MATHEMATICAL MODELLING FOR SURFACE HARDNESS IN VACUUM MOULDING OF Al-Al_2O_3 MMC

Rupinder Singh

Guru Nanak Dev Engineering College Ludhiana, Dept. of Production Engineering
E-mail: rupindesingh78@yahoo.com

Manuscript received on: October 13, 2013, acceped after revision on: February 19, 2014.

Abstract : Al-Al_2O_3, metal matrix composites (MMC) have many potential engineering applications. Not much work hitherto has been reported for modeling the surface hardness (SH) in vacuum moulding (VM) of Al- Al_2O_3, MMC. In the present study, outcome of Taguchi model has been used for developing a mathematical model for SH; using Buckingham's π-theorem for Al- Al_2O_3, MMC. Three input parameters namely vacuum pressure; component volume and grit size were selected to give output in form of SH. This study will provide main effects of these variables on SH and will shed light on the hardness mechanism in VM of Al- Al_2O_3 MMC. The comparison with experimental results will also serve as further validation of model.

Keywords : Al-Al_2O_3 metal matrix composite, vacuum moulding, hardness, Buckingham's π-theorem

1. INTRODUCTION

Al- Al_2O_3, MMC is engineered combination of the metal (Al) and hard particle/ceramic (Al_2O_3) to get tailored properties [1-2]. These are in use for the space shuttle, commercial airliners, electronic substrates, bicycles, automobiles, golf clubs, and a variety of other applications [2-5]. Many researchers have proposed different methods for Al-Al_2O_3, MMC development (like powder metallurgy, stir casting etc.) [3-7].The literature review reveals that there has been a critical need for development of cost effective Al-Al_2O_3, MMC [4, 6-7]. VM is one of the casting processes, which is distinctly different from other sand casting processes as this process requires no binders for holding the sand grains together in the mould [8]. The vacuum inside the mould results in a net pressure pushing in, holding the sand rigidly in the shape of the pattern, even after the pattern is removed. The process uses a specially designed, strong, highly flexible polymer film to seal the open ends of the sand mould and form the mould cavity [8]. The literature review reveals that lot of work has been reported on optimization of VM process [9-13]. Various process parameters like: vibration frequency, pouring temperature and plastic film thickness for the sound casting produced by VM process have been reported. But hitherto no work has been reported for modeling the SH in VM of Al- Al_2O_3 MMC. So, the present investigation has been focused to develop mathematical model for SH in VM of Al- Al_2O_3 MMC. For VM of Al- Al_2O_3 MMC, an approach to model SH was proposed and applied [14]. This model was an attempt for predicting the SH as macro model in VM and is based upon robust design concept of Taguchi technique. The model was mechanistic in sense that parameters can be observed experimentally from a few experiments for a particular material and then used in prediction of SH over a wide range of process parameters. This was demonstrated for Al-Al_2O_3, MMC; where very good predictions were obtained using an estimate of multi parameters at a time. In that study, effects of three process parameters (component volume;

vacuum pressure; and chemical composition) were revealed. Table 1 shows various input and output parameters used in experimental study.

Table 1 Various input and output parameters

Input parameters	Output parameters
• Three component volume levels (42411.50, 57726.76, 75398.22 mm^3) • Three levels of vacuum pressure (40, 54 and 68 kPa) • Three levels of grit size/ AFS No. (50, 60, 70)	SH

The relationships were studied by considering interaction between these variables. On the basis of this model, Singh and Singh (2011) studied the relationships between SH and controllable process parameters [14]. These relationships agree well with the trends observed by experimental observations made otherwise [8-13].

1.1 Description of the VM process

Fig. 1 shows the VM process schematically. VM process can be limited to as few as five items (namely: vacuum system, Film heater, vibrating table, pattern carrier and flasks) for automation or for higher production [9]. There is no need for heavy, noisy jolt squeeze equipment, ramming of slingers. Any shape or size can be produced in a VM from thin walls to thick sections, or from castings weighing ounces to several tons. Fine surface finish and excellent dimensional accuracy, no moisture related defects, no cost for binders, excellent sand permeability, and no toxic fumes from burning the binders are key advantages of VM. The major VM process variables affecting SH are shown as cause and effect diagram (Ref. Fig. 2).

The study presented in this paper is based on a previously published macro model based on Taguchi robust design [14]. As per Taguchi design SH in VM was significantly dependent on chemical composition and component volume. The case study under consideration deals primarily with obtaining optimum system configuration in terms of response parameters with minimum expenditure of experimental resources. The best settings of control factors have been determined through experiments as shown in Table2 as geometric model.

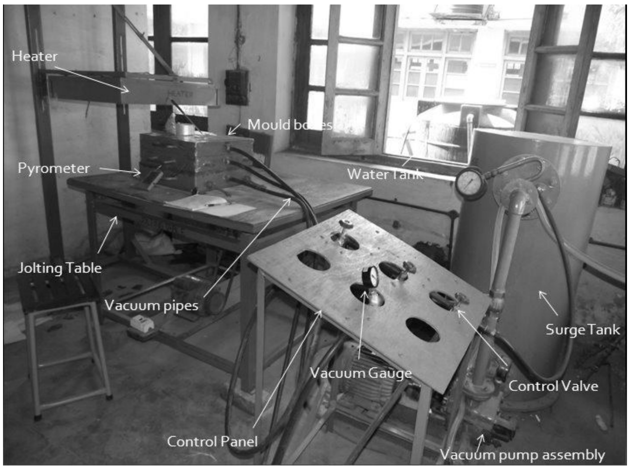

Fig. 1 VM process [14]

Table 2 Geometric model of SH for $Al\text{-}Al_2O_3$ MMC [14]

OPTIMIZED SH CONDITIONS	
Volume	42411.50 mm^3
Vacuum	40kPa
Grit size/ AFS No.	60

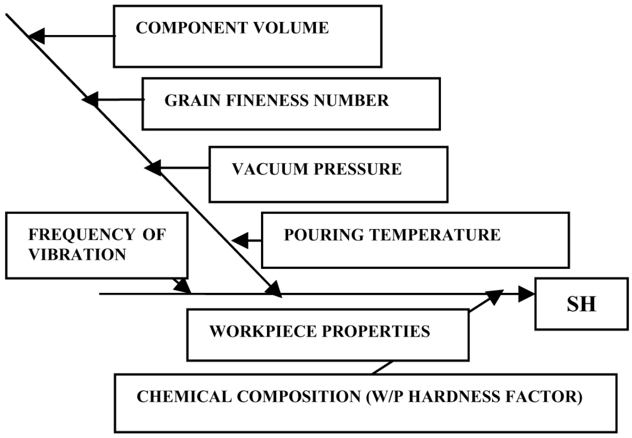

Figure 2 Cause and effect diagram of SH

Now based upon geometric model, Buckingham's π-theorem has been used to study the relationships between SH and controllable process parameters. There are three sections in this paper. Following this introduction, Section 2 describes mathematical modeling of SH. Conclusions have been drawn up in Section 3, followed by references.

2. MATHEMATICAL MODELING OF SH

The Buckingham's π-theorem proves that, in a physical problem including "n" quantities in which there are "m" dimensions, the quantities can be arranged in to "n-m" independent dimensionless parameters. In this approach dimensional analysis is used for developing the relations [15-16]. Since SH, 'H' depends upon three input parameters namely volume, vacuum pressure, grit size of moulding sand more significantly and rest three input parameters namely chemical composition, pouring temperature, frequency of vibration were not significant. Therefore by selecting basic dimensions: M (mass); L (length); T (time); and θ (temperature). The dimensions of foregoing quantities would then be:-

- The SH "H" (kgf/mm^2) Vickers hardness : $M L^{-1} T^{-2}$
- Grain fineness number "N" (μm) : L
- Component volume "V" (mm^3) : L^3
- Pouring Temperature "θ" (°C) : θ
- Vacuum pressure "P" (kgf/mm^2) : $M L^{-1}T^{-2}$
- Frequency of vibration "F" (1/sec) : T^{-1}

Now, $H = f(N, P, V, \theta, F)$ **(1)**

In this case n is 6 and m is 4. So, one can have (n-m = 2) π_1 and π_2 two dimensionless groups.

Taking H and N as the quantities which directly go in π_1 and π_2 respectively, it can be written as:

$\pi_1 = H.(V)^{\alpha 1}.(\theta)^{\beta 1}.(P)^{\gamma 1}.(F)^{\delta 1}$ **(2)**

$\pi_2 = N.(V)^{\alpha 2}.(\theta)^{\beta 2}.(P)^{\gamma 2}.(F)^{\delta 2}$ **(3)**

Substituting the dimensions of each quantity and equating to zero, the ultimate exponent of each basic dimension has been achieved, since the "π_i"s are dimensionless groups. Thus αi, βi, γi, δi, where i= 1, 2, 3, can be solved.

Solving for π_1, one gets :

$\pi_1 = (M L^{-1}T^{-2}).(L^3)^{\alpha 1}.(\theta)^{\beta 1}.(M L^{-1}T^{-2})^{\gamma 1}.(T^{-1})^{\delta 1}$ **(4)**

Here,

M : $1 + \gamma_1 = 0$

L : $-1 + 3\alpha_1 - \gamma_1 = 0$

T : $-2 - 2\gamma_1 - \delta_1 = 0$

θ : $\beta_1 = 0$

Solving, one gets :

$\alpha_1 = 0$, $\beta_1 = 0$, $\gamma_1 = -1$, $\delta_1 = 0$

Thus

$\pi_1 = H.(V)^0.(\theta)^0.(P)^{-1}.(F)^0$

$\pi_1 = H/P$ **(5)**

Similarly one gets :

$\pi_2 = (L).(L^3)^{\alpha 2}.(\theta)^{\beta 2}.(M L^{-1}T^{-2})^{\gamma 2}.(T^{-1})^{\delta 2}$ **(6)**

Solving, one gets :

α_2= -1/3, β_2= 0, γ_2= -1, δ_2= 0

$$\pi_2 = \frac{N}{V^{\frac{1}{3}}} \quad \textbf{(7)}$$

The ultimate relationship can be assumed to be of the form

$$\pi_i = f\ (\pi_j\ , \pi_k) \quad \textbf{(8)}$$

It is assumed that i =1, j = 2, k = 3. Then functional relationship is of the form

$$\pi_1 = f\ (\pi_2)$$

Or

$$H/P = f\left(\frac{N}{V^{\frac{1}{3}}}\right)$$

It has been experimentally found that H directly goes with N [14]. This means composition of casting significantly affects the SH. Therefore casting hardness factor has been taken as representative for development of mathematical equation.
Thus the equation becomes

$$H = P.\ f\left(\frac{N}{V^{\frac{1}{3}}}\right) \quad \textbf{(9)}$$

$$H = K1\left\{P.\frac{N}{V^{\frac{1}{3}}}\right\} \quad \textbf{(10)}$$

Here 'K1' represents constant of proportionality.
Now by keeping P /$V^{1/3}$ fixed, experiments were performed for different values of N, to find out 'H' and 'K1' in Eq.10. The actual experimental data for three different grit sizes have been collected and plotted in Fig. 3.

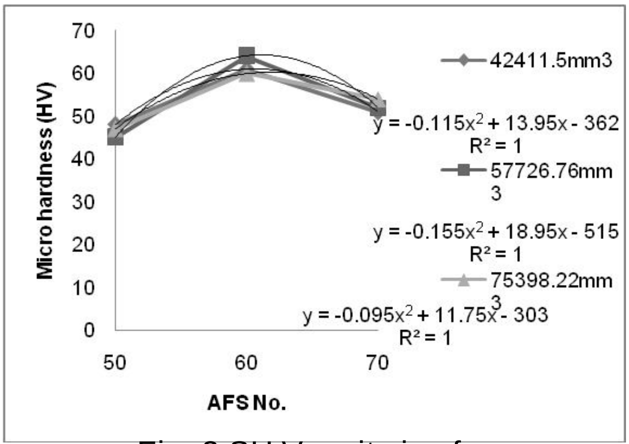

Fig. 3 SH Vs grit size for different component volumes

The data collected has been further used for finding best fitting curve. The second degree polynomial equation comes out to be best fitted curve with coefficient of co-relation equals to "1". Thus equation 10 of SH for this case becomes:

(For component volume = 42411.5mm^3)

$$SH = \frac{P}{V^{\frac{1}{3}}}\ (-\ 0.115N^2 + 13.95N - 362) \quad \textbf{(11)}$$

(For component volume = 57726.76mm^3)

$$SH = \frac{P}{V^{\frac{1}{3}}}\ (-\ 0.155.N^2 + 18.95N - 515) \quad \textbf{(12)}$$

(For component volume = 75398.22mm^3)

$$SH = \frac{P}{V^{\frac{1}{3}}}\ (-\ 0.095N^2 + 11.75N - 303) \quad \textbf{(13)}$$

Since this model is based upon Taguchi based model of SH, in which component volume and vacuum pressure are already optimized [14]. Therefore these parameters have not been varied while developing mathematical model. This model is useful in understanding effect of process parameters on SH in VM. The second degree polynomial equation has been used only to find best fit curve with coefficient of co-relation close to 1. Since the cooling rate basically affects the solidification time, which depends upon mould constant and ratio of volume to surface area of the component. In the present study three component volume, vacuum pressure and grit size were taken to see the effect on SH. The cooling rate as input parameter (based upon time-temperature curve) has not been taken separately. Further to check the internal defects of the castings, the radiography analysis was performed (as per ASTM E 155 standard) for gas holes and shrinkages at optimized conditions suggested by Taguchi design (Ref. Fig. 4). The results obtained shows that the components prepared are acceptable in accordance with ASTM E155 standard.

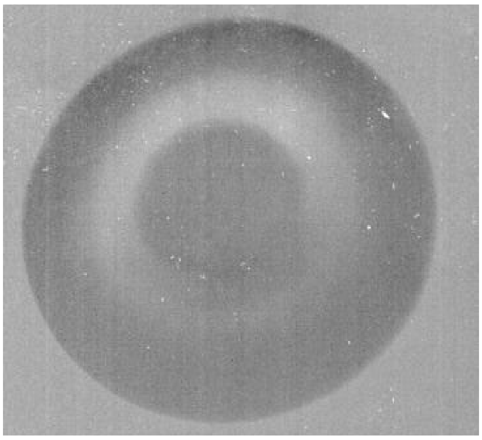

Fig. 4 Radiographic analysis of casting prepared by VM

The present results are valid for 90-95% confidence interval. For validation of this model, final observations were made under both experimental conditions (based upon Taguchi design) and theoretically developed mathematical equations. Comparison of SH result obtained experimentally agrees very well with predictions through mathematical equations. The verification experiment revealed that on an average there is 5% improvement in SH (60-61HV initial value to 64-65HV optimized value).

3. CONCLUSIONS

The Buckingham π-theorem has been used for mathematical modeling of SH in VM process. The interactions among input parameters have been considered for developing the model. The following conclusions can be drawn from this study:

- The contribution of input parameters to hardness of casting is: volume 1%, vacuum 1%, grit size 95%. The mathematical equation developed here sufficiently express all significant input parameters. As regard to mathematical model second degree polynomial equation for SH is giving best fitting curve with coefficient of co-relation H" 1.
- The verification experiment revealed that on an average there is 5% improvement in surface hardness, for selected workpiece. Also as regards to casting defects are concerned no surface defects have been observed in radiographic image at optimized vacuum moulding conditions.

ACKNOWLEDGEMENT

The author would like to thank AICTE, New Delhi for financial support.

REFERENCES

[1] Bakhtiyarov, S. I., Overfelt, R., Black, M. G., and D. J. Weiss, Design and V-Process Production of Cast Magnesium Component, Transactions of the American Foundry men's Society Journal, Vol.113, pp. 879 – 886, 2005.

[2] Hajjari, E., Divandari, M., and Arabi, H., Effect of Applied Pressure and Nickel

Coating on Microstructural Development in Continuous Carbon Fiber-Reinforced Aluminum Composites Fabricated by Squeeze Casting, Materials and Manufacturing Processes, Vol. 26(4), pp. 599- 603, 2011.

[3] Adelakin, T. K. and Suarez, O. M., Study of Boride-Reinforced Aluminum Matrix Composites Produced via Centrifugal Casting, Materials and Manufacturing Processes, Vol. 26(2), pp. 338-340, 2011.

[4] Gouwens, P. R., Vacuum Molding (The V-Process) Perspective and into the Future, Transactions of the American Foundrymen's Society Journal, pp. 98-174, 2005.

[5] Zhang, L., Li, L. and Zhu, B., Simulation Study on the LPDC Process for Thin-Walled Aluminum Alloy Casting, Materials and Manufacturing Processes, Vol. 24(12), pp.1349-53, 2009.

[6] Reddy, G. M., Prasad, K. S., Rao, K. S., Mohandas, T., Friction surfacing of titanium alloy with aluminum metal matrix composite", Materials and Manufacturing Processes, Materials and Manufacturing Processes, Vol. 27(2), pp. 92-98, 2011.

[7] Singh, R., Effect of work piece volume on statistically controlled rapid casting solution of aluminum alloys using three dimensional printing, Materials and Manufacturing Processes,pp. 377-382, 2012.

[8] Kumar, P. and Creese, R. C., Effect of Zircon Sand (added to silica sand) Surface Finish of Al-11% SiC V-Process, Transactions of the American Foundry men's Society Journal, Vol.107, pp. 95-98, 1999.

[9] Kumar, P. and Creese, R. C., Effect of Fly Ash Addition on Mechanical Properties of Al-11% Si Alloy Castings Produced by V-Process, Transactions of the American Foundry men's Society Journal, Vol. 109, pp. 1-15, 2001.

[10] Kumar, P. and Gaindhar, J. L., Off-line Quality Control for V-Process Castings, Quality and Reliability Engineering International Journal, Vol. 11, pp.175–181, 1995.

[11] Kumar, P. and Gaindhar, J. L., Solidification Time and Mechanical Properties of Al-11%Si Alloy V-Processed Castings, Transactions of the American Foundry men's Society Journal, Vol. 104, pp. 635-638, 2002.

[12] Kumar, P., Singh, N. and Goel, P., A Multi-Objective Framework for the Design of Vacuum Sealed Molding Process, Robotics and Computer Integrated Manufacturing, Vol. 15, pp. 413-422, 1999.

[13] Kumar, S., Kumar, P. and Shan, H. S., Effect of Process Parameters on the Solidification Time of Al-7% Si Alloy Castings Produced by VAEPC Process, Materials and Manufacturing Processes, Vol. 22, pp. 879-886, 2007.

[14] Singh, R. and Singh, J., Effect of Moulding Sand on MMC Development in Vacuum Moulding, International journal of materials science and engineering, Vol. 2(1), pp.121-124, 2011.

[15] Singh, R. and Khamba, J.S., Mathematical Modeling of Tool Wear Rate in Ultrasonic Machining of Titanium, The International Jol. of Advanced Manufacturing Technology, Vol. 43(5-6), pp. 573-580, 2009.

[16] Singh, R. and Khamba, J.S., Mathematical Modeling of Surface Roughness in Ultrasonic Machining of Titanium Using Buckingham-Π Approach: A Review, International Journal of Abrasive Technology, Vol.2(1), pp. 3-24, 2009.