The Study on the Presence of Volatility Smile in NSE Indices

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Abstract

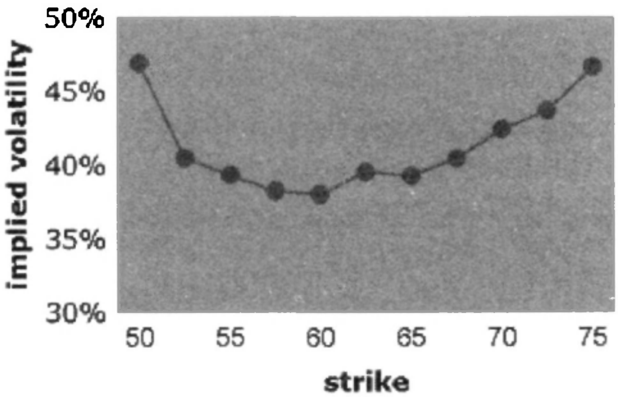
Whenever I see your smiling face

I have to smile myself

James Taylor "Your Smiling Face"©1977 Country Road Music, Inc.

If James Taylor had been an options trader, he might well have not written that second line. Options "smile" at traders, but that smile is not a source of happiness. It is in fact a source of considerable confusion and misunderstanding. In this paper, we explore how this confusion manifests in the pattern of different implied volatilities for different options on the same stock, a pattern commonly known as the volatility smile.

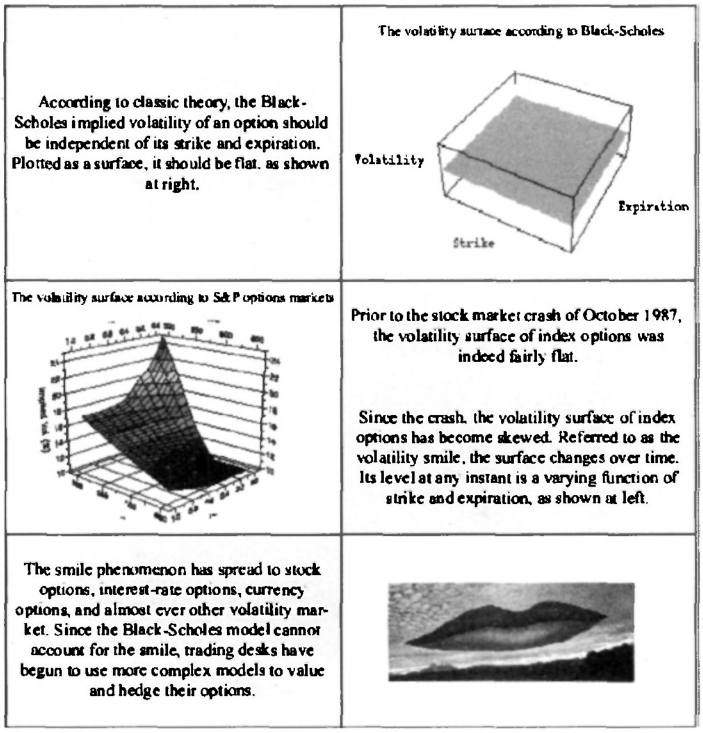
What is volatility smile? If you plot the implied volatilities of all the strike prices of options of a particular maturity (say, the December contracts), the graph will be approximately U-shaped.



On first glance, it will look like a smile. Hence, the name volatility smile.

Such a smile exists because the implied volatility of in-the-money (ITM) and out-of-the-money (OTM) options is higher than the at-the-money (ATM) options. This means that the investors are willing to pay a higher price to buy the OTM and ITM options. The basic assumption of the B&S model is that stock price returns follow a normal distribution (ND). That is, the stock price returns form a bell-shaped curve if plotted on a graph.

But this assumption is not true. The distribution of stock price returns has a fatter tail than a ND. This means profits and losses can be higher than what you can expect if the returns indeed follow an ND. The implication is that the OTM options are more likely to become the ITM options, because extreme stock price movements are possible. But such extreme price movements also mean that stocks can decline sharply, which is why ITM options are also preferred. Naturally, investors will be willing to pay a higher price for these options. And the higher price translates into higher implied volatility. Hence, the volatility smile.



The smile first appeared in options markets in 1987.

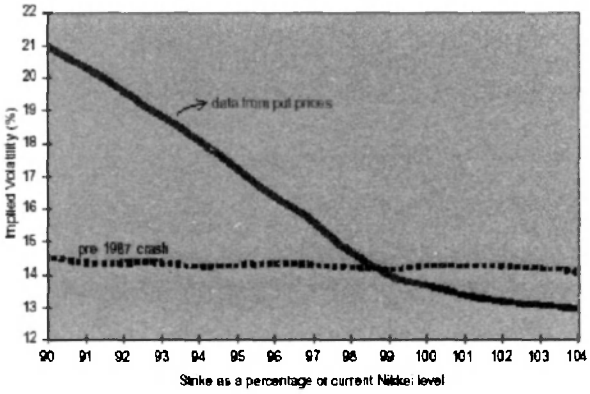


FIGURE 1. A typical implied volatility smile for the three-month options on the Nikkel index in late 1194.. The dotted lines shows the lack of skew that was common prior to the 1987 crash.

This skew or asymmetry in the implied volatilities had been absent before the infamous stock market crash of Oct. 19 1987, and had begun to appear shortly afterwards, in index markets all over the world. A similar but not identical pattern held for all expirations, so that it became common to speak of an implied volatility surface whose height varied as a function of strike and expiration.

Figure 2a shows a typical surface for S&P 500 options. The shape is fairly typical, but the exact details change from moment to moment and day to day. The lack of flatness came to be called the volatility smile, because in the currency world it really did resemble a slight upturned curve of the lips. In fact, for indexes, the skew was described as negative, since volatilities were anti-correlated with strike prices.

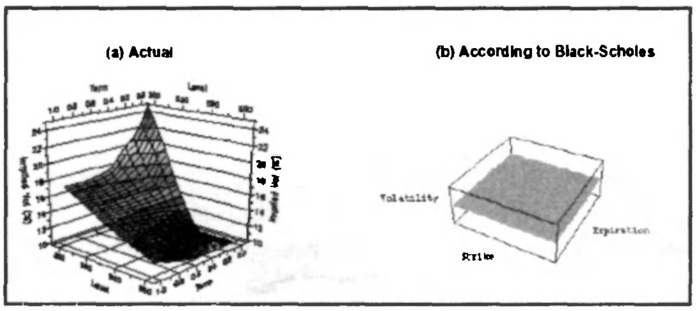


FIGURE 2. The implied volatility surface for $&P 500 options

These surfaces and curves were a challenge to theorists everywhere. The classic Black-Scholes model attributed a single lognormal volatility to an underlier at all times and all levels and therefore predicted a dull featureless plateau-like implied volatility surface, as shown in Figure 2b. According to Black-Scholes, options of all strikes should have the same volatility. But according to the smile, each option reported a different volatility for the same underlier.

What was wrong with Black-Scholes and what kind of new model could possibly match and explain this skewed surface? This wasn't just an intellectual challenge, but one of importance to the business too. Our equity derivatives desk made markets in index and single-stock options all over the world. Even if we knew the market price of a liquid option, we needed a model to hedge it. If the Black-Scholes model couldn't account for an implied volatility, it couldn't produce a reliable hedge.

The smile first appeared after the 1987 crash and was clearly connected in some way with the visceral shock of discovering, for the first time since 1929, that a giant market could drop by 20% or more in a day. Clearly low-strike puts should be worth more than high-strike calls when you thought about the higher probabilities associated with that kind of move. Over the next 15 years the volatility smile spread to most other options markets, but in each market it took its own idiosyncratic form. Slowly, and then more rapidly, traders and analyst in every product area had to model the smile.

When the volatility smile was first observed, some researchers believed that the explanation was liquidity. The true "smile" appearance meant that out-of-the-money options had the highest implied volatilities. These options were also the least liquid; hence, it was argued that the prices observed for these options of low liquidity reflected the thinness of their markets. But this explanation would suggest that highly liquid options – typically those trading nearly at-the-money – would have the same implied volatilities. In fact, they do not and never did. Moreover, when the smile turned into a skew, the moneyness argument fell by the wayside.

Other researchers believe that the smile reflects stochastic volatility. Volatility is surely not constant as assumed in the Black-Scholes model. If volatility is stochastic, researchers argue that the smile reflects the failure of the Black-Scholes model to capture the random nature of volatility. Others argue that the Black-

Scholes model, which assumes that stock prices fluctuate in a smooth and continuous manner, fails to capture the true nature of stock price movements, which are observed to have discrete jumps.

OBJECTIVES OF THE STUDY

The objective of the project is to find out whether volatility surfaces exist in case of NSE Nifty options, CNX IT Index Options and NSE Bankex Options. While investigating the existence of volatility surfaces for the SC options, this study attempts to answer the following questions:

- Whether implied volatility varies across different exercise prices and time to maturity.
- Whether implied volatility is higher for in-the-money call options (out of the money put) or for out of the money call (in the money put).
- Whether implied volatility is more for near the month options or for far the month option contracts.
- Whether for the same exercise price and time to maturity, implied volatility is more for call options or for put options.
- Whether implied volatility is higher for more liquid options or for less liquid options.

Model

To answer the above questions, the final model which has been considered for the present study is:

$$IV_{iT,t} = \alpha + \beta \left| \frac{S_t - X_{iT,t}}{S_t} \right| + \gamma D_1 + \delta T_{i,t} + \lambda D_2 + \mu \left| \frac{S_t - X_{iT,t}}{S_t} \right| * T + \theta NOC_{iT,t} + U$$

Where:

$IV_{iT,t}$: Implied volatility of an option with an exercise price of Xi and time to maturity of T on trading day t.

S_t : Closing value of NSE Nifty, CNX IT Index Options and NSE Bankex Options on trading day t.

$X_{iT,t}$	:	ith exercise price with time to maturity of T available for trading on day t.
$T_{i,t}$	:	Time to maturity of an option with an exercise price of Xi on day t.
D_1	:	D1 = 0, if call option (put option) is out of the money (in the money) on day t (that is, $St - XiT,t < 0$).
		D1 = 1, if call option (put option) is in the money (out of the money) on day t (that is, if $St - XiT,t < 0$)
$NOC_{iT,t}$	:	Number of NSE Nifty options with an exercise price of Xi and time to maturity of T traded on day t.
D_2	:	D2 = 0, if option is put.
		D2 = 1, if option is call.
U	:	Random disturbance term.

$|\frac{S-X}{X}|$ measures the extent to which an option is in the money or out of the money.

If estimated β is positive and significant it means that the deeply in the money and out of the money options are having higher implied volatility than at the money options. If estimated β is negative and significant, it means that narrower the gap between actual value of index and exercise price, higher the implied volatility.

If along with positive and significant β, estimator of γ is positive and significant, it means that though in the money and out of the money options are having higher volatility than at the money options but with the same degree to which the option is in the money or out of the money, then in the money call (out of the money put) options are having higher volatility than out of the money call (in the money put) options.

Positive and significant estimator of δ will indicate that higher the time to maturity of the option, higher the implied volatility. That is, near the month options are having higher volatility than far the month options for the same exercise price and Nifty value. If estimated ä is negative and significant, it indicates that near month option contracts have higher volatility than far the month option contracts.

Positive and significant λ will indicate that for the same exercise price and time to maturity, call options are having higher volatility than put options. Negative λ will mean that put options are having higher volatility than call options.

If estimator of μ is positive and significant, it means that with the same degree of moneyness, long-term options are having higher volatility than short-term options. Negative μ will show that short-term options are having higher volatility than long-term options (with the same degree of moneyness).

If estimated θ is positive and significant, it means that options which are more liquid are having higher volatility than options which are less liquid. Negative estimated θ will indicate that less liquid options have higher volatility than more liquid options.

The model discussed above has been tested for NSE Nifty options as supported by the analysis in the subsequent sections. This follows in the following sections.

METHODOLOGY

Data

The basic data for this study have been collected from www.nseindia.com, an official website of National Stock Exchange. The existence of volatility surfaces has been investigated using daily data on exercise prices available for trading; value of NSE Nifty, CNX IT Index Options and NSE Bankex Options; call premium for different exercise prices and time to maturity; put premium for different exercise prices and time to maturity; time to maturity for different exercise prices available for trading; and number of contracts traded for different exercise prices and time to maturity. Volatilities implied from the market prices for different exercise prices and time to maturities have been computed using DerivaGem software provided by Hull (2002).

To investigate the existence of volatility surfaces, the sample carrying one year time period from 1st January 2005 to 5th December 2005 has been chosen. From 1st January 2005 to 5th December 2005, there were total 233 days available for trading and the number of observations for which trading was available with different exercise prices and/or time to maturity were 22,441 for each call and put option (Total observations =44,882) . On an average, there were 80 observations per day for each call and put options (total 160) for which trading was available for different exercise prices and/or time to maturity. In case of NSE Bankex, the sample consists of 19,671 observations from 13th June, 2005 to 5th December, 2005.

At any point of time, there were only three contracts available with 1 month, 2 months and 3 months to expiry. The expiry date for these contracts is last Thursday of expiry month and these contracts have a maximum of three months expiration cycle. A new contract is introduced on the next trading day following the expiry of the near month contract. On the date of the start of the new option contract, there are minimum of seven exercise prices available for trading – three 'in the money', one 'at the money' and three 'out of the money' for every call and put option. The new exercise prices can be added in between for each contract.

Interpretation

Nifty Index Options

Regression Statistics	
Multiple R	0.524975283
R. Square	0.275599048
Adjusted R Square	0.275497522
Standard Error	0.18804919
Observations	42818

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	6	575.9662114	95.99437	2714.581105	0
Residual	42811	1513.903897	0.035362		
Total	42817	2089.870108			

	Coefficients	*Stadard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	0.264273733	0.003149414	83.91202	0	0.25810082	0.270446646	0.25810082	0.270447
IS-X/SI	1.998498771	0.041644016	47.99006	0	1.916875694	2.080121848	1.916875694	2.080122
D1	-0.167850011	0.001865369	-89.9822	0	-0.17150617	-0.164193852	-0.17150617	-0.16419
T	0.177472823	0.014919323	11.8955	1.40589E-32	0.148230661	0.206714986	0.148230661	0.206715
IS-X/SI*T	-9.285017439	0.257848694	-36.0096	9.1079E-280	-9.790405867	-8.77962901	-9.790405867	-8.77963
D2	-0.084764315	0.001865488	-45.4381	0	-0.088420709	-0.081107922	-0.088420709	-0.08111
NOC	-4.86877E-06	9.08439E-07	-5.35949	8.38871E-08	-6.64933E-06	-3.08821E-06	-6.64933E-06	-3.1E-06

- From the snapshot we can see that the value of beta (β) is 1.998. The P-Value corresponding to this value is 0 implying thereby that the value of beta obtained from the regression is highly significant even at 0% level. Since the beta (β) is positive and significant deeply in the money and out of the money options are having higher implied volatility than at the money options. This implies that the existence of volatility smile in Nifty index options has been substantiated.

- The value of γ is negative (-0.168) and is also significant even at 0% level. This means that although both in the money call (put) options and out of the money call (put) options are having high volatility but amongst the two out of the money options are having higher volatility as compared to in the money options.
- The value of δ is positive with its value being 0.1775 and is highly significant. This indicates that that higher the time to maturity of the option, higher the implied volatility. That is, far the month options are having higher volatility than near the month options for the same exercise price and Nifty value.
- We can see from the above results that the value of λ is -0.848. The P-Value corresponding to this value is 0 implying thereby that the value of λ obtained from the regression is highly significant even at 0% level. The negative and significant λ shows that put options are having higher volatility than call options.
- The value of μ is -9.285 and is highly significant as indicated by the P-value. Negative and significant μ shows that short-term options are having higher volatility than long-term options (with the same degree of moneyness).
- The value of θ is -0.000004869 and it's P-value shows that it is highly significant. The negative θ indicates that less liquid options have higher volatility than more liquid options.

CNX IT Index Options

Regression Statistics	
Multiple R	0.635909349
R. Square	0.4043807
Adjusted R Square	0.404297223
Standard Error	0.313891299
Observations	42818

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	6	2863.753183	477.2921972	4844.241425	0
Residual	42811	4218.071409	0.098527748		
Total	42817	7081.824593			

	Coefficients	*Stadard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	0.769433853	0.005021799	153.2187572	0	0.759591029	0.779276677	0.759591029	0.779276677
IS-X/SI	0.982437268	0.066745872	14.71907157	6.40662E-49	0.851614068	1.113260468	0.851614068	1.113260468
D1	-0.4303544	0.003136663	-137.2013543	0	-0.43650232	-0.42420648	-0.43650232	-0.42420648
T	-1.044623202	0.024360057	-42.88262615	0	-1.092369384	-0.99687702	-1.092369384	-0.99687702
IS-X/SI*T	-7.177867054	0.427692993	-16.78275579	5.19282E-63	-8.016153592	-6.339580515	-8.016153592	-6.339580515
D2	-0.057584604	0.003171529	-18.15673247	2.14394E-73	-0.063800862	-0.051368346	-0.063800862	-0.051368346
NOC	-0.008950693	0.004502305	-1.988024741	0.046815307	-0.017775298	-0.000126089	-0.017775298	-0.000126089

- The above results indicates that the value of beta (β) is 0.9824 and it's P-value indicates that it is highly significant, showing that deeply in the money and out of the money options are having higher implied volatility than at the money options. This implies that the existence of volatility smile in IT index options has been substantiated.
- The value of γ is negative (-0.430) and is also significant even at 0% level. This means that although both in the money call (put) options and out of the money call (put) options are having high volatility but amongst the two out of the money options are having higher volatility as compared to in the money options.
- The value of δ is negative with the value being -1.045 and its significance level at 0% is showing that the value is highly significant. Thus, we can conclude that near month option contracts have higher volatility than far the month option contracts.
- The value of λ is -0.058 and it's P-value shows that it is highly significant. It means that put options are having higher volatility than call options.
- The value of μ is -7.178 and is significant as indicated by the P- value. This means that short-term options are having higher volatility than long-term options (with the same degree of moneyness).
- The value of θ is -0.0089 and is significant at 5% level which means that the value is highly significant. The negative θ indicates that less liquid options have higher volatility than more liquid options.

Bankex Options

Regression Statistics	
Multiple R	0.642471314
R. Square	0.41276939
Adjusted R Square	0.412709675
Standard Error	0.310316804
Observations	59010

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	6	3993.773952	665.628992	6912.285	0
Residual	59003	5681.783515	0.096296519		
Total	59009	9675.557467			

	Coefficients	*Stadard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	0.748694932	0.004358198	171.7900102	0	0.740152845	0.757237018	0.740152845	0.757237018
IS-X/SI	0.42422027	0.048281134	8.786460356	1.58E-18	0.329589048	0.518851493	0.329589048	0.518851493
D1	-0.466572102	0.002651012	-175.9977474	0	-0.471768096	-0.461376108	-0.471768096	-0.461376108
T	-1.046119999	0.021602554	-48.42575657	0	-1.088461094	-1.003778905	-1.088461094	-1.003778905
IS-X/SI*T	-3.736427049	0.309783007	-12.06143323	1.85E-33	-4.343603023	-3.129251076	-4.343603023	-3.129251076
D2	-0.015441346	0.002798456	-5.51780875	3.45E-08	-0.020926332	-0.009956361	-0.020926332	-0.009956361
NOC	-0.024519252	0.005173378	-4.739505719	2.15E-06	-0.034569094	-0.014379411	-0.034659094	-0.014379411

- The value of beta (β) is 0.424 and is highly significant showing that deeply in the money and out of the money options are having higher implied volatility than at the money options.
- The value of γ is negative (-0.467) and is also significant even at 0% level. This means that although both in the money call (put) options and out of the money call (put) options are having high volatility but amongst the two, out of the money options are having higher volatility as compared to in the money options.
- The results indicate that the value of δ is negative i.e. -1.046 and it's P-value shows that it is very significant even at 0% level. This means that near month option contracts have higher volatility than far the month option contracts.
- The value of λ is -0.015 and the P-value shows that it also very significant at a very high level. Therefore, it can be concluded that put options are having higher volatility than call options.
- μ value is -3.736 and it's P-value shows that the value is significant. The negative and significant μ means that the short-term options are having higher volatility than long-term options (with the same degree of moneyness).
- The estimator θ carries the value -0.025 and is also significant as shown by it's P-value. The negative and significant θ means that less liquid options have higher volatility than more liquid options.

CONCLUSION

The overall results of the study show that:

Deeply in the money and deeply out of the money options are having higher implied volatility than at the money options, thereby indicating the existence of volatility smile in all the three index options.

Deeply out of the money call options are having higher volatility than deeply in the money call options. Thus, implied volatility is the highest in case of out of the money call (in the money put) options and the lowest in case of at the money options.

Implied volatility is higher for near the month option contracts than far the month option contracts.

Deeply in the money and deeply out of the money options with shorter maturity are having higher than deeply in the money and deeply out of the money options with longer maturity.

For the same degree of moneyness and time to maturity, put options are having higher volatility than call options. Thus, there exists arbitrage opportunities in case of NSE Nifty options.

High liquid options are having higher implied volatility than less liquid options.

The main reason why deeply in the money and out of the money options are having higher implied volatility than at the money options may be that deeply in the money and deeply out of the money options lack liquidity and the seller of these options demand liquidity premium (and thus, assume higher implied volatility) to price deeply in the money and out of the money options.

Thus, the results show that the shape of the volatility smile in India is similar to that which was prevailing in US before the major stock market crash of 1987. The index options in India started in 2001 only and no major stock market crash has taken place after that. Whether the relationship between implied volatility and exercise price changes from smiley shape to sneer shape (indicating inverse relationship between implied volatility and exercise price) in the future in the event of a major stock market crash has to be seen!