Portfolio Optimization using Single Index Model

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Abstract

The study here focuses on portfolio construction of mid cap companies, its optimization and throws up some important topics for further research. Issues discussed include the key inputs necessary to perform portfolio optimization, Markowitz Model, Single Index Model and the methods for evaluation of overvaluation and undervaluation of securities.

An investment portfolio is a collection of investments. Portfolios can be made up of stocks, bonds, mutual funds, cash, life insurance policies, real estate, and other investments. Portfolio objectives center on the risk–return trade off between the investor's expected return and his risk tolerance.

Modern portfolio construction came into picture in order to reduce the risk involved in investment in a fewer number of securities or financial instruments. Since, Business Cycle have a direct impact on the performance of different firms, financial markets and on the economy, if one goes for investing only in a small number of securities or in few sectors it could be riskier as there may be the possibility that these sectors are more seriously in their recession phase.

To help balance potential losses, we need to diversify our portfolio by investing a percentage of our overall portfolio in different types of financial instruments, such as bonds, cash, and international investments that may produce good returns when the value of our stock holdings is down.

How we allocate assets, or distribute our money across major classes of investments, depends on several factors, such as our tolerance for risk, desire for growth, and number of years before we wish to liquidate our assets.

Portfolio construction and optimization

Portfolio construction is a stepwise process, which requires thorough analysis and research work on several factors in order to get expected results out of an efficient portfolio. Process of portfolio construction consists of the following steps:

1) Specifying asset classes to be included in the portfolio e.g.:
   a) Money market instruments
   b) Bonds
   c) Stocks
   d) Real estate
   e) Securities on the basis of market capitalization: Large cap companies, Mid cap companies, Small cap companies.

This study is focused on mid cap companies' portfolio construction. Institutional investor rarely invests in more than the first four categories whereas individual investors include precious metal and other exotic kind of investments also.

2) Defining capital market expectations:

This step includes both historical and thorough analysis to determine the future expected return of the assets which are to be included in the portfolio.

3) Finding the efficient portfolio frontier:

Maximum expected return for any given degree of risk. Here, Sharpe's measure can be applied to find out marginal change in reward (excess return) per unit of risk (volatility).

4) Determining the optimal mix:

This step consists of selecting the portfolio that best meets our risk and return objectives within the boundary of constraints. Here, Markowitz model of portfolio optimization can be applied.

Markowitz (1952, 1959) is the father of modern portfolio theory. Markowitz formulated the portfolio problem as a choice of the mean (expected return) and variance (risk) of a portfolio of assets. He proved the fundamental theorem of mean variance portfolio theory, namely holding constant variance and maximizing

expected return, and holding constant expected return and minimizing variance. These two principles led to the formulation of an efficient frontier from which the investor could choose his or her preferred portfolio, depending on individual risk return preferences.

Evaluation of efficient portfolio

After construction of an efficient portfolio it is important to evaluate the performance of the portfolio on a regular basis and to make necessary changes required in the portfolio due to changes in market conditions and macroeconomic factors. This process is referred to as Active Portfolio Management. For the purpose of active portfolio management the market index portfolio is considered as the baseline portfolio which the model treats as passive portfolio. Perceived mispricing of the analyzed securities is what guides the composition of this active portfolio.

After mean variance portfolio theory was first developed, there was an enormous amount of work on estimating inputs. For the first time in the literature of financial economics, estimates of correlation coefficients (or alternatively covariance) were required. The principal tool developed for estimating covariance was an index model. The earliest index model that received wide attention was the single-index model, and in particular one variant of the single-index model, the market model. This was first discussed in Markowitz, but was developed and popularized by Sharpe (1967). The market model is

$$R_i = \alpha_i + \beta_i(R_m) + e_i$$

R_i = Excess return on security

R_m = Excess market return

α_i = The stock's expected return if the market is neutral, that is if the market's excess return, r_m-$r_f(R_m)$, is zero.

$\beta_i R_m$ = The component of return due to movements in the overall market, b_I is the security's responsiveness to market movements.

e_i = The unexpected component due to unexpected events that are relevant only to this security (firm specific).

From the point of view of portfolio inputs, the important characteristics of the use of the market model was that the number of estimates required was reduced, the type of inputs needed were easier for the analyst to understand, and the accuracy of portfolio optimization was increased. Even when using historical data to estimate the market model, the accuracy of the market model in estimating covariances was higher than direct estimation.

According to this model each security has two sources of risk: market or systematic risk (sensitivity to macroeconomic factors) and firm specific risk (sensitivity to firm specific factors, for example: operational risk, credit risk, financial risk etc.).This model shows the relationship between excess security return and systematic risk as well as helps in finding out whether the security is undervalued or overvalued.

METHODOLOGICAL FRAMEWORK

Companies can be divided into three categories on the basis of their market capitalization:

Large cap companies: are those whose market capitalization lies above Rs 2000 crore.

These are blue-chip companies and the investment amount required to invest in these companies is pretty high because the share price of the stocks of these companies is comparatively higher.

Mid cap companies: are those whose market capitalization lies between Rs500 – Rs2000 crore. The stocks of these companies are generally undervalued and there is a great chance of gaining high returns due to mispricing of their stocks. These firms are also in their growth phase. These firms are having moderate risk with reasonable returns.

Small cap companies: These companies' market capitalization lies below Rs.500 crore. They are having high risk with high returns.

The study here is based on portfolio construction of mid cap companies. These mid –cap companies were selected on the basis of two parameters which were as follows:

1) Companies having asset base of Rs600- Rs1200 crores.

2) Companies having ROCE greater than 25%.

38 companies were selected on the basis of the above-mentioned conditions, the companies are:

- Alembic Ltd.
- Alok Industries Ltd.
- Amtek Auto Ltd.
- Asian Paints (India) Ltd.
- Bajaj Auto Finance Ltd.
- Bajaj Hindustan Ltd.
- Balrampur Chini Mills Ltd.
- Bannari Amman Sugars Ltd.
- Bayer Cropscience Ltd.
- Bharat Forge Ltd.
- Bongaigaon Refinery & Petro
- Castrol India Ltd.
- Colgate-Palmolive (India) Ltd.
- Core Healthcare Ltd.
- Crompton Greaves Ltd.
- Dabur India Ltd.
- Deepak Fertilisers & Petrochemicals Corpn. Ltd.
- Dhampur Sugar Mills Ltd.
- Eicher Motors Ltd.
- Exide Industries Ltd.
- G I C Housing Finance Ltd.
- Gillette India Ltd.
- Godavari Fertilisers & Chemicals Ltd.

- Godrej Industries Ltd.
- Gujarat Gas Co. Ltd.
- H C L Infosystems Ltd.
- Hindustan Construction Co. Ltd.
- IPCA Laboratories Ltd.
- J C T Electronics Ltd.
- Jubilant Organosys Ltd.
- K E C International Ltd.
- Kirloskar Brothers Ltd.
- Lakshmi Machine Works Ltd.
- Lupin Ltd.
- Mafatlal Industries Ltd.
- Nagarjuna Construction Co. Ltd.
- Nestle India Ltd.
- Nicholas Piramal India Ltd.

Markowitz portfolio optimization model was applied for portfolio construction and its further optimization was done on the basis of Spreadsheet model in Microsoft Excel with the help of a software package "Solver". Reward variability ratio was calculated on the basis of MARKOWITZ MODEL. Appropriate weights were allocated to all the mid-cap companies with the help of Excel Solver. The objective of the study was to maximize the Sharpe's ratio or reward to variability ratio holding variability as a constant. It helps in the construction of an efficient or optimized portfolio, as it eliminates no. of companies which are not able to contribute to the effective returns of the complete portfolio.

Then Single index model was applied on those companies, which remained there in the efficient portfolio, on the basis of their consistent performance over the time period considered for the analysis. With the help of this model the relationship between excess returns of these securities and systematic risk can easily be find out. This model also helps in finding out whether the security is undervalued or overvalued. So, we can easily trace out the basis for inclusion of these securities into the efficient portfolio.

MODELS

Markowitz model

HARRY MARKOWITZ introduced Modern portfolio theory (MPT). Prior to Markowitz's work, investors focused on assessing the risks and rewards of individual securities in constructing their portfolios. Standard investment advice was to identify those securities that offered the best opportunities for gain with the least risk and then construct a portfolio from these. Detailing mathematics of diversification, he proposed that investors focus on selecting portfolios based on their overall risk-reward characteristics instead of merely compiling portfolios from securities that are having attractive risk-reward characteristics. In other words inventors should select portfolios not individual securities.

If we treat single-period returns for various securities as random variables, we can assign them expected values, standard deviations and correlations. Based on these, we can calculate the expected return and volatility of any portfolio constructed with those securities. We may treat volatility and expected return as proxies for risk and reward. Out of the entire universe of possible portfolios, certain portfolios will optimally balance risk and reward. These comprise what Markowitz's called an efficient frontier of portfolios. An investor should select a portfolio that lies on the efficient frontier.

Expansion of Markowitz's work was done by adding a risk-free asset to the analysis. This lead to the notion of a super efficient portfolio and the capital market line. The capital allocation line provided by risk-free asset and a broad index of the common stock is called the capital market line. Through leverage, portfolios on the capital market line are able to outperform portfolios on the efficient frontier.

Some possible risk adjusted performance measures are

1) **Sharpe's measure**: Sharpe's measure divides average excess portfolio return over the sample period by the standard deviation of returns.It measures the reward to volatility trade-off.

$$(Er_p - r_f)/\sigma_p$$

2) **Treynor's measure**: Treynor's measure gives excess return per unit of risk,but it uses systematic risk instead of total risk.

$$(Er_p - r_f)/\beta_p$$

3) **Jensen's measure(portfolio alpha)**: Jensen's measure is the average return on the portfolio over and above that predicted by the CAPM ,given the portfolio's beta and the average market return.

$$\alpha = Er_p - [r_f + \beta_p(Er_m - r_f)]$$

Here:

Er_p = Average expected return of portfolio

r_f = Risk free rate or T-bill rate

σ_p = Volatility or standard deviation of portfolio

β_p = Systematic risk or sensitivity to market movements

Er_m = Average market return

α = Unique excess return of security

Single index model

The earliest index model that received wide attention was the single-index model, and in particular one variant of the single-index model, the market model. This was first discussed by Markowitz, but was developed and popularized by Sharpe (1967). The market model is

$$R_{it} = \alpha_i + \beta_i R_{mt} + e_{it}$$

$$Er_i - r_f = \alpha_i + \beta_i(Er_m - r_f) + e_i$$

Where:

R_{it} = the excess returns of stock i in period t

α_i = The unique expected return of security i if market excess return is zero

β_i = The sensitivity of stock i to market movements

R_{mt} = The excess return of the market in period t

e_{it} = Is the unique risky return of security i in period t.

$\beta_i(Er_m - r_f)$ = Component of return due to movement in market index

In the index model we use excess return over risk free rate (r_f) rather than total returns because the level of the stock market return represents the state of the macroeconomy only to the extent that it exceeds or falls short of the rate of return on the risk free treasury.

e_f = Firm specific component, not due to market movements

Expected value of e_i is taken as zero, therefore when market is neutral i.e. market return $= r_f$, then $= Er_i - r_f = \alpha_i$

$$R_i = \alpha_i + \beta_i R_m + e_i$$

R_i = excess return

As the number of stocks included in the portfolio increases, the part of the total portfolio risk attributable to non-market factors becomes even smaller. In contrast the market risk remains, regardless of the number of firms combined into the portfolio. Law of averages can be applied to conclude that as more and more stocks are added to the portfolio, the firm specific components tend to cancel out resulting in negligible firm specific risky returns.

Therefore for a well diversified portfolio if $e_p = 0$; then

$$R_p = \alpha_p + \beta_p R_m$$

The index model is estimated by applying regression analysis to excess rates of return. The slope of the regression curve is the beta of an asset, whereas the intercept is the asset's alpha during the sample period. The regression line is also called the security characteristic line. Estimating the regression equation of the Single index model gives the security characteristic line (SCL) i.e.

$$R_i = \alpha_i + \beta_i R_m$$

Regression model

Regression equation describes the average relationship between a dependent variable and a set of explanatory variables.

Suppose there is a linear relationship between the excess return of any asset and the market index. A linear relationship, one that can be described by a straight line, takes on this form

$$R_i = \alpha_i + \beta_i R_m + e_i$$

Where, subscript i represents any asset, m represent market index. On the left hand side of the equation is the dependent variable i.e. excess return of any security. The right hand side has two parts, the explained and unexplained components of the dependent variable. Explained component reflects returns due to market specific risk or systematic risk and unexplained component reflects returns due to firm specific risk or unsystematic risk.

If we denote the variance of the excess return on the market, R_m, as σ_m^2, then we can break the variance of the rate of return on each stock into two components:

1. The variance attributable to the uncertainty of the common macroeconomic factor i.e. $\beta_i^2\sigma_m^2$
2. The variance attributable to firm specific uncertainty: $\sigma^2(e_i)$

Hence the variance of the rate of return on security i equals the sum of the variance due to the common and the firm specific components:

$$\sigma_i^2 = \beta_i^2\sigma_m^2 + \sigma^2(e_i)$$

α = Intercept of the regression line. α helps in determining whether the stock is undervalued or overvalued as it represents the extra expected return attributable to any perceived mispricing of the security. The underpriced shares have positive α whereas of the overpriced shares is negative.

β = Slope of the regression line. β shows sensitivity of the stock towards market movements or macroeconomic factors. If value of β is greater than 1 (for example 1.5) this shows that the stock is more riskythan the market i.e. stock is aggressive, means if market moves by 10% then movement in the stock will be upto 15%. And if β is less than 1 (for example 0.8), it means the stock is less risky than the market or the stock is defensive. If there is 10% movement in the market it will result in change of just 8% in the firm's total return. On the other

hand if β=1, this shows that stock is at par with the market i.e. stock and market both will move simultaneously.

DATABASE

The data for analysis have been taken from different sites and databases. Company specific returns and returns of BSE sensex have been taken from CMIE Prowess database.91 days Treasury-bills are considered as a proxy for risk free rate of return and are taken from the database named Business Beacon.

EMPIRICAL RESULTS

COMPANY	ALPHA	BETA	FIRM SPECIFIC RISK	MARKET RISK
ASIAN PAINTS	0.89913	0.545611	6.4435	0.067470136
BAJAJ AUTO	2.536544	0.851592	8.6654	0.122001311
CASTROL INDIA	-0.48786	0.986241	9.6204	0.147298217
COLGATE PALMOLIVE	0.464001	1.01268	6.9916	0.212111791
EICHER MOTORS	4.074572	1.641561	16.1638	0.242912292
GUJARAT GAS	0.492297	1.445897	8.8426	0.340524446
JUBILANT	7.411864	1.282334	15.8537	0.151542851
LAXMI	3.398845	0.973194	15.2578	0.090855705
NESTLE INDIA	0.785682	1.427019	7.5494	0.386112784

Above analysis shows that companies which are having alpha values less than zero i.e. negative, they are overvalued in terms of their stock prices with respect to market price whereas companies which are having positive alpha are undervalued with respect to market price. In other words, we can say that there exists mispricing of the undervalued securities, so there is a great chance of gaining

from the mispricing of these company stocks when in the future the value of theses stocks will rise.

In the case of Asian Paints alpha is less than 0 and beta is 0.54 which is pretty less than 1, it is an undervalued stock as well as defensive with respect to the market but firm specific risk is high in comparison to market risk, thus an investor with lower risk aversion and moderate return expectations (because lower beta value shows that there will not be much fluctuation in the value of stock) can invest in the stock of this company. But it requires proper study of the fundamentals of the company, as the firm specific risk is much higher than the market risk.

Bajaj Auto's stocks are highly undervalued and beta is also, less than 1 which shows that stock is defensive with normal future returns. Here also, firm specific risk is pretty high.

In the case of Gujarat Gas and Nestle India the alpha is positive with value of beta much greater than 1, this indicates that the stock is highly aggressive but underpriced in the market. So, one can gain a lot by investing in this kind of stock as the stock is undervalued and beta value shows that it will move a lot in the future resulting in good returns. But the risk associated with these is also very high, since both firm specific risk and market risk have higher values.

Jubilant and Eicher Motors are the most attractive as well as most risky stocks among all of these companies because stocks of these companies are highly undervalued with higher beta, so if market will move up there will be a lot to earn but if market will go down the results will be disastrous.

Colgate Palmolive is an undervalued security but its beta is almost at par with the market. So, before investing in it one should thoroughly study the trends in the market in order to gain from the undervaluation of these stocks.

As we can see from the table that the firm specific risk of most of the companies is high, one should carry out a thorough research on the companies fundamental position, financial position, operating strategies, growth prospects, stage in industry life cycle, credit rating etc before investing in these companies. One can also, eliminate the firm specific risk by diversifying its portfolio through different stocks, sectors, industries etc.

CONCLUSION

This study has attempted to review modern portfolio theory and to highlight some topics and areas of future research. An important topic, that should be discussed here, is why we bother with performance. In a perfectly efficient market we would expect performance to be random over time. While some funds might outperform appropriate passive strategy and others underperform it, the difference would be strictly random over time. On the other hand, if superior management exists, then unless this performance is reflected in higher fees, we would expect to find persistence in performance.

The facts that mutual funds do not raise fees to reflect performance and that fees as a percent of assets tends to be lower for good performing funds mean that, if superior management exists, it should be reflected in persistence in performance. The fact that performance shows persistence suggests that at least some managers have superior information and give us hope that modern portfolio theory can be used to benefit the investor.

Therefore, we find that portfolio's performance evaluation is an important task from the point of view of a rational investor. In this paper we have found that Markowitz Model is an important model for the construction of an efficient portfolio and Active portfolio management and Single Index Model help us a lot in the proper evaluation of the efficiently constructed portfolio. With the help of these models we get to know about the degree of sensitivity of these stocks towards market movements and we can also simply figure out the overvaluation or undervaluation of stocks. All these analyses help an investor in taking right decisions to invest in right/profitable kind of securities in proper amount at appropriate time and with efficient management.