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# Measuring Contributions of Labor , Capital and Technology in Indian Manufacturing Sector – A Cobb Douglas Production Function approach

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## *Abstract*

*The production function method for the measurement of potential output growth takes into account different sources of an economy's productive capacity, namely of labour, capital and total factor productivity later on including information on technological and allocative efficiency. A production function shows the technological relationship between the maximum output obtainable from a given set of inputs and the relationship between inputs in the existing state of technological change. In this paper , a Cobb Douglas form of production function has been used to measure the contribution of labour, capital and technology for Indian Economy. The basic data source of the study was Annual Survey of Industries (ASI) published by Central Statistical Organization (CSO), Government of India covering the period from 1999-00 to 2010-11. Findings of the statistical analysis are that out of five sectors, three sectors namely Public Limited Company, Government Department Enterprises and Aggregate Corporate sector had recorded increasing returns to scale and the remaining sectors namely the Private Limited company and the Public corporation recorded decreasing returns to scale.*

*Results also show that percentage share of wages was high compared to the percentage share of capital in all the sectors.*

**Key words:** *Production function, Cobb Douglas production, Productivity of Inputs , Share of labour*

## **Introduction**

The performance of the supply side of an economy is often identified with the growth rate of potential output. The use of the production function method for the measurement of potential output growth takes into account different sources of an economy's productive capacity, namely the contributions of labour, capital and total factor productivity, later on including information on technological and allocative efficiency. The production function thus constructed represents a useful and powerful tool for the macro economic analysis and evaluation of the governmental structural policies. The practical application of the production function method requires making certain assumptions, particularly

on the functional form of the production technology, returns to scale, and characteristics of the technological progress.

Production function approach to the productivity measurement is more advantageous because it can handle the problems arising due to non-separability of inputs and output, non-neutral technical change,

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non-constant returns to scale and non-proportionality of input prices and their respective marginal productivity in an explicit manner. A production function shows the technological relationship between the maximum output obtainable from a given set of inputs and the relationship between inputs themselves in the existing state of technological change. In this approach to productivity measurement, various components of productivity can be estimated directly by econometric estimation. The production function can be used to measure the efficiency of the production technology, returns to scale, the degree of economies to scale, the degree of capital intensity of the technology and the degree of substitution between factors of production.

As technical change is presently largely accepted to be an engine of economic growth, researchers have tried to include it explicitly in the economic growth models either as an exogenous or endogenous factor of influence.

## Methodology

The basic data source of the study was Annual Survey of Industries (ASI) published by Central Statistical Organization (CSO), Government of India covering the period from 1999-00 to 2010-11. The ASI data were available only up to this period. All the referred variables were normalized by applying Gross Domestic Product (GDP) deflator. GDP at current and constant prices was obtained by referring to Economic Survey, published by Government of India, Ministry of Finance and Economic Division, New Delhi.

Net value added was taken as output. Labour input consisted of both workers directly involved in production and persons other than workers like supervisors, technicians, managers, clerks and similar type of employees. In productivity measurement, the fixed capital was taken into account in calculating capital inputs. Wages included remuneration paid to both workers and non-workers.

## Tools of Analysis

### a. Cobb-Douglas production function

One of the most commonly estimated functional forms in the Cobb-Douglas production (C-D) function is written as:

$$V = A(t)K^{\hat{a}} L^{\hat{a}} e^u$$

Where  $\hat{a}$  and  $\hat{a}$  are coefficient of labour and capital,  $A(t)$  is the efficiency parameter and  $u$  is the stochastic disturbance term following usual properties. Some functional form has to be given to the term  $A(t)$  before the production function can be estimated. The most commonly used in practice has been  $A(t) = Ae^{\dot{e}t}$  where  $\dot{e}$  is the measure of the technical change in output per period [ $e$  measures the proportionate change in output per period when input level are held constant]. It is very important here to point out the limitations of this representation of the technical change. It assumes neutral technical progress and that the technical progress is exogenous and disembodied (this neglects the usefulness of investment for technical progress).

This function is linear in the logarithmic of the inputs, output and time. Thus,

we have:

$$\ln V = a + \hat{a} \ln L + \hat{a} \ln K + \dot{e}t + \mu_i$$

The estimation of this equation yields values of  $\hat{a}$ ,  $\hat{a}$ , and  $\dot{e}$ .  $\dot{e}$  provides estimates of TFPG and is the rate of exponential technological change. Sum of the partial elasticities ( $\hat{a} + \hat{a}$ ) indicates the extent of economies or diseconomies to scale. The returns to scale are constant, increasing or decreasing if the value of  $\hat{a} + \hat{a}$  is equal to unity, more than unity or less than unity respectively.

The marginal product of labour ( $MP_L$ ) and capital ( $MP_K$ ) can be obtained by applying the following formulae:

$$\begin{aligned} MP_L &= \partial V / \partial L = \hat{a} V / L \\ MP_K &= \partial V / \partial K = \hat{a} V / K \end{aligned}$$

Since profit maximization entails that the marginal productivity of labour is equal to the real wage rate and the marginal product of capital is the real price per unit of capital, it would imply that:

$$MP_L = w/p = \hat{a}V/L.$$

The Share of labour in total output:

$$\hat{a} = (w/p).(L/V). \text{ Similarly } MP_K = r/p = \hat{a}(V/K).$$

And the share of capital in total output

$$\hat{a} = (r/p).(K/V)$$

### b. Elasticities of substitution

The elasticity of substitution or the marginal rate of technical substitution ( $MRTS_{LK}$ ) can now be written as

$$\hat{\sigma} = \frac{\% \Delta(K/L)}{\% \Delta(w/r)} = \frac{\Delta \ln(K/L)}{\Delta \ln(w/r)}$$

where  $w$  is the wage rate and  $r$  is the rental rate of capital. Different values of  $\hat{\sigma}$  have different implications for the distribution of income.

If  $\hat{\sigma} = 1$ , any change in  $K/L$  will be matched by a proportional change in  $w/r$  and the relative income shares of capital and labor will stay constant. Any increase in the capital-labor ratio over time will be exactly matched by a percentage increase in the  $MRTS$  and an identical percentage increase in  $w/r$ . As a result, constant shares of output are allocated to capital and labor even though the capital-labor ratio may change over time. During the post-World War II period, the long-term trend in factor shares in the United States appears to have been roughly constant while the capital-labor ratio has been steadily increasing.

If  $\hat{\sigma} > 1$ , then a given percentage change in  $K/L$  will exceed the associated percentage change in  $w/r$ . For example, an increase in the capital stock would raise the ratio  $K/L$  but lower  $w/r$  by a smaller percentage, hence the share of capital in total

income would rise as the capital-labor ratio increased. The opposite result occurs when  $\hat{\sigma} < 1$ : an increase in the ratio  $K/L$  would tend to lower capital's share.

## Results and discussion

The technical progress of the sectors were analyzed by calculating the marginal productivity of labour ( $MP_L$ ), the marginal productivity of capital ( $MP_K$ ), the marginal rate of technical substitution of labour for capital ( $MPTS_{LK}$ ) and the capital intensity ( $K/L$ ). Marginal rate of technical substitution of labour for capital ( $MPTS_{LK}$ ) shows the rate at which one input (e.g. capital or labor) may be substituted with another while maintaining the same level of output. The marginal productivity or the coefficient of capital ( $MP_K$ ) may be defined as the ratio between a change in output in of industry for a given time period and change in capital. The Marginal productivity of labour ( $MP_L$ ) may be defined as the ratio between a change in output for a given period and change in amount of labour. Capital intensity  $K/L$  is nothing but the capital intensity of the technology.

### a. Growth of $MP_L$ ratios

The trends in the growth of the marginal productivity of labour ( $MP_L$ ) ratios of the corporate manufacturing sector is presented in table-1

$MP_L$  ratios of various sectors during the period shows that among sectors on an average public limited company had recorded the maximum ratio (4.1520) followed by public corporation (4.0248), government department enterprises (3.9543) and private limited company (1.9443). The variations in  $MP_L$  ratios might be due to wage differentials across the sectors. And this average growth of sectors had made aggregate sectors to grow at 4.1067 on an average.

Table-1

MP<sub>L</sub> ratios of corporate manufacturing sectors

Year	Public Limited company	Private Limited company	Government department enterprise	Public corporation	Aggregate corporate sector
1999-2000	3.766	2.189	5.26	3.03	3.765
2000-2001	3.653	2.058	4.365	2.878	3.765
2001-2002	3.804	1.927	3.524	3.181	3.652
2002-2003	4.595	2.562	0.736	5.666	4.405
2003-2004	4.482	2.211	7.89	4.575	4.706
2004-2005	4.519	2.299	3.156	4.878	4.405
2005-2006	4.067	2.715	6.417	3.666	4.104
2006-2007	4.293	2.08	2.84	3.03	3.878
2007-2008	4.218	2.124	0.157	3.757	4.292
2008-2009	3.615	2.343	8.416	5.666	4.141
2009-2010	3.54	1.97	0.736	2.181	3.991
2010-2011	5.272	1.751	4.891	3.03	4.179
Mean	4.152	1.9443	3.9543	4.0248	4.1067
Ó (S.D)	0.5141	0.6528	2.8378	2.7078	0.3134
C.V	12.38	33.88	71.76	67.28	7.63

Source: calculations are based on ASI data

### b. Growth of MP<sub>k</sub> ratios

Table-2 presents details regarding MP<sub>k</sub> ratios of the corporate manufacturing sector from 1999-2000 to 2010-2011.

Table-2

Year	Public Limited Company	Private Limited Company	Government Department Enterprises	Public Corporation	Aggregate Corporate Sector
1999-2000	3.274	0.547	2.578	5.397	0.375
2000-2001	3.11	0.547	1.314	4.965	0.371
2001-2002	2.881	0.563	0.312	5.181	0.371
2002-2003	3.732	0.519	1.959	9.606	0.446
2003-2004	3.699	2.321	2.09	6.854	0.517
2004-2005	3.928	0.612	0.696	8.149	0.656
2005-2006	3.012	0.793	3.609	5.882	0.72
2006-2007	3.634	0.497	2.06	1.673	0.836
2007-2008	3.437	0.634	0.158	5.906	0.978
2008-2009	2.619	0.667	3.944	1.159	1.095
2009-2010	2.455	0.59	3.905	3.13	1.241
2010-2011	4.256	0.607	3.609	5.99	1.65
Mean	3.3364	0.7417	2.1862	5.3239	0.6768
σ (S.D)	0.54286	0.5035	1.3794	2.4383	0.417
C.V	16.27	67.88	63.09	45.79	61.61

Source: calculations are based on ASI data

The  $MP_k$  ratios of various sectors during the reference period showed that it was positive in all the sectors. This shows that capital contributed positively to output. Inter sectoral group analysis indicated that on an average, the public corporation has recorded the maximum productivity performance of 5.3239 followed by the public limited company (3.3364), the government department enterprises

(2.1862), the private sector (0.7417).

#### c. Growth of K/L (capital intensity) ratios

The capital intensity ratios (K/L) of the corporate manufacturing sector from 1999-2000 to 2010-2011 is given in table-3.

Table 3

Year	Public Limited Company	Private Limited Company	Government Department Enterprises	Public Corporation	Aggregate Corporate Sector
1999-2000	0.969	2.734	4.85	1.276	6.309
2000-2001	0.989	2.901	7.916	1.33	6.935
2001-2002	1.144	3.396	1.333	1.439	7.141
2002-2003	1.229	2.765	2.538	1.511	7.052
2003-2004	1.287	3.44	4.541	1.807	7.847
2004-2005	1.296	3.647	9.833	1.942	7.79
2005-2006	1.523	4.258	8.529	2.179	8.416
2006-2007	1.557	4.043	1.5	6.66	8.769
2007-2008	1.672	4.783	1.5	7.75	9.712
2008-2009	1.991	5.4	1.556	4.06	1.137
2009-2010	2.459	6.512	1.762	5.036	1.396
2010-2011	2.647	8.986	1.176	4.561	1.51
Mean	1.5636	44.54	3.9195	3.2959	6.1678
σ (S.D)	0.547	1.8335	3.184	2.2612	3.0443
C.V	34.98	4.116	81.23	68.6	49.35

Source: calculations are based on ASI data.

Capital intensity(capital labour ) ratios of various sectors during reference period showed that the average capital intensity across the intra sectoral group analysis was maximum (4.4054) in the private limited company. The minimum ratio of 1.5636 was found in the public limited company. The K/L ratios of private limited company from the beginning of the period to the end of period had shown an increase

from 2.734 to 8.986 which showed that higher quantum of fixed assets had been accumulated for a given units of labour.

#### d. Growth of $MRTS_{LK}$ ratios

The  $MRTS_{LK}$  of various sectors is presented in table-4

Table -4

Year	Public Limited Company	Private Limited Company	Government Department Enterprises	Public Corporation	Aggregate Corporate Sector
1999-2000	1.15	2.135	2.04	3.569	1.004
2000-2001	1.174	2.003	3.321	3.375	1.014
2001-2002	1.32	1.87	0.341	3.699	9.844
2002-2003	1.231	2.51	0.375	6.696	9.877
2003-2004	1.211	1.979	0.377	5.66	9.103
2004-2005	1.15	2.237	3.852	5.693	6.715
2005-2006	1.35	2.635	1.002	4.254	5.7
2006-2007	1.181	2.03	4.9	3.197	4.638
2007-2008	1.227	2.06	0.315	9.747	4.388
2008-2009	1.38	2.276	8.81	7.582	3.782
2009-2010	1.441	1.911	4.641	2.494	3.216
2010-2011	1.238	1.69	8.5	3.629	2.532
Mean	1.169	1.9443	3.2064	4.9598	5.1511
Ó (S.D)	0.334	0.6589	3.0685	2.1556	3.1675
C.V	28.57	33.88	95.69	43.46	61.49

Source: calculations are based on ASI data

The  $MRTS_{LK}$  ratios of various sectors during the period under study showed that all the ratios were positive for all the sectors including the aggregate of all sectors. Intra sectoral group analysis indicated that the mean  $MRTS_{LK}$  was maximum in the the public corporation (4.9598) followed by the government department enterprises (3.2064), the private limited company (1.944) and the public

limited company (1.1690). This has made the aggregate of all sectors to show the substitution mean level of 5.1511.

#### e. Production function estimates

The estimates production function for various sectors is presented in table-5

Table-5

Sector	A	Capital (a)	Wages (B)	Economics of scale(s)	R2	D.W. Statis	a/S	b/S
Public limited company	76.544** (-2.464)	0.274** (2.149)	0.766** (2.149)	1.040	0.88	1.725	26	74
Public limited company	-43.11* (-14.457)	0.189** (2.189)	0.547* (14.41)	0.7413	0.97	2.379	26	74
Government department Enterprises	10.835 (0.987)	0.460** (2.578)	0.578**** (5.265)	1.038	0.66	0.834	44	56
Public Corporation	73.468*** (4.353)	0.311 (1.457)	0.397*** (4.487)	0.697	0.74	1.613	43	57
Aggregate Corporate Sector	73.018*** (6.839)	0.375 (6.701)	0.765** (2.567)	1.14	0.86	0.493	33	67

Source : Calculations are based on ASI data. Foot note: figures are parentheses in t-values

\*\*significant at 5% level

\*\*\*significant at 10% level

Efficiency parameter 'A' is positive and statistically significant in four sectors namely, the public limited company, the private limited company, the public corporation,

and aggregate of all the corporate sector. The implication is that the organizational efficiency is high, positively contributes to output and its contribution was explicitly significant in output generation.



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The elasticity of capital with respect to output ( $\hat{\alpha}$ ) is positive in all sectors and statistically significant in three out of five sectors. An encouraging feature noticed from the results is that wage coefficient ( $\hat{\alpha}$ ) is positive in all sectors and statistically significant.

The sum of the coefficients ( $\hat{\alpha} + \hat{\alpha}$ ) implies that out of five sectors, three sectors namely the Public limited company, the Government Department Enterprises and Aggregate of all the Corporate sector had recorded increasing returns to scale ( $\hat{\alpha} + \hat{\alpha} > 1$ ) and the remaining sectors namely Private Limited Company and Public Corporation recorded decreasing returns to scale ( $\hat{\alpha} + \hat{\alpha} < 1$ ). The percentage share of factor inputs presented in the table indicated that percentage share of wages ( $\hat{\alpha} / S$ ) was high when compared to the percentage share of capital ( $\hat{\alpha} / S$ ) in all the sectors.

### Conclusion

Based on the study, it is concluded that the production function is purely a technical relation, which connects factor inputs and outputs. It describes the Laws of proportion i.e. the transformation of factor inputs into outputs at any particular time period. The production function represents the technology of a firm or an industry, or the economy as a whole and it includes all the technically efficient methods of production.

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