic data source of the study was Annual Survey of Industries (ASI) published by Central Statistical Organisation (CSO), Government of India covering the period from1998-99 to 2010-11.All the referred variables were normalised by applying Gross Domestic Product (GDP) deflator. The GDP at current and constant prices were obtained by referring to Economic Survey, published by Government of India, Ministry of Finance and Economic Division, Delhi.

Tool of analysis- Stochastic frontier production function

A stochastic frontier production function as proposed by Battese and coelli (1992) is defined as :

$$\iota = f(X\iota\beta)\varepsilon_{ei}$$

Where Yi, is the output vector for the i^{th} firm, X_i is a vector of inputs, β is a vector of parameter and ει is an error term. In this model, a production frontier defines output as a function of a given set of inputs, together with technical inefficiency effects. Furthermore, this model specifies that these inefficiency effects are modeled by other observable explanatory variables and all parameters are estimated simultaneously. The stochastic element of this model allows some observations to lie above the production function, which makes the model less vulnerable to the influence of outliers than with deterministic frontier models.

The stochastic frontier is also called composed error model, because it postulates the error term εi as two independent error components:

$$\varepsilon_i = v_i + u\iota$$

When a symmetric component is normally distributed, $v_i \sim (\text{N}, \sigma_v^2)$ represents any stochastic factors that is beyond the firm's control affecting the ability to produce on the frontier such as luck or weather. It can also account for measurement error in Y or minoromitted variables. The asymmetric component, in this case distributed as a half- normal $u_i \sim (\text{N}, \sigma_v^2), u_i > 0$ can be interpreted as pure technical inefficiency. This component has also been interpreted as an unobservable or latent variable ; usually representing managerial ability.

The parameters of v and u can estimated by maximizing the following log-likelihood function :

$$\ln (Y \sim \beta, \lambda, \sigma^2 = \frac{N}{2} \ln [\frac{2}{\pi}] - \text{Nin}\, \sigma + \sum_{i-1}^{N} In\, [1 - F(\varepsilon_i \lambda \sigma^{-1})] + \frac{1}{2\sigma^2} \sum_{i=1}^{N} \varepsilon_1^2$$

Where,

$$\varepsilon_I = Y_1 - f(X_i, \beta)$$

$$\sigma^2 = \sigma_u^2 + \sigma_v^2$$

$$\lambda = \sigma_u / \sigma_v$$

F = the standard normal distribution function
N = Number of observation
Given the assumptions on the distribution of v and u, Jondrow et al. (982) showed that the conditional mean of u given ε is equal to

$$\text{E} (u_i \setminus \varepsilon_i) = \frac{\sigma_u \sigma_v}{\sigma} [\frac{f\ (\varepsilon i\ \lambda\sigma)}{1 - f\ (\varepsilon i\ \lambda\sigma)} - \frac{\varepsilon i\ \lambda}{\sigma}$$

Where f and F are the standard normal density and distribution functions evaluated at $\varepsilon_i \lambda / \sigma$

Measures of technical efficiency (TE) for each firm can be calculated

$$TE_i = \exp (-E[u_i\ / \varepsilon]) \text{ so that } 0 \leq TE \leq 1$$

The Cobb- douglas stochastic frontier production function in logarithm form is as follows:

$$\text{In VAi} = 1n\ \beta_0 + \beta_1\, 1n\, C + \beta_2\ 1n\ L_i + \beta_3\ 1n\ E_i + \varepsilon_I$$

Where VA represents Net value added per year. Independent variables are C (capital) and L (number of labourers). Parameters β_0 denotes the technical efficiency level and β_I is elasticities of the various inputs with respect to output level.

The productive efficiency of the firms were calculated by applying the Stochastic frontier production approach of 4.1c version. The results show the summary statistics of the variables, maximum likelihood estimates and technical efficiency for rural, urban and aggregate industries of India for the reference period under study.

a) Rural Industries

As for primary investigation the summary statistics results of the selected variables of rural industries are presented in the following table-1.

Table-1

Summary Statistics of variables Rural Industries

Variable	Mean	Std Deviation	Minimum	Maximum	C.V
Net Value Added (NVA)	2.2454	0.2395	2.00	2.67	10.67
Invested Capital (K)	2.1152	0.1417	1.99	2.40	6.70
Number of workers (L)	2.0393	0.0743	1.97	2.18	3.64

Source: calculations are based on ASI Data

Foot Note: C.V - co –efficient of variation

Mean values of input variables indicate that the industry's main factors of production were both capital and labour since there were not much differences in their mean values. The magnitude of variability (C.V) also substantiated this point since the co-efficients are less for both the inputs.

Table-2 show the maximum likelihood estimates of rural industries of India in the context of its productive efficiency.

Table-2

Maximum Likelihood estimated of stochastic frontier production function – Rural Industries

Variable	Co-efficient	Std-error	t – ratio
Intercept	-1.1175	1.5383	-0.7638
Ln K	1.3952 ***	0.7383	1.890
LnL	0.2565	1.5093	0.1699
σ^2	0.0009 ***	0.0005	1.8758
γ	0.9999	0.00002	.00004
μ	0.0167	0.0487	0.3436
η	0.1018**	0.0350	2.7879

Source : Calculations are based on ASI Data
Foot Note: ** - Significant at 5 % level
*** - Significant at 10 % level

The maximum likelihood estimates for productive efficiency of rural industries show that in single output case, parameters of capital input was positive and statistically significant. Hence capital is main input factor for these industries as its value was higher than labour. The co-efficients of σ^2 and γ were statistically significant though the sign of them differs. It reveals that estimated levels of output considerably differ from their potential levels due to factors, which are within the control of the industries. The estimated value of γ indicated the absence of efficiency gap that exists between the actual and potential level of performance which is mainly due to technical efficiency of the industries. The statistically insignificant co-efficient of μ term indicated that it followed a normal distribution and the positive and statistically significant co-efficient of η indicated that efficiency increases in getting production overtime. The summation of the elasticities of factors of production, indicated return to scale of 1.65. The value of return to scale greater than unity suggested that increasing returns to scale prevails. One percent increase in inputs (labour and capital) resulted in an increase 1.16 percent in output level for the stochastic frontier.

Table-3 presents the year-wise technical efficiency of rural industries during the period 1998-99 to 2010-11.

Table-3

Technical efficiency – Rural Industries

Year	Efficiency Scores
1998-99	0.879
1999-00	0.935
2000-01	0.922
2001-02	0.893
2002-03	0.962
2003-04	0.980
2004-05	0.999
2005-06	0.985
2006-07	0.989
2007-08	0.939
2008-09	0.995
2009-10	0.899
2010-11	0.968
Mean	**0.950**
Average inefficiency score	**.053**

Source: calculations are based on ASI data

Average technical inefficiency score=1-average efficiency/ average efficiency

In terms of technical efficiency, the rural industries recorded an average efficiency of 0.950 (95.0 percent). The table also reveals that the technical efficiency of rural industries have not shown any decline but showed mixed trend. The average technical inefficiency was observed as 0.053, which was negligible.

a) Urban Industries:

The following table-4 provides details regarding the summary statistics of variables selected for urban industries.

Table-4
Summary Statistics of variables – urban industries

Variables	Mean	Std Deviation	Minimum	Maximum	C.V
Net Value Added (NVA)	2.1293	0.1620	1.96	2.41	7.608
Invested Capital (K)	2.1525	0.1153	2.00	2.35	5.357
Number of workers (L)	2.0049	0.0324	1.97	2.06	1.616

Source: calculations are based on ASI Data

Foot Note : C.V- co –efficient of variation

It is clear from the table that the mean values of input variable both labour and capital were the main factors of production in urban industries. The co-efficient of variation figures showed that the magnitude or extent of variability in the growth of these variables were 5.35 percent and 1.616 percent respectively. This indicated that labour was the main factor without much variation in its contribution to the growth of net value added. In other words these industries, no doubt can rely more on labour force for the growth of its output.

Table -5 gives details regarding the maximum likelihood estimates for productive efficiency of urban industries.

Table-5

Maximum Likelihood Estimates of stochastic frontier production function – Urban industries

Variable	Co-efficient	Std-error	t – ratio
Intercept	-1.489*	1.1986	-1.2423
Ln K	0.8443**	0.3437	2.456
LnL	0.9348	0.9989	0.9850
σ^2	.0002	.0002	1.1634
γ	0.6599	0.0192	1.1356
μ	0.0218	0.01919	1.1355
η	0.228***	1.1129	2.0192

Source: Calculations are based on ASI Data
Foot Note: * - Significant at 1% level
** - Significant at 5 % level
*** - Significant at 10 % level

From the table it is clear that since the co-efficients of both labour and capital were positive, the urban industries can improve its productive efficiency by the combined influence of both labour and capital. In a single output case, parameter of capital input was positive and statistically singnificant. The co-efficient of σ^2 and γ were statistically insignificant. This revealed the fact that the estimated levels output considerably differed from their potential levels due to factors which were not with in the control of the industries. This is evident from the value of γ, which indicated the presence of efficiency gap existed between the actual and potential level of performance.

The co-efficient of η indicated that efficiency increases in getting production over time. The sum of the elasticities of labour and capital was 1.78. It indicated increasing returns to scale of 1.780. One percent change in input would bring about 1.78 percent change in output level for the stochastic frontier. Since the co-efficient of labour was more than capital ,these industries were labour- intensive.

Table-6 shows year-wise technical efficiency of urban industries during the period under study.

Table-6
Technical efficiency – urban industries

Year	Efficiency Score
1998-99	.932
1999-00	.901
2000-01	.865
2001-02	.865
2002-03	.897
2003-04	.949
2004-05	.987
2005-06	.966
2006-07	.962
2007-08	.982
2008-09	.986
2009-10	.933
2010-11	.956
Mean	**.937**
Average in efficiency score	**.068**

Source: calculations are based on ASI data
Foot note: Average technical inefficiency score=1-average efficiency/ average efficiency
Based on the efficiency scores it was observed that the average efficiency score was 0.937 (93.7 Percent). The average inefficiency score was estimated as 0.068. Which explains the fact that 0.68 percent of inefficiency prevails in these industries .

a) Aggregate Industries :

The summary statistics of variables selected for aggregate industries are presented in table-7

Table.7

Summary statistics – Aggregate industries

Variable	Mean	Std Deviation	Minimum	Maximum	C.V
Net Value Added (NVA)	2.1776	0.1955	2.00	2.52	8.98
Invested Capital (K)	2.1337	0.1285	2.00	2.38	6.02
Number of workers (L)	2.0185	0.0493	1.97	2.11	0.02

Source: calculations are based on ASI Data

Foot Note: C.V - co –efficient of variation

In the aggregate industries no doubt that both capital and labour are significant inputs equally since there was no much gap in the growth of their mean values. The extent of variation was estimated to be 6.02 and 0.02 percent respectively in capital and labour.

Table-8 below gives details on maximum likelihood estimates of aggregate industries.

Table-8

Maximum Likelihood Estimates of Stochastic frontier

Production function – Aggregate industries

Variable	Co-efficient	Std-error	t – ratio
Intercept	-1.695*	0.4699	-3.608
Ln K	1.270*	0.0799	15.88
LnL	0.5797	0.3619	1.602
σ^2	0.0008***	0.0004	1.865
γ	0.0048	0.0631	0.0760
μ	0.0009	0.0261	0.0339
η	0.2217	0.4688	0.4729

Source: calculations are based on ASI Data
Foot Note : * - Significant at 1% level
*** - Significant at 10 % level

The log likelihood estimates of stochastic frontier model show that the co-efficients of both labour and capital were positive. In a single output case, parameter of capital was statistically significant. Hence capital is the main, input factor for aggregate industries as its co-efficient was higher than labour input. The co-efficients of σ^2 and γ were positive revealing the fact that the estimated levels of output differ from their potential level due to factors which are with in the control of industries at the aggregate level. This is evident from the insignificant co-efficinent of μ. Statistically insignificant η indicated that efficiency may decline in getting desired level of output in due course. The sum of elasticities of both the inputs was more than one (1.849) indicating increasing returns to scale. Since the co-efficient of capital was more than labour the industries at the aggregate level is capital intensive.

Table - 9 explains the technical efficiency scores for the aggregate industries of India.

Table.9
Technical Efficiency – Aggregate Industries

Year	Efficiency Score
1998-99	0.987
1999-00	0.990
2000-01	0.990
2001-02	0.991
2002-03	0.994
2003-04	0.996
2004-05	0.997
2005-06	0.997
2006-07	0.998
2007-08	0.998
2008-09	0.997
2009-10	.978
2010-11	.993
Mean	**0.993**
Average inefficiency score	**.007**

Source: calculations are based on ASI data
Foot note: Average technical inefficiency score=1-average efficiency/ average efficiency

It is evident from the table that the mean technical efficiency of aggregate industries score was 0.993. It shows the maximum efficiency attained by these industries to the extent of 99.3 percent. The inefficiency score calculated was 0.07. It explained the fact that the inefficiency present in these industries was negligible.

Conclusion

Both urban and rural Industries were enjoying increasing returns to scale. This had no doubt made the aggregate industries also work under increasing returns to scale . The technical efficiency of rural industries have not shown any decline but showed mixed trend. The inefficiency present in aggregate industries was zero. But in as indicated earlier about their inefficiency in future , these industries can become more efficient by increasing output using the existing resources or by reducing costs given the current level of production. Labour was the main factor without much variation in its contribution to the growth of net value added in urban industries. Where as capital was the main input factor for aggregate industries. The main factors of production were both capital and labour in rural industries.

Reference

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