Quantitative Models For Pareto Analysis & Cell Formation

Jaideep Gupte*

ABSTRACT

The motivation of the paper stemmed from the need to obtain sample products from the product population on the basis of quantity, revenue and routings. This approach is also called as Multi Criteria Pareto Analysis. Mixed Integer Programming Models have been developed for Multi Criteria Pareto Analysis. Single Linkage Cluster Analysis method has been implemented to obtain product clusters on the basis of Product Quantity, Revenue and Routings.

Keywords: cluster, Pareto, revenue, routing.

1. Introduction

The layout of a facility determines how to arrange, locate and distribute the equipment and support services in a manufacturing facility for minimization of overall production time, maximization of operational and arrangement flexibility, maximization of turnover of work-in-process and maximization of factory output in conformance with production schedules (Tompkins et al, 2003). Modern manufacturing facilities experience significant changes in product designs, process plans, demand volumes, product mix, product life cycles and production routings. Since the traditional facility layouts – Product (or Focused Factory), Functional (or Process), Production Line (or Flowline) or Product Family (or Cellular) – are unsuitable for these operating conditions, manufacturers require new methods to design hybrid layouts that combine the manufacturing focus of one or more of these traditional layouts.

2. Review and Extension of Fundamental Theory for Facility Layout

2.1 P-Q Analysis

2.1.1 Literature Review and Critique

Product-Quantity (P-Q) Analysis uses a single criterion – Production Quantity (or Volume) – to sort and segment the complete product mix produced in a facility to design a layout for the facility. Muther (1973) states "For a facility planner or layout analyst, this Volume-Variety Analysis has a significant meaning. It is a basis for deciding which fundamental type of production system or layout arrangement to use – line production, jobshop or combination or split of two or more basic arrangement". He provides only empirical guidelines for using the analysis to determine an appropriate layout for a facility: (1) large quantities of relatively few products or varieties that tend to favor production lines or product-focused layouts will appear at the left end of the P-Q Analysis curve, whereas, at the other end of the curve will appear

***Mr. Jaideep Gupte**
Department of Management , IILM,
New Delhi, India.
IILM Institute of Higher Education,
Lodhi Institutional Area, Lodhi Road,
New Delhi, 110003
Email : jgupte@gmail.com
Jaideep.gupte@iilm.edu

many different products with small quantities that tend to be produced in a process (or jobshop) layout and (2) a "shallow" P-Q Analysis curve suggests a general layout for all the items with universal handling methods, whereas, a "deep" curve suggests dividing the products and the production areas into two or more different layouts and handling systems.

Since Muther's textbook was published, contemporary textbooks have not extended this method. Tompkins et al. (2003) states that "(if) 85% of the production volume is attributed to 15% of the product mix (then) a facilities plan should consist of a mass production area for 15% of high-volume items and job shop arrangement for (the) remaining 85% of the product mix". Heragu (1997) states that "a company that makes multiple products may generate most of its revenues, say 80%, from 20% of its products". Lee (1994) states "A small set of products account for a large volume".

However, the P-Q Analysis method ignores *at least two* other criteria that are critical for product mix segmentation and layout selection for an HVLV facility:

(1) *Revenue (or Sales):* This is important since a primary goal of a facility layout is to maximize the throughput and to minimize the WIP inventory of high-revenue products. Although Volume is a primary determinant of material handling traffic and costs, it is possible that a high-volume product is not a high-revenue product, and vice-versa.

(2) *Variety:* This is important since a primary goal of a facility layout is to promote unidirectional flow of materials and minimize backtracking or crisscrossing of flow paths for a diverse product mix with numerous dissimilar routings.

Contemporary textbooks do not discuss any models or heuristics that extend the original P-Q Analysis method. Tompkins et al. (2003) only states that "Pareto's Law may not always describe the product mix of the facility. However, knowing Pareto's Law does not apply is valuable information because if no product dominates the production flow, a general jobshop facility (layout) is suggested". The authors present a Volume-Variety Layout Classification Chart that classifies the traditional layouts on two axes: Volume and Variety. However, the "Volume" axis carries no units, except for "Low", "Medium" and "High" labels, and the "Variety" axis carries no units, except for "Low", "Medium" and "High" labels to indicate the number of different manufacturing routings in the product mix.

Table 1 presents essential data for a machine shop that is adequate for various analyses and algorithms utilized for facility layout.

Table 1 P, Q, R and $ Data for the Product Mix of Fools-R-Us Inc.

Part #	Part Name	Operation Sequence	Prod. Qty.	Sales $
1	Slider (a)	6→9→10→11→12	40	$10,000
2	Slider (b)	4→6→9→10→11→12	45	$25,000
3	Press Brace	5→8→9→10	80	$50,000
4	Bracket #1	4→7→9→10	15	$5,000
5	Table	3→7→10→12	100	$30,000
6	Damper	1→7→9→10	20	$10,000
7	Bracket #2	1→8→9→10	30	$5,000
8	Support	4→7→9	30	$20,000
9	Housing	2→7→9	70	$40,000
10	Flange	2→9	15	$20,000
11	Shaft	3→9→10→12	10	$10,000
12	Base	3→6→4→10→12	90	$35,000
13	Spacer	4→6→4→10→12	75	$45,000

However, P-Q Analysis could introduce potential sampling errors into the layout design, since it does not consider similarities in product routings during the sample selection process. Figure 1 shows the P-Q Analysis Chart for the product mix in Table 1. Using the standard cut-off rule of 80% of Total Volume, six products – 8, 7, 6, 10, 4, 11 – get eliminated from the Pareto sample. However, while the Pareto sample based on this single-criterion sampling method includes the pairs of products - (12, 13) and (1, 2) – which have similar routings, it fails to include another pair of products with similar routings – (5, 11).

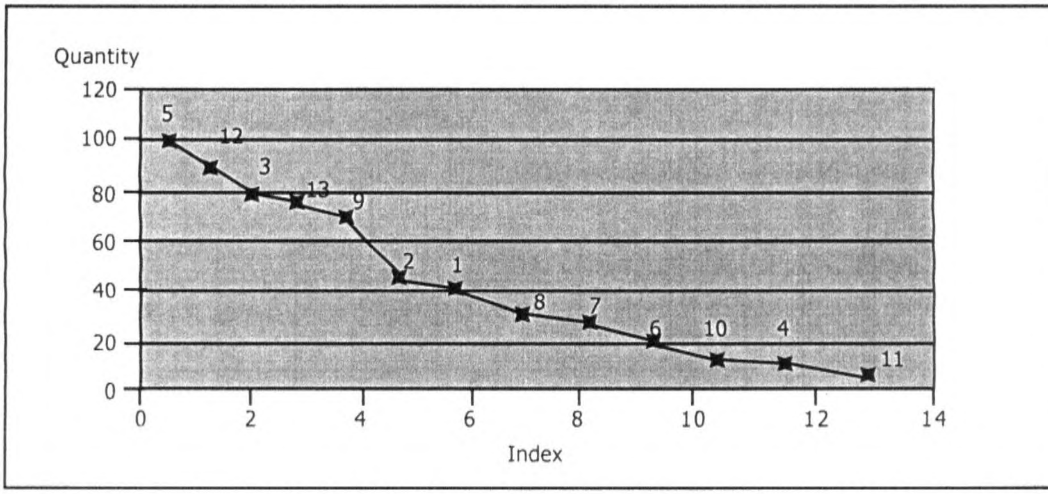

Figure 1 P-Q Analysis Chart

2.1.2 Revisionist Theory

2..1.2.1 Revisionist Theory for P-Q Analysis: P-Q-R Analysis

To incorporate Variety into P-Q Analysis, we propose a new sampling approach called P-Q-R (Product-Quantity-Routing) Analysis that maps the diversity in the routings of the different products. A sample of P-Q-R Analysis Chart is generated as shown in Figure 2. The clustering dendogram in Figure 2 shows groups of products that have identical or similar routings. Similar routings group at low values of distance on the Y-axis and are the basis for the design of flowlines or manufacturing cells for those products. Beside the clustering dendogram, a second graph is drawn with Distance on the Y-axis and the Cumulative Production Quantity on the X-axis. For any value of distance on the Y-axis, the Cumulative Production Quantity of only those products with similar routings that get grouped into families is plotted. This allows the layout analyst to know the total production volume of products that have been grouped at any desired level of routing similarity used for product grouping. Note that, instead of the CPQ = 28 which was the original cut-off value in Figure 1, it is the two values of CPQ = 26 and CPQ = 34 that correspond to two potential alternatives for product groupings. Having identified such groupings of similar routings, one could implement Muther's recommendation that "within a grouping of more or less similar products, splits based on quantity are practical. In such cases, the planner is frequently better off to remove completely the jobbing work from the mass production areas" (Muther, 1973).

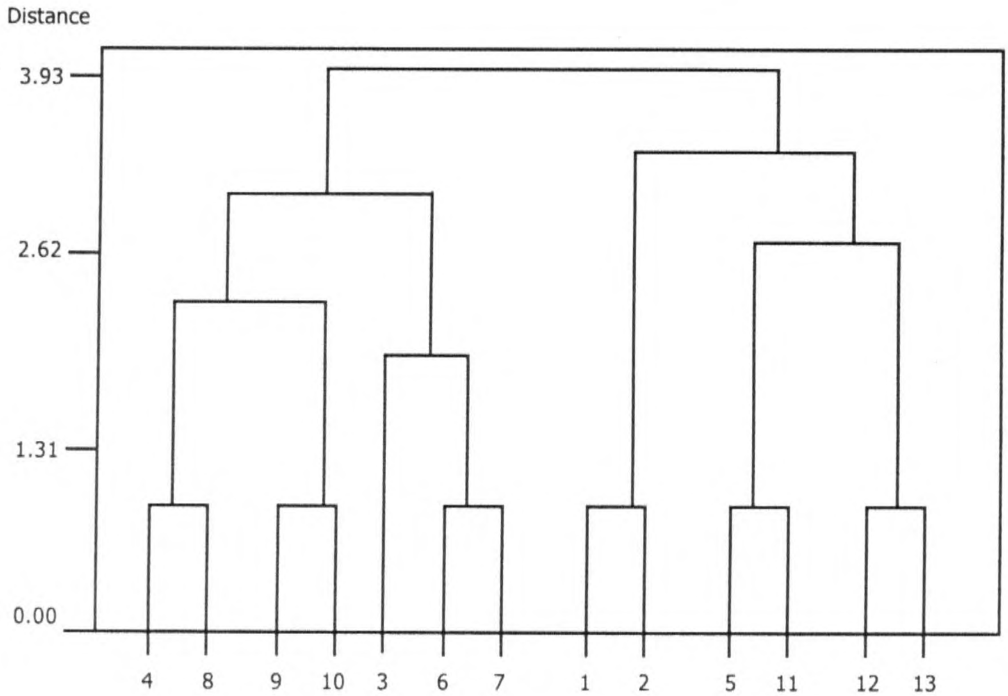

Figure 2(a) Cluster Analysis Dendogram for P - R Analysis

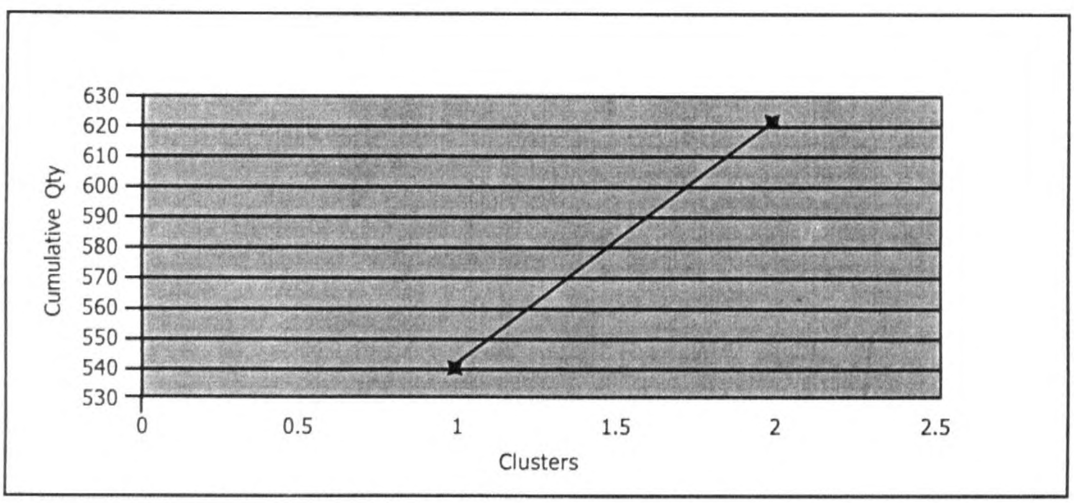

Figure 1 P-Q Analysis Chart

Figure 2(b) Pareto Analysis based on Volume embedded in P-R Analysis of Figure 2(a)

2.1.2.2 Revisionist Theory for P-Q Analysis: P-Q-$ Analysis

Heragu (1997) states that the design of a facility layout based on the entire population of products is computationally intensive as the number of products increases. P-Q Analysis is a sampling approach to reduce the number of products utilized for layout design that uses a single criterion – Volume – to select a sample of products from a large population of products (or product mix) produced in a facility. Unlike P-Q Analysis, P-Q-$ Analysis is a bicriterion sampling approach that will design a layout to simultaneously minimize *both* the material handling costs for high-volume products and delay-induced WIP costs for high-value products. This is because, in any high-variety low-volume manufacturing facility, the product mix could contain products in one or more of the following segments: (High Revenue, High Volume), (High Revenue, Low Volume), (Low Revenue, High Volume), (Low Revenue, Low Volume).

Since P-Q Analysis has the structure of a standard Knapsack problem, we propose a mixed integer programming model for P-Q-$ Analysis that has a similar formulation. The formulation is mathematically expressed as follows.

$$\text{Minimize } z = \sum_i x_i - m - c$$

Subject to :

$$\sum_i q_i \; x_i > p_q - \sum_i q_i \qquad (1)$$

$$\sum_i \$_i \; x_i > p_s - \sum_i \$_i \qquad (2)$$

$$\$_i \; x_i > -m.q_i + c - M(1 - x_i) \qquad (3)$$

$$q_{max} . m < c \qquad (4)$$

where the decision variables in this model are defined as follows:

$$x_i = \begin{cases} 1 & \text{if product i is selected in the sample} \\ 0 & \text{otherwise} \end{cases}$$

m = slope of the constraint function in equation (3)

$m > 0$

c = intercept of the constraint function in equation (3) on the y axis which is the revenue axis.

$c > 0$

The constraint function in equation (3) determines whether a product is included or excluded in the sample. If a product is selected in the sample, it corresponds to the point above the line,

If the product is not selected in the sample, it corresponds to the point below the line.

The constants in this model are defined as follows:
M = A very large number.
N = *Number of products in the population.*
$\$_i$ = Revenue of product i
q_i = Quantity (or Volume) of product i.
p_s = proportion of revenue of products with respect to the total revenue of the product population that is explained by the sample of products selected.
p_q = proportion of quantity with respect to the total quantity of the product population explained by the sample of products selected.

The objective function seeks to minimize the number of products included in the sample. Constraints (1) and (2) set a predetermined bound on the minimum proportion of the cumulative quantity and cumulative revenue of the entire product mix that is explained by the sample. Constraint (3) determines the slope *m* and y intercept *c* of a cutting plane that partitions the Q-$ space into two zones, one containing the products included in the sample and the other containing the remaining products that were excluded from the sample. The cutting plane is a straight line for the above model. Constraint (4) is a valid inequality.

Figure 3 illustrates the differences between the P-Q-$ Analysis and P-Q Analysis samples for the product mix in Figure 1(a). The equation for the cutting plane is: $ = 235.294Q + 23529.412. Points to the right of this line are included in the sample and account for 80% of Total Quantity (.8*620= 496) and 80 % of Total Revenue (.8*305000 = 244000). The products included in the sample are {2, 3, 5, 8, 9, 10, 12, 13}. These products are located on or above the straight line drawn on the Q-$ plot corresponding to the constraint function in equation (3).

For the purpose of comparison, the sample of products chosen using P-Q Analysis with an 80% of Total Quantity yielded the following sample of products – {5, 12, 3, 13, 9,} – which explained 0.77 % of Total Revenue. It can thus be argued that the P-Q analysis procedure is insufficient for the sample selection as the products included in the sample according to the P-Q analysis do not account for a predetermined proportion of cumulative revenue of entire product mix.

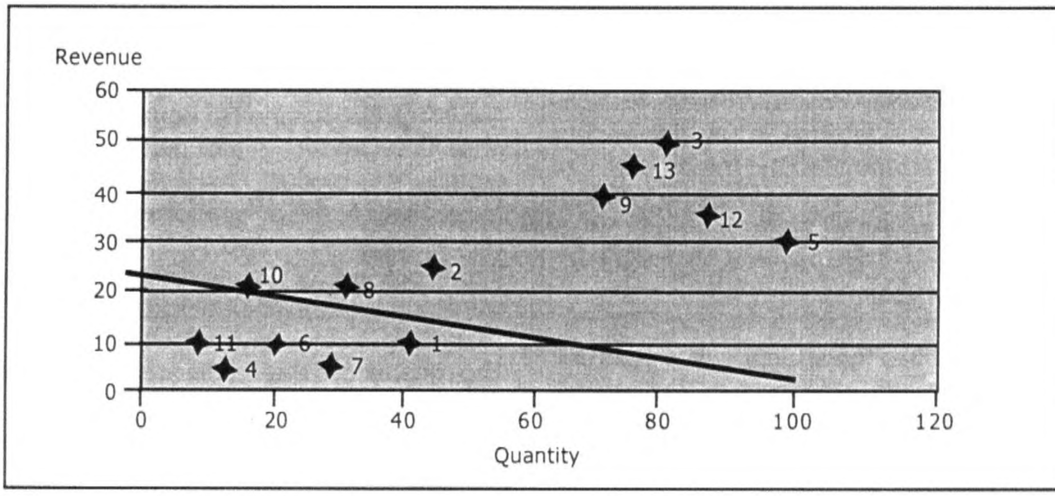

Figure 3 P-Q-$ Analysis

The above Figure 3 is a pictorial representation of the P-Q-$ analysis performed from the above mixed integer linear model which selects a sample based on a linear cutting plane. The straight line in the above Figure 3 represents the cutting plane.

2.2 P-Q-R-$ Analysis Representation In a Single Plane

Since the mixed integer programming model discussed above for the P-Q-R-$ analysis is intractable for large industrial data sets, we propose in our research to represent three defining criteria of all products namely: quantity, revenue and routing in a single plane in order to study the tricriterion product sampling problem. We propose to develop computationally viable models for the tricriterion product sampling problem on the basis of this representation. This representation enables us to map and hence locate the quantity, revenue and dissimiler values between pairs of products on a single plane and simultaneously explain the order in which products are to be picked for sample selection based on the three criteria: quantity, revenue and routings of products.

In this representation, products are represented on an x-y plot where x represents the quantity of product and y represents the revenue of product. The Product Routing analysis (also called as P-R analysis) is performed on the product population. It is observed that the P-R is a single linkage clustering problem. Single linkage cluster analysis is the symmetric traveling salesperson problem (TSP).Therefore the P-R analysis problem can be modeled as a symmetric TSP.

In our problem the products represent the cities in a TSP tour. The total distance of the TSP tour is represented by the total dissimilarity between the products which is to be minimized. The TSP tour of order in which products are to be picked while selecting the sample in a manner that their total dissimilarity between them is minimized. The following Figure 4. represents a TSP tour drawn on a Q-$ plot. Product is represented by a point on the plot whose x and y coordinates are the quantity and revenue values of the product. Routing criteria is represented by performing the P-R analysis on the product population. This sequence is the order in which sample selection is to be carried.

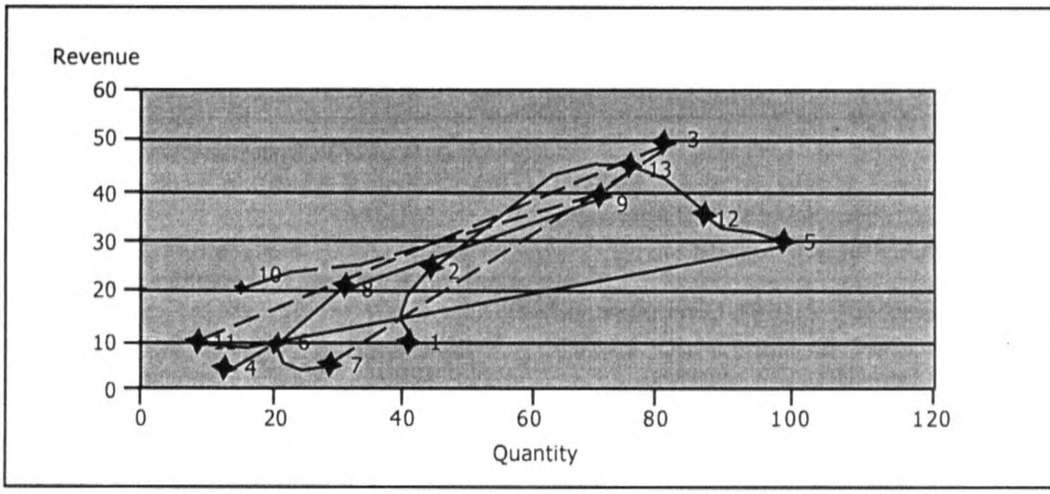

Figure 3 P-Q-$ Analysis

The input data required for the above representation is the quantity and revenue values of products in the entire population and the Levenshtein distance values between each pair of products in the population. The Levenshtein distance matrix of the products is analogous to the TSP matrix in a sense that a city in the TSP problem corresponds to the product in the distance matrix. The Levenshtein distance value between a pair of products corresponds to the distance between a pair of cities. The P-R analysis problem being a single linkage cluster analysis problem (modeled as a TSP) is solved by giving the Levenshtein distance matrix of the products as an input into STORM (a software package). A TSP tour of products along with a total dissimilarity value of products is obtained as an output of STORM. The software (STORM) employs an insertion heuristic method to solve the TSP. The length of the TSP tour was observed to be 15.6 based on Levenstein distance values.

This approach thus explains the order in which products are to be picked for the sample selection in a manner that the total dissimilarity value between the products is minimized. The dissimilarity value between the products in this case is a function of routings of the products.

The same representation can be used to obtain a sequence of products for sample selection to minimize the total dissimilarity between the products where in the dissimilarity value is a function of three criteria: quantity, revenue and routings of products.

3. Conclusions & Future Directions

This paper explains the methods and models to obtain a sample of products from a large product population based on three distinct criterion: product quantity, product revenue and product routing. The selected sample of products would be used to design the facility layout of the plant and help in overall resource planning and strategic planning of the company. Future research in this area would include the application of Artificial Neural Network techniques to select the Product sample from the population and provide strategic planning measures for the Industry.

References

Al-Hakim, L. (2002). A note on 'Maximally weighted graph theoretic facility design planning'. International Journal of Production Research, 40, 495-505.

Askin, R.G. & Zhou, M. (1998). Formation of independent flowline cells based on operation requirements and machine capabilities. *IIE Transactions*, 30, 319-329.

Francis, R.L., Mcginnis, L.F., and White, J.A. (1992). Facility Layout and Location: an Analytical Approach. Englewood Cliffs, N.J : Prentice-Hall, Inc.

Garey, M.R., & Johnson, D.S. (1979). *Computers and Intractability: A Guide to the Theory of NP-Completeness*. San Francisco, CA: Freeman.

Goetschalckx, M. (1992). An interactive layout heuristic based on hexagonal adjacency graphs. *European Journal of Operational Research*, 63, 304-321.

Heragu, S. (1997). Facilities Design. Boston, MA: PWS Publishing company.

Lee, Q. (1997). Facilities and Workplace Design. Norcross, GA: Engineering and Management Press.

Muther, R. (1973). Systematic Layout Planning. Boston, MA : Industrial Education Institute.

Meyers, F. (2000). Manufacturing Facilities Design and Material Handling. NJ : Prentice Hall, Inc.

Tompkins, J.A., White, J.A., Bozer, Y.A. & Tanchoco, J.M.A. (2003). Facilities Planning. Hoboken, NJ: John Wiley.