Projecting Teledensity in India using Logistic and Gompertz Curve Models

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Abstract

Just five years ago, India had only 36 million telephones for more than 1000 million people. From 2000-01 to 2005-06, India added more than 100 million new telephone subscribers. Presently, Indian telecommunication network with around 140 million telephone connections is one of the largest and the fastest growing network in the world. As a consequence, teledensity number of telephones per 100 inhabitants in the country has improved considerably. It has increased by more than 3.5-fold in last five years, from 3.55 in 2000-01 to 12.66 in 2005-06. The Government of India aims to raise the teledensity in the country to more than 40 in next five years. This study tries to verify whether teledensity in India could reach a level of around 40 by the year 2010-11. For this, we projected the future path of teledensity in the country using S-shaped logistic and Gompertz curve models. Annual data of teledensity from 1990-91 to 2005-06 have been used for the same. It is found that the teledensity in the country is likely to cross the mark of 40 in 2010-11.

Keywords: *Forecasting; Growth Curve Models; Teledensity; Telecommunications*

Introduction

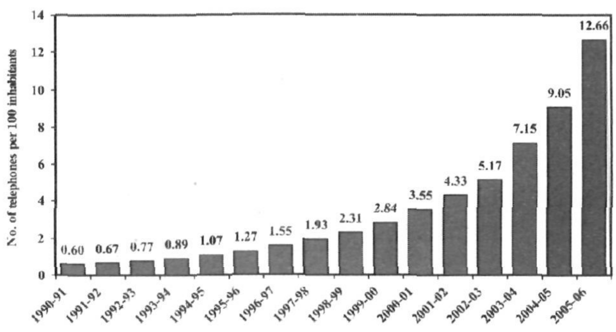
Teledensity, number of telephones per 100 inhabitants, has been used as an important indicator of economic prosperity of a country (see, for example, Hardy, 1980; Gille, 1986; Cronin et al., 1993; Saunders et al., 1994; and Mbarika et al., 2002). It has been shown that the teledensity is a reliable predictor of productivity of an economy (Cronin et al., 1993). While analyzing the teledensity in the French and Spanish economies, Berry (1983) found that the growth in teledensity precedes economic development and argued that the ultimate cost of underestimating the significance of teledensity would be quite high. Jussawalla (1988) supports the findings of Berry (1983) and claims that growth of teledensity promotes resource mobilization. Clarke and Laufenberg (1983) argued that a variety of social benefits (health and social service delivery, education, handling of natural and social disasters, etc.) in addition to economic benefits is positively correlated with the teledensity. There is no doubt that the teledensity is an important indicator of social and economic development of a country. Although growth and development of telecommunication services and teledensity has been exhaustively studied in developed countries, there are few such studies for developing countries, in general, and India, in particular. Therefore, it would be of interest to analyze the level and growth of teledensity in India and project the future path of the same.

The telephone industry in India has experienced an astonishing growth particularly during recent years. The growth in telephone subscriber base in the country has accelerated from around 20% per year during 1990s to more than 30% per year during last five years. Presently, Indian telecommunication network with around 140 million telephone connections is one of the largest and the fastest growing network in the world. As a result, teledensity in the country has improved significantly from 0.60 in 1990-91 to 12.66 in 2005-06 (Figure 1). It is interesting to note that the teledensity has increased by more than 3.5-fold in last five years. Rapid increase in teledensity during recent years is mainly the result of deregulation, liberalization, and competition introduced in the telecommunication market. In this regard, market oriented regulation that encouraged competition in terms of both the number of competitors and the breadth of services that they can offer has also played a key role. India's *Unified Licensing for Basic and Cellular Mobile Services* has enabled operators to use the most cost-efficient access technology. Competition among operators has resulted in rapid fall in telephone tariffs. The tariffs for local calls, particularly for cellular, have fallen considerably during recent time. The peak long distance tariff between Delhi and Mumbai has come down from Rs. 30 per minute in 2000 to less than Rs. 2.40 per minute in 2004. Recently, Mahanagar Telephone Nigam Limited (MTNL) has decided to provide telephone services between Delhi and Mumbai at the rate of local calls. In the same fashion, international call charges have also come down drastically from Rs. 61.20 per minute in 2000 to Rs. 7.20 per minute from April 2004 onwards for the American continent. Mobile telephony prices have dropped from Rs. 16 per minute to Rs. 1.20 per minute. The public sector operators Bharat Sanchar Nigam Limited (BSNL) and MTNL have launched the 'One-India Plan' with effect from March 1, 2006. This new plan will enable the customers of BSNL and MTNL to call from one end of India to other at the cost of Rs. 1.00 per minute, any time of the day to any phone. Presently, telephone usage charges in India are among the lowest whereas growth in subscriber base is among the highest in the world. Encouraged by the recent boom in telecom sector as well as to reap the economic and social benefits of telecommunication services, the government of India has set a target to achieve the country's teledensity of more than 40 in next five years. This study tries to verify whether teledensity in India could reach a level of around 40 by the year 2010-11. For this, we projected the future path of teledensity in the country using annual data from 1990-91 to 2005-06.

The technological diffusion literature shows that the spread of an innovation over time typically follows a sigmoid or *S*-shaped curve (see, Griliches, 1957; Mansfield, 1961; Artle and Averous, 1973; Mahajan and Peterson, 1985; Rogers, 1995; Barros and Cadima, 2000; Geroski, 2000; and Botelho and Pinto, 2004). In line with this, we hypothesize that the growth in teledensity over time in India follows an *S*-shaped curve. This hypothesis is based on the fact that adoption of telephone proceeds slowly at first, accelerates as it spreads throughout the potential adopting population, and then slows down as the relevant population becomes saturated. For a time series like teledensity, it is expected that the time series converge to a certain maximum as time passes by. There are a number of different functional forms that can describe *S*-shaped curves, for example, the logistic, Gompertz, logarithmic logistic, log reciprocal, simple modified exponential, general modified exponential, and cumulative normal functions. Among these, the logistic and Gompertz functions are the two most widely used functional forms. Therefore, this study uses these two functions to model and project the development of teledensity in India.

The study is organized into the following sections. Section 2 deals with the model to project future teledensity. Model estimation and projection of teledensity up to the year 2010-11 is presented in Section 3. Section 4 contains concluding remarks.

Figure 1. Teledensity in India from 1990-91 to 2005-06



The Model to Project Future Teledensity

As discussed in the previous Section, in any society, teledensity is expected to increase slowly at initial stage when only a few members of the social system opt for telephones whereas, over time, due to network externality, dissemination of information, reduction in prices, etc., it increases rapidly as many people opt for the services. Finally, during the maturing phase, its growth rate slows down as it converges to a certain maximum. Therefore, we hypothesize that the growth in teledensity over time in India follows an *S*-shaped curve. Although there are number of different functional forms that can describe *S*-shaped curves, this study uses the two most widely used functions logistic and Gompertz to model and forecast the development of teledensity in India.

2.1 Logistic Model

Teledensity can be represented by the logistic function as,

$$Td(t) = \frac{\alpha}{1 + \beta e^{f(t)}}$$

where $Td(t)$ is teledensity at time t (ranges from 0 to α as t ranges from $-\infty$ to $+\infty$), $f(t)$ is a function of t (ranges from $+\infty$ to $-\infty$ as t ranges from $-\infty$ to $+\infty$), and α is the saturation level. Parameters α and β are positive. To transform the model in a linear form, equation **(1)** can be written as,

$$\frac{\alpha}{Td(t)} - 1 = \beta e^{f(t)}$$

Taking natural logarithm on both sides, equation **(2)** can be transformed in a linear form as follows,

$$\ln\left(\frac{\alpha}{Td(t)} - 1\right) = \ln\beta + f(t)$$

This can further be simplified by making a reasonable assumption about the functional form of $f(t)$. Therefore, to estimate the model using Ordinary Least Squares (OLS) method, equation **(3)** can be written as,

$$\ln\left(\frac{\alpha}{Td(t)} - 1\right) = \beta_1 + \beta_2 t + \beta_3 t^2 + \varepsilon_t$$

where β_1, β_2, and β_3 are parameters to be estimated using OLS and ε_t is a disturbance term with zero mean and constant variance.

2.2. Gompertz Model

Gompertz function is of the form,

$$Td(t) = \alpha e^{-\beta e^{f(t)}}$$

where $Td(t)$ and $f(t)$ have their previous meaning and parameters α (saturation level) and β are positive.

In line with the logistic function, equation **(5)** can also be transformed in a linear form as follows,

$$\ln\left[\ln\left(\frac{\alpha}{Td(t)}\right)\right] = \ln\beta + f(t)$$

Equation **(6)** can further be simplified by making a reasonable assumption about the functional form of $f(t)$. Therefore, to estimate the model using OLS method, equation **(6)** can written as,

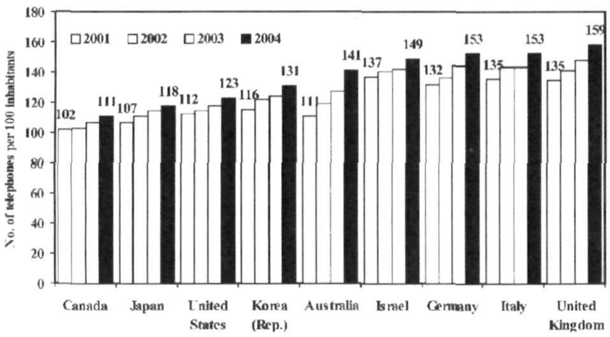
$$\ln\left[\ln\left(\frac{\alpha}{Td(t)}\right)\right] = \beta_1 + \beta_2 t + \beta_3 t^2 + \varepsilon_t$$

where β_1, β_2, and β_3 are parameters to be estimated using OLS and ε_t is a disturbance term with zero mean and constant variance.

Now, models **(4)** and **(7)** can easily be estimated by OLS method provided we know the saturation level, α. If we analyze the teledensity in developed countries, we find that the saturation level in India could be anywhere between 100 and 160 (Figure 2). Therefore, both models **(4)** and **(7)** should be estimated for different saturation levels (e.g., 100, 110, 120, 130, 140, 150, and 160 telephones per 100 inhabitants) not only to illustrate the different possible paths of teledensity but also to find out the most appropriate saturation level. The Mean Absolute Percentage Error (MAPE)[1] over the sample period could be used to find out the most appropriate model and saturation level. The F-test can be used to test the validity of restriction on coefficients (e.g., whether $\beta_3 = 0$ is valid or not can be tested using F-test).

The F-test statistic is calculated as $\frac{(R_{ur}^2 - R_r^2)/r}{(1-R_{ur}^2)/(N-k)}$ where R_{ur}^2 is the value obtained from the unrestricted regression model, is the value obtained from the restricted regression model, r is the number of restrictions imposed, N is the number of observations and k is the number of parameters in the unrestricted regression model. This statistic follows F-distribution with r, N-k degrees of freedom. If calculated F value is greater than the tabulated one, we reject the null hypothesis of restriction on coefficients. Therefore, MAPE and F-test should be used to find out the most appropriate saturation level and model to project the future teledensity in India.

Figure 2. Teledensity in selected developed countries



Model Estimation and Projection of Teledensity

Models **(4)** and **(7)** are estimated using the annual data of teledensity in India from 1990-91 to 2005-06 by OLS method. The variable is taken as 1 for 1990-91, 2 for 1991-92, 3 for 1992-93,......, and 16 for 2005-06. Both the models have been estimated for seven different saturation levels (100, 110, 120, 130, 140, 150, and 160) once by imposing restriction on $\beta_3 = 0$ and afterwards without imposing any restriction on β_3. According to F-test, the null hypothesis of is rejected for both the models for all the selected saturation levels from 100 to 160. Since $\beta_3 = 0$ for any model for any saturation level, Table 1 reports the estimation results for all the models without imposing any restriction on β_3. According to the R^2 values, models fit the data very well. All the estimated parameters have the expected sign and most are highly significant.

To choose the most appropriate model and saturation level, we also compared the predicted values with the actual value of teledensity over sample period. The MAPE, reported in Table 1, is in the range of 2.09 to 2.16 for the logistic models and 3.60 to 4.52 for the Gompertz models. According to both R^2 and MAPE, logistic models fit the data better than the Gompertz ones. Figure 3 presents the future teledensity trend in India up to the year 2010-11 using the estimated logistic models. It shows that the teledensity in 2010-11 would be somewhere between 40 and 44. Since according to both MAPE and R^2, teledensity of 150 appears to be the most appropriate saturation level for India, further analysis in this paper will primarily be based on the estimated logistic model at saturation level of 150 telephones per 100 inhabitants as shown in equation **(8)**.

$$Td(t) = \frac{\mathbf{150}}{1 + e^{\mathbf{5.6517 - 0.117}t - \mathbf{0.0052}t^2}}$$

$$\Rightarrow Td(t) = \frac{\mathbf{150}}{\mathbf{1 + 284.775}e^{-\mathbf{0.117}t - \mathbf{0.0052}t^2}}$$

where 1 is for 1990-91, 2 for 1991-92, 3 for 1992-93,......, and so on.

On the basis of the estimated logistic model for the saturation level of 150 telephones per 100 inhabitants, projected path of teledensity in India is presented in Figure 4. The analysis reveals that the inflection point (the point at which the growth rate is maximum) of the curve will occur after two years from now during the year 2007-08. This means that the rate of growth of teledensity will continue to increase till 2007-08 and decline in the same will start from 2008-09 onwards. Teledensity in India will cross the mark of 100 telephones per 100 inhabitants in 2015-16 whereas it will reach 95% of the saturation level in 2020-21. It is estimated that there will be 142.7 telephones for 100 people in 2020-21.

Based on the estimated teledensity as given in equation **(8)** and population estimates as given in the *World Population Prospects: The 2004*

Revision Population Database published by the United Nations Population Division, future telephone subscriber base in India is estimated and presented in Figure 5. Telephone subscriber base in India is expected to increase rapidly during the next five years at the rate of around 30% per annum. It is projected to increase from 140 million in 2005-06 to 513 million in 2010-11. This is in line with the expectation of the Department of Telecom, Ministry of Communications and Information Technology, Government of India. The Department of Telecom expects to have around 500 million telephone connections in India at the end of 2010. Furthermore, it is projected that India will have more than 1300 million telephone connections at the end of 2015-16.

Table 1.
Parameter estimates of the logistic and Gompertz models (with t-statistic in parentheses)

Model	Estimate
Saturation level, α = 100	
Logistic (4)	β_1 = 5.2388 (169.9), β_2 = -0.1143 (13.7), β_3 = -0.0055 (11.5); R^2 = 0.9989; MAPE = 2.16
Gompertz (7)	β_1 = 1.6282 (98.5), β_2 = -0.0062 (1.4), β_3 = -0.0030 (11.6); R^2 = 0.9958; MAPE = 4.52
Saturation level, α = 110	
Logistic (4)	β_1 = 5.3362 (176.0), β_2 = -0.1151 (14.0), β_3 = -0.0054 (11.6); R^2 = 0.9989; MAPE = 2.13
Gompertz (7)	β_1 = 1.6481 (105.4), β_2 = -0.0068 (1.6), β_3 = -0.0028 (11.7); R^2 = 0.9960; MAPE = 4.04
Saturation level, α = 120	
Logistic (4)	β_1 = 5.4249 (181.4), β_2 = -0.1157 (14.3), β_3 = -0.0054 (11.6); R^2 = 0.9988; MAPE = 2.16
Gompertz (7)	β_1 = 1.6659 (111.8), β_2 = -0.0073 (1.8), β_3 = -0.0027 (11.8); R^2 = 0.9962; MAPE = 3.89
Saturation level, α = 130	
Logistic (4)	β_1 = 5.5064 (186.3), β_2 = -0.1162 (14.5), β_3 = -0.0053 (11.6); R^2 = 0.9989; MAPE = 2.11
Gompertz (7)	β_1 = 1.6819 (117.8), β_2 = -0.0077 (2.0), β_3 = -0.0026 (11.9); R^2 = 0.9963; MAPE = 3.86
Saturation level, α = 140	
Logistic (4)	β_1 = 5.5817 (190.6), β_2 = -0.1166 (14.7), β_3 = -0.0053 (11.6); R^2 = 0.9990; MAPE = 2.14
Gompertz (7)	β_1 = 1.6964 (123.5), β_2 = -0.0080 (2.2), β_3 = -0.0026 (12.0); R^2 = 0.9964; MAPE = 4.17
Saturation level, α = 150	
Logistic (4)	β_1 = 5.6517 (194.6), β_2 = -0.1170 (14.9), β_3 = -0.0052 (11.6); R^2 = 0.9990; MAPE = 2.09
Gompertz (7)	β_1 = 1.7097 (128.8), β_2 = -0.0083 (2.3), β_3 = -0.0025 (12.1); R^2 = 0.9966; MAPE = 3.83
Saturation level, α = 160	
Logistic (4)	β_1 = 5.7172 (198.3), β_2 = -0.1173 (15.0), β_3 = -0.0052 (11.6); R^2 = 0.9990; MAPE = 2.11
Gompertz (7)	β_1 = 1.7219 (133.9), β_2 = -0.0085 (2.5), β_3 = -0.0024 (12.2); R^2 = 0.9967; MAPE = 3.60

Figure 3. Projection of teledensity in India using logistic models

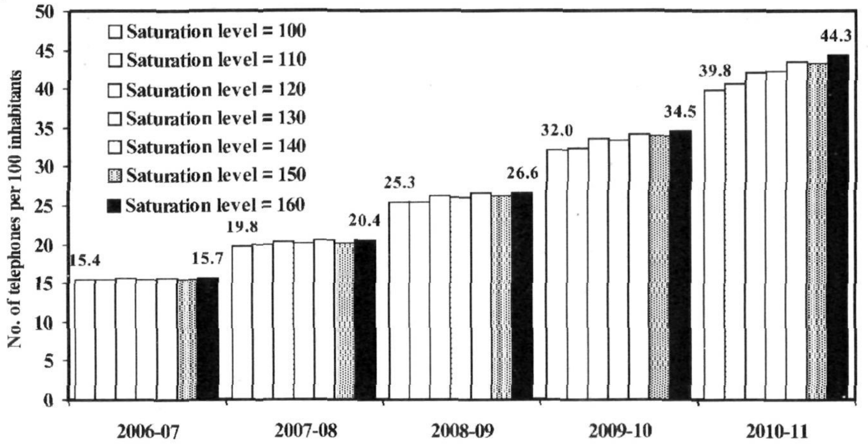


Figure 4. Projected path of teledensity in India (based on logistic model for saturation level of 150)

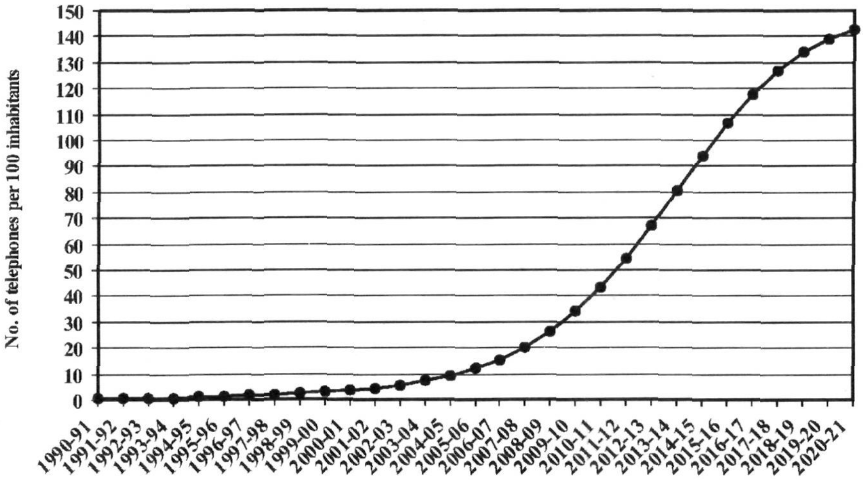
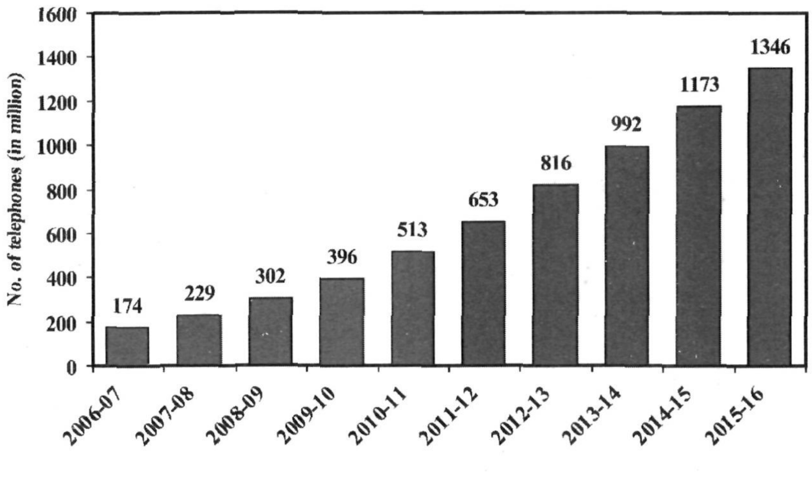


Figure 5. Future telephone subscriber base in India



Concluding Remarks

Telephone industry in India has witnessed the most fundamental structural and institutional reforms since 1991. The industry is now open to unrestricted competition in almost all segments. The opening of the industry has not only stimulated the growth but also helped a great deal towards maximization of consumer welfare. The tariffs have been falling continuously across the board as a result of increased competition.

Presently, Indian telecommunication network with around 140 million telephone connections is one of the largest and the fastest growing network in the world. As a result, teledensity in the country is improving rapidly. It is interesting to note that the teledensity has increased by more than 3.5-fold in last five years. In this paper, the growth in teledensity in the country has been analyzed using the logistic and Gompertz curve models. The result shows that the logistic model is better than the Gompertz model to describe the path of teledensity in India. It is found that the teledensity in the country is likely to be more than 40 telephones per 100 people in 2010-11. It is projected that telephone subscriber base in India will increase from 140 million in 2005-06 to 513 million in 2010-11 and 1346 million in 2015-16. There is no doubt that the rapid expansion of telephone services will provide economic, logistic and strategic challenges to the operators. As operators expand coverage into rural and semi-urban areas, they will be confronted with the daunting tasks of developing a countrywide infrastructure and improving and maintaining the quality of service. The expected high growth in telephone subscriber base in India will have at least two implications. First, telephone operators should be ready with contingency plans to deploy and operate infrastructure including customer care, billing, applications, etc., faster than they might have initially planned. Second, infrastructure and handset suppliers and vendors, and application service providers should be prepared to respond to such plans.

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