FREQUENCY DISTRIBUTION OF QUARTZ GRAIN-SPHERICITY AND QUARTZ GRAIN-ROUNDNESS IN SIEVE FRACTIONS

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Introduction: From a study of the two-dimensional projection sphericity, ψ, and two-dimensional projection roundness, P, of clastic quartz grains belonging to the river, beach and aeolian-dune environments and to the sieve fractions -45, $+50$; -50, $+60$; -60, $+70$ and -70, $+80$ of A.S.T.M. sieve series, Sahu and Patro (1970, p. 65) and Patro and Sahu (1971, p. 38), basing on the parametric test i.e., significance of skewness, β_1, and kurtosis, β_2, concluded the following: ψ ($=b/a$ where b is the diameter of the largest inscribed circle and a is the diameter of the smallest circumscribing circle of the grain projection) and P ($= \sum_{i=1}^{N} r_i/NR$, where $r_i =$ the radius of curvature of individual corners in the grain projection, $N =$ the number of corners measured and $R = b/2$) data of quartz grains in a sieve fraction are non-normally distributed both on number as well as weight frequency basis; though a -log ψ transformation fails to normalize ψ distribution, log $[\psi/(1-\psi)]$ transformation produces the best normalized distribution; and -log P as well as -log $[P/(1-P)]$ transformation of P data normalize its distribution, but -log P is preferable because of its simplicity.

The detection that the ψ and P are nonnormally distributed does not disclose all the information regarding the variate's frequency distribution. As for example, though the β_1 and β_2 statistics measure the departure of the shape of frequency curve of an observed distribution from that of the normal density, they are incapable of isolating the contributions of individual sphericity or roundness class frequencies to the nonnormality. Such information on individual class frequencies is basic to interpret the roundness and sphericity frequency distributions in terms of the dynamic conditions operating in the transporting and depositional media. Further, a knowledge of the deficiency or excessiveness of observed frequencies as compared with the expected normal density frequencies in a particular sphericity or roundness class or classes, if consistent, may be of practical utility in environmental identification. The loss of information, resulting from the indiscriminant nature of β_1 and β_2 statistics to the discrepancies in the individual class frequencies, can, however, be supplemented by the nonparametric Chi-Square, χ^2, test, which besides testing the agreement between the observed and theoretical distributions, can also locate the exact place(s) of discrepancy between the observed and expected frequencies in the case of a non-normal distribution. The β_1 and β_2 statistics and the χ^2, measuring the departure of two different aspects of an observed distribution from those of the normal distribution, are complementary in nature (Griffiths, 1967, p. 260) and their application to the same data is not redundant.

In view of the preceding discussion the ψ, -log ψ, log $[\psi/(1-\psi)]$ and P, -log P and -log $[P/(1-P)]$ data (see, Patro, 1970, Appendix-2, pp. 183-186) of clastic quartz grains belonging to the samples R1 (100) of Mahanadi River, B76 (100) of Puri Beach, RD51(50) of River Dune, and BD96(50) of Beach Dune environments and to the sieve fractions investigated earlier by Sahu and Patro (1970), were subjected to χ^2 test to locate the areas of disagreement between the observed individual ψ and P class frequencies and their corresponding expected normal density frequencies; and to check the stability of the transformation functions log $[\psi/(1-\psi)]$ and -log P and -log $[P/(1-P)]$ to normalize sphericity and roundness data respectively. The number in parenthesis following the sample number indicates the number of grains studied in each of the four sieve fractions obtained from the particular sample.

Calculation of X^2 and testing the 'Goodness of Fit' of normal density to the observed data: For details of laboratory sampling, projection and measurement techniques Sahu and Patro (1970) may be referred. The ψ, -log ψ and log $[\psi/(1-\psi)]$ data of sphericity and P, -log P and -log $[P/(1-P)]$ data of roundness belonging to each sieve fraction were grouped into 7 to 8 categories with uniform class interval. The number of sphericity or roundness values falling in a particular class gives its observed number frequency (f_o), while the expected frequency (f_e) for the same class is obtained by multiplying with the total observed number frequency the area under the standard normal curve between the two standard scores (z) corresponding to the upper and lower limits of the particular sphericity or roundness class. After finding the f_o and f_e for all the sphericity as well as roundness classes, the X^2 is calculated according to the following formula:

$$U = \sum_{i=1}^{k} (f_o - f_e)_i^2 / f_e$$

Where U = the observed Chi-Square; $(f_o - f_e)_i$ = the difference in a pair of f_o and f_e belonging to the ith sphericity or roundness class; k = the number of pairs of f_o and f_e compared. Details involved in the X^2 calculation are given in Hoel (1964, p. 167). To test the goodness of fit of the theoretical distribution to the observed, the U is compared with the theoretical X^2 with .05 probability for k-3 degrees of freedom. $U \geqslant X^2$ rejects normal density as a befitting model to the observed data, while $U < X^2$ indicates a good fit of the normal density to the observed data.

Results and Discussion: Table I shows the tabulation of f_o and f_e, calculation and testing U for its significance. Six such tables, three corresponding to the three sphericity functions and rest three corresponding to the three roundness functions, were obtained. These, excepting Table I, are omitted to save space. However, the results contained in them concerning sphericity are summarised in Table II and those concerning roundness in Table III.

Table II shows that the distribution of -log ψ data is nonnormal and that of ψ as well as log $[\psi/(1-\psi)]$ data is normal. The results of the parametric (Sahu and Patro, 1970 and Patro and Sahu, 1971) and nonparametric tests with ψ data are contradictory in nature. The reason for their disagreement becomes explicit by comparing the number of significant β_1 values with that of significant β_2 values and their levels of significance given in Table II of Sahu and Patro (1970, p. 58). Such a study clearly indicates that kurtosis or peakedness of the data is the sole cause of nonnormality of ψ distribution in the parametric test. Kurtosis measures the departure in the length and height of tails rather than the shape of the hump of an observed frequency curve from those of the normal curve (Kenney and Keeping, 1964, p. 102). Due to its concern with the tails of a frequency curve peakedness remains undetected by the X^2 criterion, whose estimation involves summation of f_o, inevitably in the tail classes of ψ distribution, as a consequence of pooling of f_e in these classes to satisfy the condition $f_e \geqslant 5$. Pooling of f_o apparently obscures the otherwise significant discrepancies between the unpooled f_o and f_e and results in a good fit of the observed ψ data to the normal density.

TABLE I

FREQUENCY DISTRIBUTION OF *P* DATA (NORMAL H_0)

Size grade	Sample No.	observed (f_o) and expected (f_e) frequencies	Number frequency of *P* classes						No. of observations	Observed Chi-square *U*	Degree of freedom
−45, +50	R1	f_o	18	23	39	20			100	3.63(ns)	1
		f_e	16.85	31.16	32.22	19.77					
	B76	f_o	10	16	32	42			100	24.59*	2
		f_e	8.23	23.33	35.44	24.31	8.69				
	RD51	f_o	7	11	30	2			50	13.30*	1
		f_e	6.46	18.54	18.54	6.46					
	BD96	f_o	5	15	18	12			50	0.57(ns)	1
		f_e	6.79	14.84	17.34	11.03					
−50, +60	R1	f_o	14	28	39	19			100	1.46(ns)	1
		f_e	15.62	31.59	33.57	19.22					
	B76	f_o	9	27	37	26			100	9.80*	2
		f_e	13.35	28.33	33.50	19.23	5.59				
	RD51	f_o	9	7	14	20			50	23.60*	2
		f_e	11.79	11.62	12.21	8.63	5.75				
	BD96	f_o	4	15	16	15			50	1.12(ns)	1
		f_e	4.67	14.06	18.87	12.40					
−60, +70	R1	f_o	16	54	30				100	7.13*	1
		f_e	24.82	44.68	26.04	4.46					
	B76	f_o	11	23	40	26			100	9.61*	2
		f_e	11.12	28.61	36.06	19.35	4.85				
	RD51	f_o	3	7	15	9	16		50	7.90*	2
		f_e	6.69	6.53	14.17	13.14	9.47				
	BD96	f_o	7	6	28	9			50	11.66*	1
		f_e	6.05	14.79	17.83	11.33					
−70, +80	R1	f_o	14	19	51	16			100	13.99*	1
		f_e	14.69	30.93	34.05	20.33					
	B76	f_o	14	35	48	3			100	19.58*	1
		f_e	22.66	38.75	29.41	9.18					
	RD51	f_o	5	6	7	14	18		50	5.21(ns)	2
		f_e	5.02	8.04	11.94	11.95	13.05				
	BD96	f_o	5	7	4	15	13	6	50	8.56*	3
		f_e	5.95	7.27	9.99	10.65	8.33	7.81			

*=Significant at 5% level   ns=Nonsignificant

TABLE II

SUMMARIZED CHI-SQUARE (OBSERVED) TABLE FOR SPHERICITY DATA FOR COMPARISON OF TRANSFORMATION FUNCTIONS

Transformation function	Number of non-significant *U*s	Number of *U*s significant at 5% Level	Total
Arithmetic (ψ)	15	1	16
$-\log\psi$	7	9	16
$\log[\psi/(1-\psi)]$	15	1	16

TABLE III

SUMMARIZED CHI-SQUARE (OBSERVED) TABLE FOR ROUNDNESS DATA FOR COMPARISON OF TRANSFORMATION FUNCTIONS

Transformation function	Number of non-significant *U*s	Number of *U*s significant at 5% level	Total
Arithmetic (P)	5	11	16
$-\log P$	15	1	16
$-\log[P/(1-P)]$	15	1	16

The concurrence of the results of the nonparametric and parametric tests as to the goodness of fit of normal frequency distribution to the log $[\psi/(1-\psi)]$ data, shows that it is a more reliable and stable normalizing transformation.

Table III shows that the distribution of P data is nonnormal and that of -log P as well as -log $[P/(1-P)]$ data is normal. These results comply with those of the parametric test of Sahu and Patro (1970).

It is of interest to note that the summation of f_o in the tail portions did not result in a good fit of the normal density to the P distribution, unlike the case with ψ distribution. A comparative study of the number of significant β_1 values with that of significant β_2 values and their levels of significance given in the Table III of Sahu and Patro (1970, p. 59) reveals that the skewness or asymmetry, but not the peakedness, of the data is the sole cause of nonnormality of P distribution in the parametric test. Consequently pooling of f_o in the tail classes fails to fit normal density to the P data in the X^2 test.

Though the results pertain to quartz grains belonging to the particular sieve fractions and to the particular environments studied, they hold true for other sieve fractions and other environments as well if the sampling is restricted to pure quartz sands.

Conclusions: 1. The present study shows that to ensure the minimum number of classes ($k \geqslant 5$) required to perform X^2 test for goodness of fit to normal density, sphericity and roundness classes with .1 unit class interval are to be used when the sphericity and roundness observations in each sieve fraction are 100 or more; with observations $\geqslant 50$ but < 100, classes with .05 unit class interval are to be used.

2. The normal density is a good fit to the observed frequency distribution of two-dimensional projection sphericity, ψ, as well as the transformed log $[\psi/(1-\psi)]$ data of quartz grains in sieve fractions. The ψ data, without any transformation can be considered as normally distributed in quick routine work; for greater accuracy the log $[\psi/(1-\psi)]$ transformation should, however, be applied.

3. The frequency distribution of the two-dimensional projection roundness, P, of quartz grains in sieve fractions is log normal and -log P is the simplest normalising transformation.

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