Option Pricing Models in the Indian Options Market - An Empirical Study

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ABSTRACT

The Black Scholes option pricing formula assumes that the underlying price follows a continuous distribution. However there is evidence of jumps in stock prices as in real data. This paper studies option pricing using three models viz. Black Scholes Model, Merton's Jump Diffusion Model and the Gram Charlier Model on the NSE's Nifty call options. The study concludes that the models generally overprice out-of the-money options and underprice in-the-money options. However, none of the models priced options close to market price.

1.0 INTRODUCTION

The seminal paper of Black and Scholes not only solved an important problem for practitioners but also gave academic researchers a new area to explore. Since then the number of mathematical models for finding the option price has been multiplying at a frenetic pace. Each model is one up on the others because of certain advantages very specific to that model. However there has not been one model that exactly prices the options because of the unpredictable nature of the market, nay the nature of man, the person who trades on the exchange. There have been substantial parallel advances in time series econometrics; in particular, the development of numerically intensive techniques compatible with the continuous-time models commonly employed in derivatives research. Deficiencies of the Black Scholes model has made people consider many extensions recently. The modern quantitative ?nance literature discusses for example local volatility models (e.g. Derman et al.(1994)), stochastic volatility models (e.g. Heston (1993)) and exponential L´evy models (e.g. Madan et al. (1991)). Recent research has made use of Monte Carlo simulation, implied trees and many other mathematical techniques that have increased the complexity of the calculation process. The current study focuses on studying three models, the original Black-Scholes model, a modified version of Merton's Jump Diffusion Model and the Gram Charlier model which is a correction of the Black Scholes model. In the current study we estimate implied volatilities of 1-month options on the Index (NIFTY) and compare it with the actual volatilities to find that the market is generally more volatile than the estimated parameter for more than half the period of study. Also evident is that the historical volatility values are higher than the implied volatility values in many of the first 12 months.

2.0 LITERATURE REVIEW

There have been legions of articles documenting the various deviations of option prices from those given by the Black-Scholes formula (for some recent examples, see Alexander (2004), Bakshi, Cao and Chen (1997), Heston (1993), and Heston and Nandi(2000)).The typical approach in explaining these deviations is to point out the inaccuracy of the lognormal distribution implied by the geometric Brownian motion (GBM) assumption, and then to posit some more complicated set of dynamics in order to improve this accuracy. The more complicated set of dynamics is then solved, often by employing some Fourier transform, given boundary conditions as dictated by the nature of the derivative.

Robert C. Merton (1975) suggested that the antipathetical process to the continuous stock price motion was a "jump" stochastic process defined in continuous time. He analyzed each and every assumption of Black-Scholes and came up with a totally new way of looking at option pricing i.e. assuming each observation to be a jump (a non-local change) from the previous one. The model in this paper suggested a direction for more, careful empirical research. Moreover, since the same analysis applied to options could be extended to pricing corporate liabilities in general, the results of such further research would be of interest to all students of Finance. Gurdip Bakshi, Charles Cao, and Zhiwu Chen (1997) conducted a comprehensive empirical study on the relative merits of competing option pricing models and examined several alternative models from three perspectives: 1) internal consistency of implied parameters/volatility with time-series data, 2) out-of-sample pricing, and 3) hedging. Marco Neumann (1998) used a mixture of lognormal distributions which is actually a linear combination of two or more lognormal distributions. The empirical performance of the mixed lognormal option pricing formula was by construction at least as good as the Black/Scholes formula, since the latter is a special case of the former. This approach incorporated the possibility of future extreme underlying price movements. Two shortcomings of the Black/Scholes model are avoided when using the mixed lognormal model. First, the constant Black/Scholes volatility is replaced by a randomly changing volatility by the mixed lognormal model, leading to improved prices for options with later maturities. Second, the strange Black/Scholes pricing pattern with respect to the moneyness of the options (which is also related to the volatility smile) disappears when using the more flexible mixed lognormal distribution. S.G.Kou (2002) proposed a double exponential jump-diffusion model for option pricing. In particular, the model simple enough to produce analytical solutions for a variety of option-pricing problems, including call and put options, interest rate derivatives, and path- dependent options and the equilibrium analysis and a psychological interpretation of the model are presented. However, two puzzles emerged from many empirical investigations: the Leptokurtic Feature that the return distribution of assets may have a higher peak and two (asymmetric) heavier tails than those of the normal distribution, and an empirical phenomenon called Volatility Smile in option markets. Chen (2002) documented that, contrary to the implication of the Black-Scholes model, the implied volatilities that were generated by the model vary systematically across moneyness levels (known as the "volatility smile" puzzle). The literature attributed the problem to two unrealistic features of the Black-Scholes model: the assumed stochastic process of the price of the underlying asset and the continuous rebalancing in the absence of transaction costs. In the paper, he constructed an alternative valuation procedure to price S&P 500 call options, by using a histogram from past S&P 500 index daily returns and found that the implied volatilities that are generated by the model did not exhibit substantial relationship to moneyness levels. Consistent with the absence of the smile, payoffs to holding options were also not related to moneyness levels. The findings indicated that the model was more appropriate than the Black-Scholes model to value S&P 500 call options and implied that the Black-Scholes model underpriced in- and out-of the money call options relative to at-the-money options. Kristin E. Fink and Jason Fink(2005) demonstrated how Monte Carlo simulation may be employed to simulate option values when the underlying process follows Heston's stochastic volatility process, and motivated the example by demonstrating the significant improvement of a properly specified stochastic volatility model over the Black Scholes model. Both theoretically and empirically, the Heston model outperformed the Black-Scholes model. However, the

Heston model, along with a number of others, was found to be much more accurate in describing observed option prices than the Black-Scholes model.

Damodaran(2002) has mentioned about the various studies of market efficiency that have uncovered numerous examples of market behavior that are inconsistent with existing models of risk and return and do not go with rational explanations. The persistence/constant presence of some of these patterns of behavior suggests that the problem, in at least some of these anomalies, lies in the models being used for risk and return rather than in the behavior of financial markets. The effects are categorized as the temporal anomalies. There are a number of peculiarities in return differences across calendar time that are not only difficult to rationalize but are also *suggestive of inefficiencies. The anomalies* can be further divided into two categories on the basis of what they are classified. The first category is under the characteristics of the firm like size of the firm, market value of equity, price earnings ratios and price book value ratios

Studies have consistently found that smaller firms (in terms of market value of equity) earn higher returns than larger firms of equivalent risk, where risk is defined in terms of the market beta.

Investors have long argued that stocks with low price earnings ratios are more likely to be undervalued and earn excess returns. For instance, Benjamin Graham, in his investment classic "The Intelligent Investor", uses low price earnings ratios as a screen for finding under valued stocks. Studies that have looked at the relationship between PE ratios and excess returns confirm these priors. The only explanation that can be given for this phenomenon, which is consistent with an efficient market, is that low PE ratio stocks generate large dividend yields, which would have created a larger tax burden in those years where dividends were taxed at higher rates.

Another statistic that is widely used by investors in investment strategy is price book value ratios. A low price book value ratio has been considered a reliable indicator of undervaluation in firms. In studies that parallel those done on price earnings ratios, the relationship between returns and price book value ratios has been studied. The consistent finding from these studies is that there is a negative relationship between returns and price book value ratios, i.e., low price book value ratio stocks earn higher returns than high price book value ratio stocks.

The other type includes the time dimension of the anomalies. They are:

a. January Effect

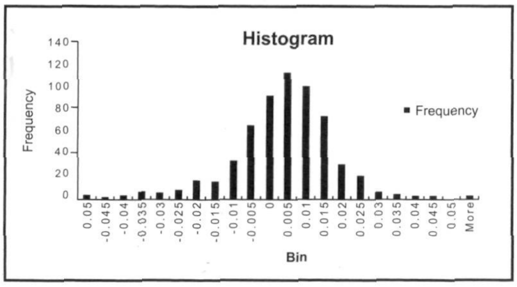
Returns in January are significantly higher *than returns in any other month of the year.* This phenomenon is called the year-end or January effect, and it can be traced to the first two weeks in January. The relationship between the January effect and the small firm effect adds to the complexity of this phenomenon. The January effect is much more accentuated for small firms than for larger firms, and roughly half of the small firm premium, described in the prior section, is earned in the first two days of January. The January effect is often attributed to the turn of the tax calendar; investors sell off stocks at year's end to cash in gains and sell losing stocks to offset their gains for tax purposes. Once the New Year begins, there is a rush back into the market and particularly into small-cap stocks.

b. The Weekend Effect

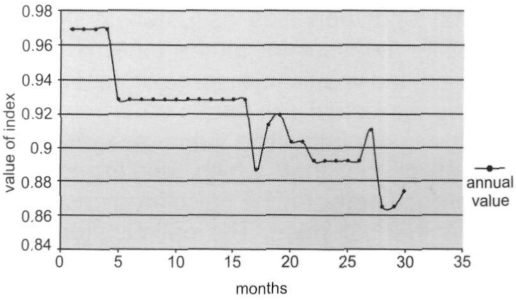
The weekend effect is another return phenomenon that has persisted over extraordinary long periods and over a number of international markets. The weekend effect describes the tendency of stock prices to decrease on Mondays, meaning that closing prices on Monday are lower than closing prices on the previous Friday. For some unknown reason, returns on Mondays have been consistently lower than every other day of the week. In fact, Monday is the only weekday with a negative average rate of return. There are some who have argued that the weekend effect is the result of bad news being revealed after the close of trading on Friday and during the weekend. The weekend effect is fairly strong in most major international markets especially in the Asian markets.

c. The Prior Return or Momentum Effect

Prior stock returns have been shown to have explanatory power in the cross section of common stock returns. Stocks with prices on an upward (downward) trajectory over a prior period of 3 to 12 months have a higher than expected probability of continuing on that upward (downward) trajectory over the subsequent 3 to 12 months. This temporal pattern in prices is referred to as momentum. Jegadeesh and Titman (1993) show that a strategy that simultaneously buys past winners and sells past losers generates significant abnormal returns over holding periods of 3 to 12-months. The abnormal profits generated by such offsetting long and short positions appear to be independent of market, size or value factors and has persisted in the data for many years.



The histogram suggests to us the findings of the skewness and kurtosis. The presence of a high frequency in the bin area of 0 and 0.005 suggests that the phenomena of high kurtosis.



The jump drop graph shows us the trend observed i.e. the fall and then stabilizes. The graph is also suggestive of the continuous improvement in the volatility in the market. This could be seen as a reason for the wild fluctuations in the market that happened in the market during the period of study as well as in the current situation.

3.0 METHODOLOGY

The models considered in the study can be said to represent the key categories of models, i.e. Black-Scholes Model, a modified version of Merton's Jump Diffusion Model and the Gram Charlier Model, which is actually a correction of the Black Scholes Model. Each model has certain key characteristics and assumptions which affects the results obtained thereby. The estimation of parameters for each of the models is very important and decisive for the models to give accurate results and hence is covered in this section.

A. BLACK SCHOLES MODEL

The Black Scholes equation for pricing a call is given by

$$c = S^*N(d_1) - K^* e^\wedge(-r^*t)^*N(d_2) \qquad (1)$$

where

$d_1 = \ln(S/K)+(r+\sigma^2/2)^*t$

$d_2 = d_1 - \sigma^*\sqrt{t}$

The parameters d_1 and d_2 are used to denote a probability which is made use of to calculate the distribution values (Z-values). The implied volatility is found out for the first month and then it is assumed to be the volatility measure for all the other months. The implied volatility value was found out by taking the actual call prices data from nifty for the first month. The value of sigma was found to be that which minimized the difference between the calculated price and the actual call price. The optimization was done through SOLVER in EXCEL.

B) MERTON'S JUMP DIFFUSION MODEL

The model used in this case is the modified version of Merton's actual formula. The correction is made in the fact that the jump intensity was found out from the actual distribution so the value of the underlying changed each time the simulation was carried out. The equation used to simulate the prices of the underlying, in this study NIFTY, is given as

$$\frac{dS}{S} = (\alpha-\lambda^*k)^*dt + \sigma^*dZ + dq \qquad (2)$$

Where α is the instantaneous expected return on the stock; α^2 is the instantaneous variance of the return, conditional on no arrivals of important new information (i.e., the Poisson event does not occur.); and dZ is a standard Gauss-Wiener process. The function q(t) is the independent Poisson process described in (1). dq and dZ are assumed to be independent. X is the mean number of arrivals per unit time. $k\text{-}\varepsilon^*(\gamma\text{-}1)$ where (γ-l) is the random variable percentage change in the stock price if the Poisson event occurs and is the expectation operator over the random variable γ. The "dZ" part describes the instantaneous part of the unanticipated return due to the "normal" price vibrations, and the "dq" part describes the part due to the "abnormal" price vibrations. If $\lambda = 0$ (and therefore, dq = 0), then the return dynamics would be identical to those posited in the Black and Scholes and Merton papers. Equation (1) can be rewritten in a somewhat more cumbersome form as

$$\frac{dS}{S} = (\alpha-\lambda^*k)^*dt + \sigma^*dZ + (\gamma\text{-}1),$$

if Poisson event does not occur

$$= (\alpha-\lambda^*k)^*dt + \sigma^*dZ + (\gamma\text{-}1),$$

if the Poisson event occurs (2')

where, with probability one, no more than one Poisson event occurs in an instant, and if the event does occur, then (γ-l) is an impulse function producing a finite jump in S to S $^*\gamma$. The resulting sample path for S(t) will be continuous most of the time with finite jumps of differing signs and amplitudes occurring at discrete points in time. If α, λ, σ and k are constants, then the random variable ratio of the stock price at time t to the stock at time zero (conditional on S(t) /S) can be written as

$$S(t)\Big/S = e^{[(\alpha-\frac{1}{2}\sigma^2-\lambda^*k)t+\sigma^*Z(t)]}\,Y(n) \qquad (3)$$

where Z(t) is a gaussian random variable with a zero mean and variance n equal to t; Y(n) =1 if n = O; $Y(n) = \Pi^n_{j=1} Y$ for n > 1 where the Y are independently and identically distributed and n is Poisson distributed with parameter Xt.

For this distribution the probability and the intensity of the jumps are very important. The probability of the jump was found out using the returns of the nifty closing values. For each month, the mean was found out. The returns were subtracted by 1 and then sorted in the ascending order to find out the lowest jumps. The smallest 1% i.e.15 values were considered as the sample and then the average of these values gave the probability of the jump for each month. The other input was the intensity of the jumps. This parameter was initially assumed to be some value and the prices for the first month were found out. Then the value was found out for the other months using the same procedure as used in the Implied Volatility Estimation in Black Scholes model.

C) GRAM CHARLIER MODEL

The Gram-Charlier series uses the moments of the real distribution to incorporate the effects of non-normal skewness and kurtosis into the Black-Scholes option pricing formula. To allow for moments of higher order in the returns distribution, Corrado and Su (1996) found an approximate probability density function using a Gram- Charlier expansion of the normal density function. The Black and Scholes (1973) model is then adjusted in an intuitive way by introducing third and fourth moments as higher order terms of the expansion. The series is truncated after the fourth term, noting that for practical purposes the first four moments of the underlying distribution should capture most of the effect on option prices (Jarrow and Rudd 1982).

The expression for Q3 in Black Scholes must be altered from

$$Q_3 = 1/3!*S*\sigma*\sqrt{t}[2*\sigma*\sqrt{t}-d)n(d)-\sigma^2*t*N(d)]$$

to

$$Q_3 = 1/3!*S*\sigma*\sqrt{t}[2*\sigma*\sqrt{t}-d)n(d)+\sigma^2*t*N(d)] \tag{4}$$

Then, using this result, the call option price is given by

$$C = C_{BS} + \mu_3 Q_3 + (\mu_4-3)\,Q_4 \tag{5}$$

where CBS is the Black-Scholes price of the call option, Q3 is given by equation (3), and

$$Q_4=1/4!*S*\sigma\sqrt{t*}[d_2-1-3\sigma\sqrt{t(}d-\sigma\sqrt{t)}*n(d)+\sigma^3*t^{3/2}N(d)] \tag{6}$$

With $d = \dfrac{(\ln(S/K) + (r+\sigma^2/2)*t)}{\sigma*\sqrt{t}}$ and

$$\mu = \ln(S)+(r-\sigma^2/2)*t$$

and μ_3 and μ_4 are the skewness and kurtosis respectively for the distribution and

n(d) stands for probability density function and

N(d) stands for cumulative distribution function.

The important parameters used in this model are the skewness and the kurtosis of the distribution. These parameters were found by using Nifty data. The skewness and kurtosis was found out for each month similar to the calculation for the mean, i.e. by taking 6 years closing values and finding the skewness using functions SKEW and KURT in EXCEL.

4.0 DATA

The data for the study was taken from the website of NSE (National Stock Exchange).The index, S&P CNX Nifty, is a well diversified 50 stock index, accounting for 21 sectors of the economy. It is used for a variety of purposes such as benchmarking fund portfolios, index based derivatives and index funds. The data on one month expiring options for each month's first day was obtained for 30 months from 01-01-2005 to 29-06-2007. The data was then sorted out because of the existence of contracts that were not traded highly i.e. number of contracts < 100. The data collected was for options expiring in one month. The data was then cleansed and then only the closing prices, the strike prices, the day of expiry and the day of the contract were given as input to the program that calculated the value of the option prices.

The following data was required for the study:

- The closing values of NIFTY for all the days from January 1999 to the last date of analysis period i.e. 29-06-2007.
- The strike price, open, high, low, close, settle price, number of contracts traded and the value of the contracts for Nifty European call options being traded in the F&O exchange for all the days from the start of January 2005 to the end of June 2007.
- The risk free rate i.e. the Treasury bill yields for a period .This data was obtained from RBI website.

5.0 RESULTS & DISCUSSION

The comparison of the models was done using the following three yardsticks. Each model's price values were also compared with the historical prices.

A. MONEYNESS

The moneyness is defined as the ratio of the strike price (K) of an option and the spot price (S) (stock price on the day of pricing the contract).The moneyness was calculated for each contract and plotted against the mean absolute errors of the models with the market price of the option. The strike price relative to current value of the index of an option determines whether that contract is in-the-money, at-the-money, or out- of-the-money. There in the case of a call option is less than the current market price of the underlying security, it is said to be in-the-money.

In the present study the limits for in-the-money are specified as moneyness (m) $<$ 95%, and out-of-money for m $>$ 1.05.On the other side the criteria for deciding under/over pricing is three or more contracts should be below/above the ideal price ratio i.e.1.

B. PRICE COMPARISON

In this section, the prices obtained by using the models were plotted against the strike prices and the models were checked for their accuracy or closeness to the market price. The charts for all the months were plotted and categorized based on whether the calculated prices were greater or lesser than the historical prices. The conclusion from the methodology was whether the models overpriced or underpriced the options. Another key observation from the method was that the method gave us the periods of the study that were important for the analysis and made the conclusions more evident.

C. IMPLIED VOLATILITY ESTIMATE

In this method, the implied volatility for each model was calculated by comparing the calculated prices with the actual prices and then minimizing the difference. This was done for each contract of each month. The implied volatility was then plotted against the historical volatility estimated as the standard deviation of returns over a historical period, to conclude certain important aspects of market volatility.

ANALYSIS & FINDINGS

The first importance was given to justify the need for jump diffusion model for the pricing of the options. This was done by checking the plot of nifty returns. Fig.1 shows the presence of jumps in the data. The jumps have been highlighted in black circles. This established the rationale for employing the Jump Diffusion model. Another important justification was needed for the need of the Gram Charlier model. This was done by plotting a histogram of the frequency of returns based on the various ranges they were classified into. The histogram showed the twin effects i.e. high kurtosis and negative skewness. An interesting observation regarding the jumps in the actual market data was shown by plotting the lowest jumps for each month and it was noticed that the jump drop i.e. the percentage jump in the prices appears to be increasing and it followed a particular fashion i.e. it stays constant for sometime,then increases and again stays constant.

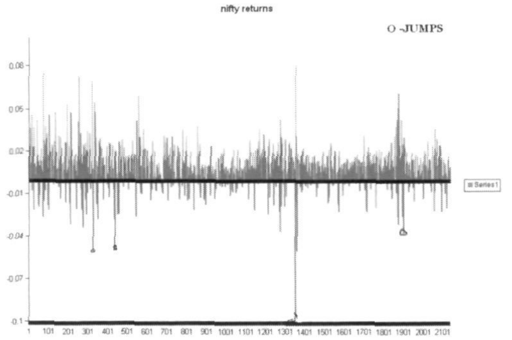


Fig 1 .Plot of Nifty returns against the time period of study.

The above findings were based on the data generally. Then we switched to the three yardsticks we had chosen to compare the models. The first yardstick was the Implied Volatility Measure. This measure found out the comparison between the implied volatility and the historically observed volatility for all months. In this we made a broad classification of the patterns observed. The table is given below:

CLASSIFICATION	MONTHS
1 implied volatility > historical volatility	2006-April, May, July 2007-March
2 The implied volatility < historical volatility	2005-February, March, April, May, June, July, August, September, November 2006-January, February, October, November 2007-April and June
3a Implied volatility is continuously increasing and meets the historical volatility at some point	2005-none 2006-June,August 2007-none
3b Implied volatility is continuously decreasing and meets the historical volatility at some point	2005-January, October. 2006-March, September, December 2007-January, February, May
3c Implied volatility is varying	2005-December 2006-November, December 2007-None

Table 1: Table of categorization of charts

One key observation made here was that for more than half the time of study the implied volatility was lesser than the actual historically observed volatility. One trend observed in the charts plotted of the implied and historically observed volatilities is that as the strike price increases, the implied volatility tends to stabilize or changes very less. The figure below shows the behavior of the implied volatility when it was compared month wise .The results are that in the initial months of the period of study; the implied volatility is less than the historically observed volatility. Also observed is that the highest value the implied volatility gets is 0.4 in the month of May 2006.

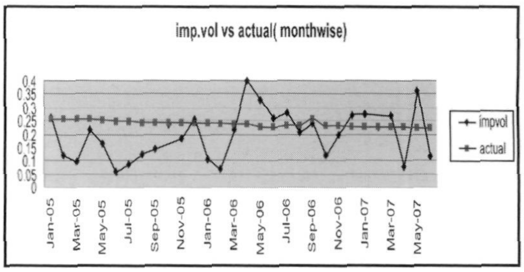
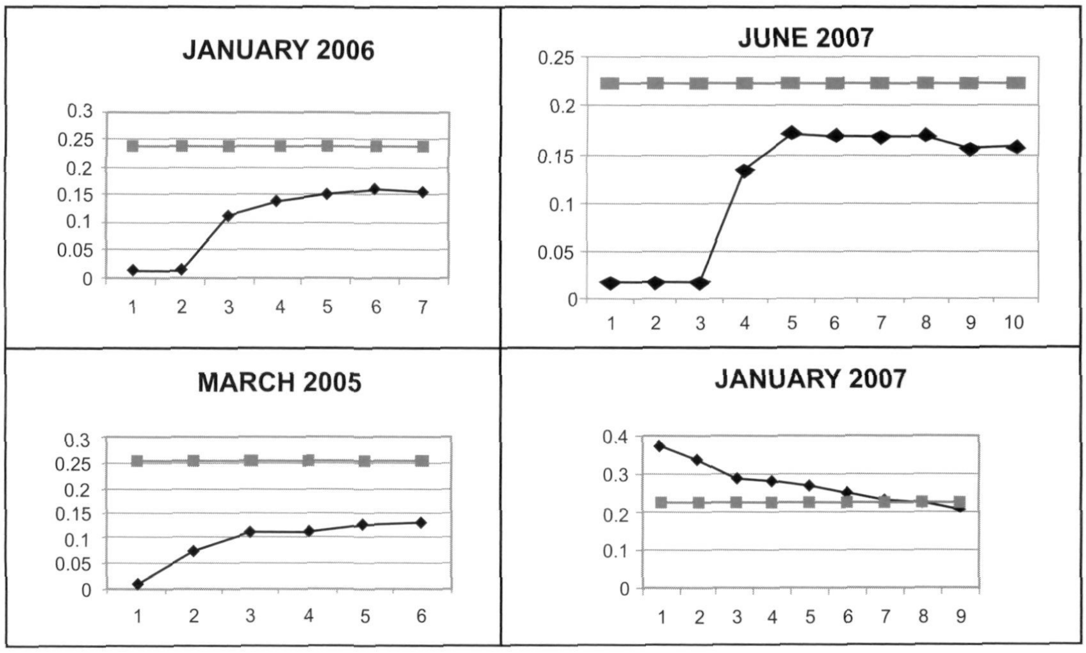


Fig 2: Chart of Implied Versus Historical Volatility (Month wise)



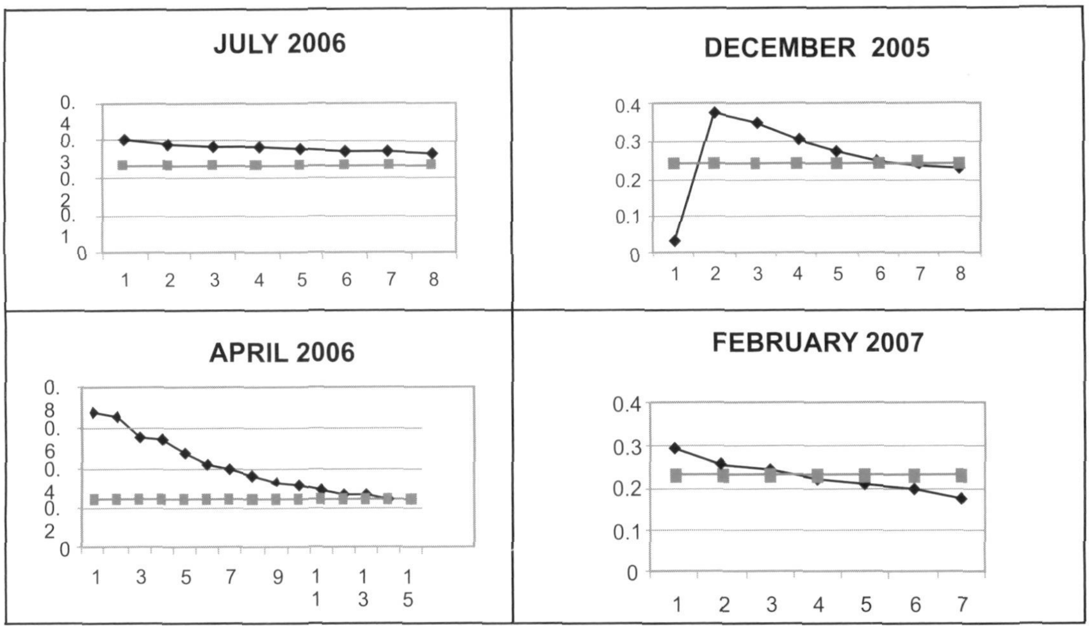


Figure 3: The Implied and Actual Volatilities

The second yardstick is the price comparison of all the models. The plots of option prices by all the models and the actual observed price were broadly classified into three categories. The categories are:

1. The actual market price is lower than the prices given by the models.
2. The actual market price is higher than the prices given by the models.
3. The market price and the price predicted by the models are almost similar.

We found that 60% of charts fell in the first category. Thus we can conclude that the models are generally overpricing the options.

In the second category we have the plots of the months April 2006, May 2006, July 2006 and March 2007.This suggests to us that the actual prices are affected by certain additional factors not considered by the models like sudden announcements or short-selling by traders.

The third category is of much interest because we can very clearly make certain conclusions as to when which model is pricing very close or almost equal to the actual price. One observation in this category is that the price found using the jump diffusion model is generally higher than the others and the other models fall behind. One reason for the above observation could be that the jump parameters are not always held true by the actual prices although they were derived from the actual data. The prices are however not very wide apart.

From the analysis of the three categories of charts we can conclude that none of the models could exactly match the actual historical values.

It was observed that the models (BLACK SCHOLES, GRAM CHARLIER and JUMP DIFFUSION) were in most cases giving call prices that were close to each other.

In general, the period of study is one which witnessed lot of boom in the markets, especially in the field of options trading which was due to the increased activity in the corresponding equity markets. The period of January to March in the year 2006 shows a steady increase in the value of the index, corresponding to which the options price follow suit.

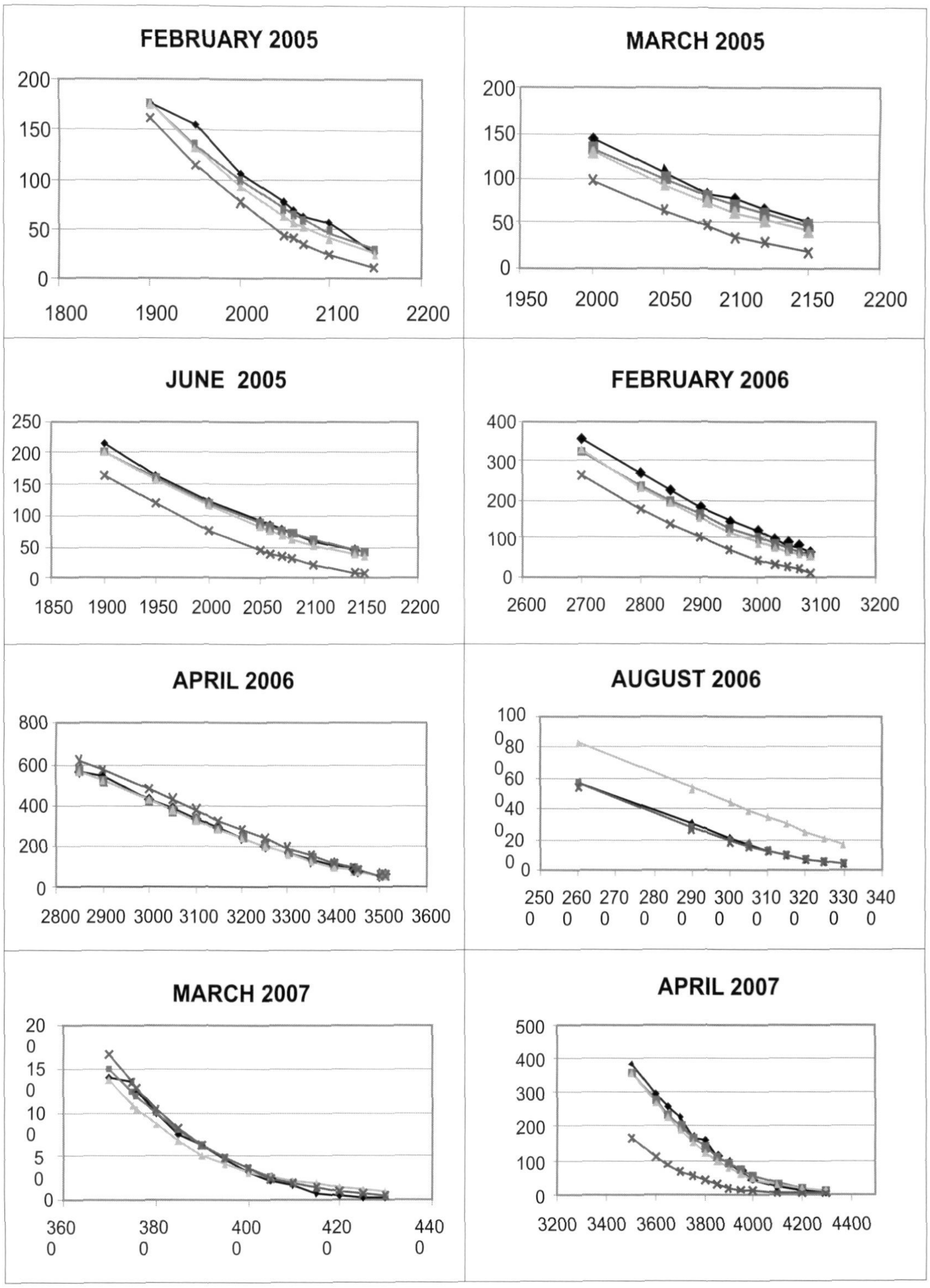


Figure 4: Price Comparison Method

The third and final yardstick used to compare the models is the very popular 'moneyness' concept. The moneyness method gives the broad classification for the overpricing and underpricing of the models when compared with the actual data. The plots of the errors of each model with respect to the moneyness of each contract give us the information that in most of the cases the *out of-the-money contracts are overpriced* and the in-the-money contracts are underpriced. An observation about the moneyness of the options is that most of *them are in- the-money generally, and when* they are out-the-money they tend to be very high/low.

Pricing		Moneyness	
		out of money	1
Overpriced	A	at the money	2
Underpriced	B	in the money	3

Table 2: CONVENTIONS

The analysis found using the conventions gave us key statistics for each model that help us codify specific behavior of the models with respect to the moneyness yardstick.

Jump Diffusion	**Overpriced**	**Underpriced**
Out-of the money	18	2
At-the-money	9	0
In-the-money	1	4

Table 3: Jump Diffusion Model

Black Scholes	**Overpriced**	**Underpriced**
Out-of the money	15	1
At-the-money	10	2
In-the-money	2	5

Table 4: Black Scholes Model

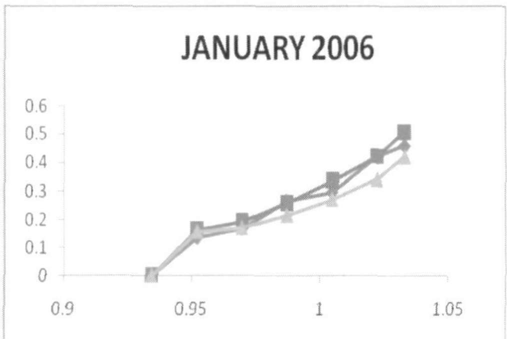
Gram Charlier	**Overpriced**	**Underpriced**
Out-of the-money	15	2
At-the-money	8	3
In-the-money	2	7

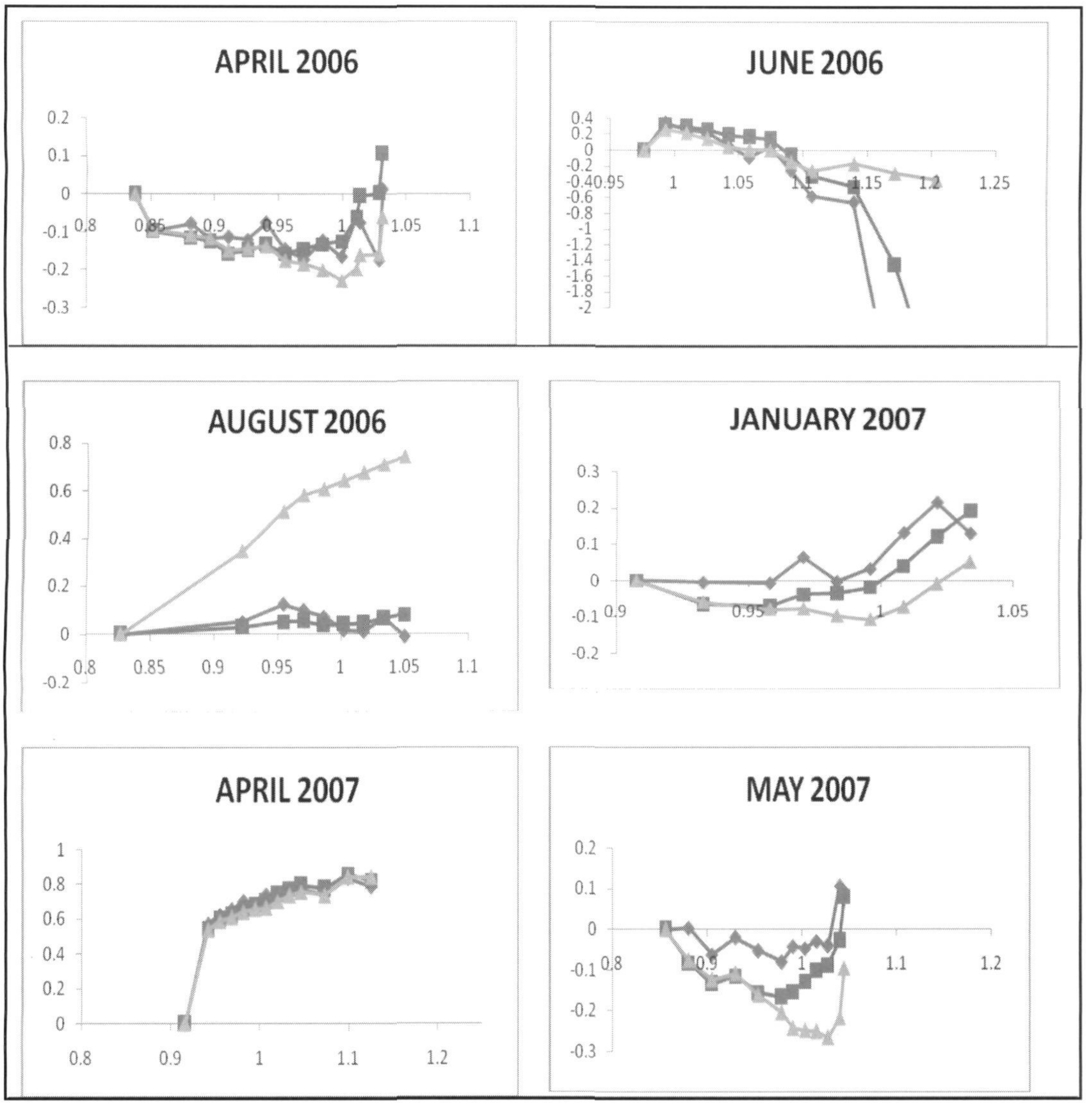
Table 5: Gram Charlier Model

The above tables give us information that the jump diffusion model, Black Scholes model and the Gram Charlier model generally overprices out-of-money contracts.

This leads us to a general conclusion regarding the models that all the models overprice out-of the- money contracts.

In almost all the months, the mean error is around 5%.There are however exceptions to this as seen in the chart for August 2006 where the error is as high as 25%.Also noteworthy is the fact that the Black Scholes model overprices at-the-money options which makes it different from the other models.





6.0 CONCLUSION

The conclusion from the present study is that all the models generally overprice out-of the-money options and underprice in-the-money options. The study also presents the conclusions that all the models used for the study did not price the options as the historical prices. This leads us to the conclusion that the actual prices are not found using any of the above models and also that there are many other factors and parameters that affect the option price other than those taken care by the models. The study also confirms the belief that there are jumps in the actual market price. The implications of the results are important to participants in the options market and for the exchange authorities. Since institutional investors and funds need to hedge their risks of investment, a sound pricing model assumes enormous importance. This study was carried out on data from one index viz. the Nifty. It could be analyzed for individual stocks having option contracts on them. The study was carried out only for one month expiry contracts. It could be extended to find the performance of the selected models for other maturities e.g. two month, three month, one year, etc. Lastly, other models such as the Heston-Nandi stochastic volatility model and other newer models can be experimented with.

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