NOTES

TSUNAMIS AND EARTHQUAKES: WHAT PHYSICS IS INTERESTING?

[Reprinted with permission from American Institute of Physics from "Physics Today", v.58, no.16, June 2005, pp.10-11, for the benefit of our readers — Editor]

In 1960, my elementary school in Wellington, New Zealand, was evacuated for fear of the tsunami from the great Chile earthquake, the largest in the past century. My school was about six feet above sea level, but (to our disappointment) no substantial wave arrived, and the disruption was brief. The tsunami from that earthquake crossed the Pacific Ocean and devastated downtown Hilo on the main island of Hawaii.

Great earthquakes are infrequent and large tsunamis are even less frequent, but the recent events in Sumatra late last December have revived our awareness. Much has been written about the human tragedy and the importance of appropriate warning systems, but my goal here is to talk about the interesting physics of these events (*see* also page 499 of this issue). Tsunamis are well understood. Earthquakes are not that well understood. Of course that means that tsunamis are difficult to predict. But once they are generated, their subsequent behaviour is not difficult to estimate because we now have detailed information about the ocean floor.

Tsunamis present a wonderful opportunity to explain basic physics at work. Instructors can enrich a physics class with a topic that can catch the students' attention as well as convey some very nice ideas, many of which are elementary. Regrettably, fluid dynamics is not well covered in standard physics curricula, but the ideas have natural connections to basic conservation laws, optics, and quantum mechanics. They can also be used to enliven a class in differential equations.

Tsunamis

Tsunamis are water waves in which the restoring force is gravity and the wavelength is greater than the ocean depth. Unlike shorter wavelength disturbances, the fluid motion extends throughout the water column. For a wave of surface amplitude h, the pressure difference that drives the fluid horizontally away from beneath a crest is about ρgh, where ρ is the water density and g is the gravitational acceleration. This pressure difference is spread over a horizontal distance that is roughly λ, the wavelength. For an ocean depth D, the resulting horizontal motion arising from the horizontal driving force acting for a period T can be estimated from $F = ma$ and must be about ghT/λ. By continuity, this flow must be larger than the vertical particle velocity (roughly h/T) by the ratio of λ/D. Equating these estimates, we find a period T roughly equal to $\lambda/(gD)^{1/2}$ or, equivalently, the wave speed is $(gD)^{1/2}$.

Despite the crudeness of this derivation, the result is nearly exact, even if the ocean floor is of nonuniform depth, provided that any large variation in depth occurs on a large length scale compared to the wavelength. For a 4 km deep ocean, the predicted wave speed is about 200 m/s and a wave packet can cross a 5000 km ocean in about seven hours. The wave speed does not depend on wavelength, so the initial disturbance will cross the ocean undispersed and largely unaltered in form until it approaches the shore. As a delightful and simple application of the WKB approximation (named after Eugene Wigner, Hendrik Kramers, and Leon Brillouin and equivalent to the semiclassical approximation in quantum mechanics), one can show that as a wave travels into water of different depth D, the wavelength scales as $D^{1/2}$ and the amplitude of the wave scales as $D^{-1/4}$. The regional variation of ocean depth acts as a lens to refract the waves, just as a lens refracts light. Abrupt changes in ocean depth cause partial or even complete reflections, and waves can diffract around islands and coastlines. As the tsunami approaches the shore, it increases dramatically in amplitude and eventually becomes nonlinear as the wave steepens and its large energy is confined to an ever decreasing water mass. Details of the coastline also matter, and resonances can arise with the sloshing frequencies of bays. The motions resemble those that occur if you tilt a basin of water or enter a partly filled bathtub, and they can cause the sequence and amplitude of arriving waves to be more complicated than the deep-sea waveform. These complications prevent easy scaling laws for the resulting wall(s) of water but don't invalidate my claim that the fundamental principles are well understood.

Tsunamis can be generated by any long-wavelength disturbance of the ocean surface, either directly (such as by impacts from space) or by disturbing the ocean floor through landslides or undersea volcanoes or large earthquakes. The largest earthquakes often occur where the ocean floor is being carried down into Earth's interior. During a large earthquake beneath the ocean floor, the floor is displaced both vertically and horizontally. Although horizontal displacements are often larger, they are unimportant for tsunami generation except to the extent that the sloping ocean floor also forces a vertical displacement of the water column. Upward displacements in one area are approximately balanced by downward displacements elsewhere, because Earth is close to incompressible, so the wave troughs are as important as the crests. The displacement of the water happens rapidly relative to the time it would take for a wave to disperse the resulting ocean surface displacement. Even though the tsunami speed may seem fast, it is slow compared to the time scale of the earthquake rupture.

Earthquakes

One often hears talk of an earthquake epicenter, but in a very large earthquake the net ground displacement is not confined to a small region. There are three length scales to consider. The largest of these is the length of the rupture, call it L. In the great Sumatran earthquake, L was about 1200 km and extended along a gently curving line, roughly North-South, which follows the plate boundary. Earthquakes begin at a particular point on the rupture surface and then propagate, like the propagation of a crack, though they typically take advantage of a zone of previous rupture. The rupture speed is a few kilometers per second or less, somewhat less than the propagation speed of shear waves in rock. The intermediate length scale, call it W, is the width of the rupture zone and is perpendicular to L but in the plane of the rupture surface. In December's quake, W was variable, but averages to about 150 km, roughly in the East-West direction. The area $A = LW$ is of great importance in defining the magnitude of the quake. The smallest but nonetheless important length scale is d, the net displacement that occurs on the rupture surface during the quake. It was up to around 10 to 20 m in the December quake, but varied along the rupture surface. In the thrusting motion that accompanied the Sumatran quake, the displacement was perpendicular to L and parallel to W so that points tens of kilometers to the East and West of the plate boundary actually came closer together by approximately 10 m during the quake.

I will simplify the discussion by talking only of one characteristic large length scale $A^{1/2}$, of order a few hundred kilometers in December's quake. From the shear modulus, μ, of the rock, we can construct a typical stress $\mu d/A^{1/2}$. It does not vary greatly from large to small earthquakes and is usually around 100 bars or even less, about four orders of magnitude smaller than the shear modulus. In other words, large and small earthquakes differ primarily in the size of the area of rupture A and not in the stresses or stress drop. A rough measure of the volume in which the stress is stored is $A^{3/2}$. Since the stress and strain do not vary much from large to small earthquakes, it follows that the energy associated with the quake scales as that volume $A^{3/2}$. In the language of acoustics, seismic waves are quadrupolar (with no net force or torque) and their amplitude in the far field of the wave source is proportional to $A^{1/2}d$ or equivalently A, since d is proportional to $A^{1/2}$. The standard magnitude scale (often still called the Richter scale) is based on the base ten logarithm of that amplitude, so there is a factor of $10^{3/2}$ or roughly a factor of 30 increase in energy per unit of earthquake magnitude. That factor can also be understood in terms of the longer wave train (lower frequencies) created in an earthquake of large magnitude.

The Sumatran earthquake was eventually assigned a magnitude of 9.0 by the US Geological Survey, but seismologists do not entirely agree on that value because of the complicated and extended nature of the rupture.

Immediately after the quake, the ocean surface was disturbed over roughly the same area A as the area of the rupture surface. The gravitational energy gained by creating a surface ocean disturbance of amplitude h (but mean of zero) is $1/2\rho g h^2$ per unit area. Since h is proportional to d, the energy of the tsunami scales roughly as Ad^2 and this scaling factor increases 100-fold for one unit increase of earthquake magnitude, so tsunami energy increases even more rapidly than earthquake energy as one increases the earthquake magnitude. The energy in a tsunami is still considerably lower than the energy in the earthquake that created it, even if one assumes $h = d$, because $\rho g A^{1/2}/\mu$ is less than 1. The wave propagates away from the initially disturbed area, to East and West in this instance, and the polarity of the wave (whether the first arrival is a crest or a trough) depends on the location of the affected coastline relative to the original disturbance. There is no overall preference for the first arrival to be positive or negative. The recent tsunami was the first to be directly observed f rom space, and its waveform is a wonderful example of the simple dynamics I've discussed.

Earth Ringing and Wobbling

Earthquakes as big as the one in Sumatra set Earth ringing. At the time of the 1960 Chile quake, our ability to study the ringing was limited, but a number of detectors were already in place to study Earth's normal modes. That quake provided the impetus for studying them

Instrumentation is now sufficiently sensitive and broadband that the detection of normal modes is easy The December earthquake is a boon for assessing the behavior of these modes and how they decay over time About 20 years ago, predictions were made for the change in Earth's rotation and wobble arising from large earthquakes For the Sumatran quake, the predicted spin-up of Earth is about one part in 10^{11} in angular velocity, which corresponds to a decrease in the length of day (LOD) by a few microseconds Since Earth's spin angular momentum must be conserved in such an event, the decrease requires a corresponding fractional decrease in Earth's polar moment of inertia Large earthquakes do not necessarily speed up Earth rotation—the 1964 Alaskan earthquake was predicted to increase the length of day Unfortunately, the predicted effects have not been detected because of other larger known effects involving angular momentum transfer among solid Earth, the oceans, the atmosphere, and Earth's liquid core There is also a steady tidal background increase in LOD of around 10^{9} per year as Earth's spin angular momentum is transferred to the Moon's orbital angular momentum

If Earth's great earthquake zones were randomly distributed, then there would be no net tendency for great earthquakes to spin up Earth However, the zones are not randomly distributed and their gravitational effects mean that Earth is not randomly oriented By the theory known as true polar wander (Euler's equations with a small amount of dissipation), Earth always migrates toward the state in which the axis of maximum moment of inertia coincides with Earth's rotation axis As a result, the cumulative effect of many large earthquakes tends to spin up Earth However, the work done by convection between earthquakes keeps everything in balance

Why Don't We Know it All?

Why are earthquakes less well understood than tsunamis? One way to appreciate the difference is to ask Can we write down their equations? Tsunamis have well-understood equations The seismic waves produced by earthquakes are also well understood But the earthquake itself—the rupture process, the energetics (both elastic and gravitational), and the regional stress balance—do not have an equation So tsunamis and ëarthquakes provide interesting physics in different ways, illustrating and exercising principles we know and love but also demonstrating how far we have to go to understand some of the complex phenomena that lie at the interface of materials science, continuum mechanics, and the behaviour of planets

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MODELING THE SUMATRA-ANDAMAN EARTHQUAKE REVEALS A COMPLEX, NONUNIFORM RUPTURE

[Reprinted with permission from American Institute of Physics from "Physics Today", v 58, no 16, June 2005, pp 19-21, for the benefit of our readers — Editor]

Data from a global network of seismometers were available within minutes of last December's Sumatra earthquake. Constructing a detailed self-consistent picture of where, when, how fast, and how much the sea floor moved has taken months.

What is the appropriate scientific response to a human tragedy? Thorne Lay of the University of California, Santa Cruz asked himself and his colleagues that question in the days following the 26 December 2004 earthquake off the coast of Sumatra Like the rest of us, he and other geophysicists saw disturbing images of thousands of bodies floating in the devastation from the tsunami A flurry of e-mails and a New Year's Eve conference call from Lay to his colleagues soon initiated a collective effort from the seismological community to analyze what happened Their hope was to replace the usual race to publication among competing groups with a more concerted response a single account that would provide a complete and robust characterization of the earthquake

That account, now published in a collection of three papers in Science,[1–3] coauthored by 40 researchers from 23 universities and institutes in 7 countries, confirms that the Indonesian earthquake was indeed astonishing-the largest anywhere in 40 years A thousand kilometers from the fault zone, the ground in Sri Lanka vibrated with