

## COMPARATIVE STUDY OF FORCE ANALYSIS AND HEAT GENERATION IN GRINDING

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**Abstract:** Heat generation in grinding process is a complicated phenomenon compared to other metal cutting processes. High specific energy is required in grinding in contrast to other machining processes. Before analyzing heat generation process, detailed force analysis is needed as forces play an important role for calculating specific energy in grinding. Several authors have worked on the force models in different ways. Due to plastic deformation and rubbing action, both temperature of the job and grinding wheel becomes very high. This high temperature causes burn, crack formation and tensile residual stress in the job. Two comparative force models and temperature analysis of the job have been presented in this paper. The temperature equation may be solved either by numerical process or some other methods to depict the temperature distribution inside the job.

**Key words:** Grinding, Modeling, Work fraction, Specific energy, Force, Temperature.

### 1. Introduction

Grinding is a commonly used method for finish machining. Grinding wheel wear, dynamic performance of the machine tool, geometric accuracy and surface quality of the workpiece are greatly influenced by grinding forces. For this, some considerable research works were done by a number of researchers in this area.

Based on the experimental results, tangential grinding force was modeled in terms of shear deformation force and frictional force by Malkin and Cook [1][2]. Another grinding force model was established on the basis of grinding experiments, and the formula to evaluate normal chip formation force was proposed by Li and Fu [3]. Badger and Torrance [4] proposed two kinds of grinding force models; the first is based on Challen and Oxley's two dimensional (2-D) plane strain slip line field theory, and the second one is based on the three dimensional (3-D) pyramid shape asperity model suggested by Williams and Xie. There are several other mathematical models of force analysis available.

Designing a technological process, such as grinding entails to deal with heating problem of the workpiece to be ground. During grinding, large amount of mechanical energy is transformed into heat, which is accumulated in the contact zone between grinding device and the workpiece. Therefore, it is of considerable industrial interest to understand the generation of heat and its conduction in order to minimize energy losses and to increase efficiency.

Classical modeling of grinding problem was done using coupled system of partial differential equations (PDEs). Andrews et al. [5] and Gu et al. [6] have shown how to calculate the evolution of interconnected temperature field in the wheel, the workpiece and the applied fluid. Most of the works reported in the literature on thermal aspects of grinding deal with experimental and numerical analysis, as detailed in ref. [7-9] among others. In this paper, only the PDE involving heat conduction and heat generation due to plastic deformation and rubbing action is considered.

### 2. Mathematical Model of Grinding forces

In this paper, authors shall discuss two force models as given by Tang et al. [11] and Patnaik et al. [10]. Both models are based on the model proposed by Malkin [12]. The model proposed by Tang [11] does not include any ploughing force, but model made by Patnaik [10] involved ploughing force.

According to Tang (2009) model,

$$\begin{aligned} F_n &= F_{n,ch} + F_{n,sl} \\ F_t &= F_{t,ch} + F_{t,sl} \end{aligned} \quad (1)$$

However, according to Patnaik (2010) model force has three components,

$$F = F_{ch} + F_{sl} + F_{pl} \quad (2)$$

Suffix 'n' indicates normal force component, and suffix 't' indicates tangential force component. Suffix 'ch' indicates chip formation force, 'sl' stands for sliding or friction force and suffix 'pl' denotes ploughing force.

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Accordingly, specific grinding energy is divided into specific chip formation energy,  $u_{ch}$ , and specific sliding energy,  $u_{sl}$ . The specific chip formation energy  $u_{ch}$  as given by Malkin and Cook [2] can be determined by equation 3.

$$u_{ch} = F_{t,ch} V_s / V_w a_p b \quad (3)$$

Where  $V_s$  is the grinding wheel velocity,  $V_w$  is work piece feed velocity, as shown in Fig. 1,  $a_p$  is the grinding depth and  $b$  is the grinding width.

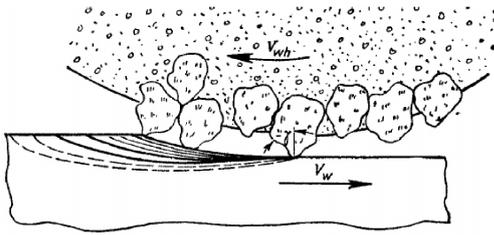


Fig.1 Cutting action of abrasives in grinding

### 2.1 Chip formation force

Specific chip formation energy,  $u_{ch}$  can be separated into two parts: static chip formation energy,  $u_s$  and dynamic chip formation energy,  $u_d$ .

$$u_{ch} = u_s + u_d \quad (4)$$

Here, static chip formation energy,  $u_s$  is a constant which is determined by experiment according to work material and grinding wheel material. Dynamic specific chip formation energy  $u_d$  is determined by element material, grinding wheel material and process parameters.

Experimental results obtained by Shinji et al. [13] indicate that shear strain rate is given by,

$$= 10v \sin \phi \cos \phi / a_c \cos(\phi - \phi_0) \approx vk/a_c \quad (5)$$

where,  $\phi$  is the shear angle,  $v$  is the cutting velocity,  $a_c$  is cutting depth,  $k$  is a constant. From equation (5), it is clear that cutting depth and cutting velocity  $v$  have remarkable influence on shear strain rate in shear zone.

It is generally agreed that grinding process of metal is similar to turning and milling, and material is removed by shear process [12]. Hence, conclusion deduced from cutting analysis can be applied to the research on chip deformation force in grinding. Depth of cut in metal cutting corresponds to average uncut chip thickness,

$a_g$  in grinding. Average uncut chip thickness as proposed by Shinji [13] is given by

$$a_g = a_{gmax} / 2 = (a_p^{0.25} v_w^{0.5}) / [(Cr)^{0.5} d_e^{0.25} v_s^{0.5}] \quad (6)$$

where,  $C$  is the number of effective abrasive blades in unit area of grinding wheel,  $r$  is the ratio of chip width to chip thickness and  $d_e$  is the equivalent diameter of wheel.

Substituting equation (6) in equation (5),

$$= k[(Cr)^{0.5} d_e^{0.25} v_s^{1.5}] / (a_p^{0.25} v_w^{0.5}) \quad (7)$$

Experimental data indicated [14] that shear flow stress is in direct proportion to logarithmic shear strain rate approximately. So, dynamic chip formation energy,

$$U_d = K_2 \ln(\gamma / \gamma_0) \quad (8)$$

Where,  $K_2$  and  $\gamma_0$  are constants and determined from experiments.

#### 2.1.1 Calculation of chip force formation

Chip formation energy is expressed by the following equation,

$$u_{ch} = u_s + K_2 \ln\{k[(Cr)^{0.5} d_e^{0.25} v_s^{1.5}] / (a_p^{0.25} v_w^{0.5} \gamma_0)\} \quad (9)$$

Considering  $K_1 = u_s + K_2 \ln\{k[(Cr)^{0.5} d_e^{0.25} v_s^{1.5}] / \gamma_0\}$ , equation (9) yields,

$$u_{ch} = K_1 + K_2 \ln\{v_s^{1.5} / (a_p^{0.25} v_w^{0.5})\} \quad (10)$$

$$\text{Now, } F_{t,ch} = (u_{ch} v_w a_p b) / V_s = K_1 (v_w b a_p) / v_s + K_2 (v_w b a_p) / v_s \ln\{v_s^{1.5} / (a_p^{0.25} v_w^{0.5})\} \quad (11)$$

In equation 11, the first term is static tangential chip formation force and the second term is dynamic tangential chip formation force. Setting  $K_3$  and  $K_4$  as ratio of static normal chip formation force to static tangential chip formation force and the ratio of dynamic normal chip formation force to dynamic tangential chip formation force respectively, considering,  $K_3 = K_1 / K_2$  and  $K_4 = K_2 / K_2$ , equation 11 yields,

$$F_{n,ch} = [K_3 + K_4 \ln\{v_s / (a_p^{0.25} v_w^{0.5})\}] (v_w a_p b / v_s) \quad (12)$$

### 2.2 Sliding force

In geometric dynamic analysis of grinding grains, using

parabolic function to approximate cutting path, deviation between grinding wheel radius and radius of curvature of cutting path, Hahn [15] defines,

$$= \pm 4V_w / (d_e v_s) \quad (13)$$

Positive sign is for up grinding and negative sign is for down grinding. Experimental data as indicated by Kannappan and Malkin [16] shows that the average contact pressure  $p$  between workpiece and abrasive grains approximately linearly increases as deviation of radius of curvature. So the relationship is given by,

$$P = p_0 = 4p_0 v_w / (d_e v_s) \quad (14)$$

Where,  $p_0$  is a constant and can be determined from experiments.

There may be elastic, elastoplastic or plastic contact between workpiece and abrasive grains. So, frictional co-efficient,  $\mu$  varies with average contact pressure. According to friction binomial theorem, frictional co-efficient is,

$$\mu = A_0 / W + \dots = \mu_p + \dots \quad (15)$$

Here,  $W$  is the normal load,  $A_0$  is the contact area, and  $\dots$  are coefficients which are determined by physical and mechanical properties of contact surface. Tangential and normal sliding force are as follows [2]:

$$F_{t,sl} = \mu p b A (d_e a_p)^{0.5} \text{ and } F_{n,sl} = p b A (d_e a_p)^{0.5} \quad (16)$$

Here,  $A$  is the area ratio of grinding wheel wear surface. So, the final equation for sliding force is expressed as,

$$F_{t,sl} = b A [ \dots + (4 p_0 v_w) / (d_e v_s) ] (d_e a_p)^{0.5} \quad (17)$$

$$F_{n,sl} = [ (4 b A p_0 v_w) / v_s ] (a_p d_e)^{0.5} \quad (18)$$

Different mechanisms for material removal in grinding are shown clearly in Fig.2.

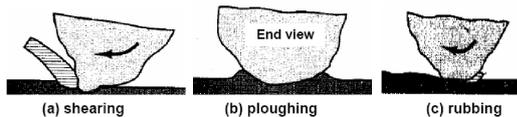


Fig. 2. Mechanism of material removal in grinding

### 2.3 Ploughing force component

Another mechanism associated with abrasive

machining process is ploughing (Fig.2b). Ploughing energy is expended by deformation of workpiece material without removal. Ploughing is associated with slide flow of material from cutting point, but it can also include plastic deformation of material passing under cutting edges. Initially, grit makes elastic contact, which is assumed to make a negligible contribution to total energy, followed by plastic deformation (ploughing) of workpiece. After analysis, as given by De Vathaire et al. [17] the force equation for ploughing becomes

$$F_{t,pl} = K_5 (v_w / v_s)^{a_0} (d)^{b_0} (a_p)^{c_0} C (a_p d_e)^{0.5} \quad (19)$$

$$F_{n,pl} = K_6 (v_w / v_s)^{a_0} (d)^{b_0} (a_p)^{c_0} (a_p d_e)^{0.5} \quad (20)$$

Where,  $d$  is the diameter of grit or grain and  $C$  is the grain distribution of grinding wheel (number of grains per unit area).

### 3. Temperature Distribution Model

From force analysis, it is observed that the entire force components do some work or these components contribute to the specific energy in grinding process. Now, for both chip formation and ploughing, some plastic works are needed. During plastic deformation, obviously temperature increase occurs at plastic zone due to dissipation of plastic work. The majority of plastic work (>90%) is converted into heat before cracking appears. The problems that involve both heat generation due to plastic work and heat conduction, it is advantageous to define a new factor called as plastic work fraction is given by,  $\dots = Q / Wp$ , where  $Q$  is the heat production density rate and  $Wp$  is plastic work density rate. Generally, converted plastic work fraction is assumed to be constant in the range of 0.9-1.0. So, the PDE for temperature distribution of the workpiece can be written as,

$$\nabla^2 T + (u_{ch} + u_{pl}) + u_{sl} = C_p \frac{\partial T}{\partial t} \quad (21)$$

The term  $(u_{ch} + u_{pl}) + u_{sl}$  is responsible for heat generation in the workpiece. Equation (21) can be solved by FEM or any other numerical method to get the heat distribution in the workpiece.

### 4. Conclusion

Grinding is an interesting phenomenon to work on. Force models discussed in this paper are mainly based on the experimental results. The exact solution for

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grinding process is still unavailable. Theory of plasticity can be used to determine the exact force models of grinding process. Especially, the ploughing and friction phenomenon require special attention as these two processes have not been modeled accurately. Several research works have been done to find out the temperature distribution of grinding workpiece taking grinding zone as moving heat source. Several exact solutions, numerical models and experimental data are available for temperature analysis in grinding process. Still, there are scopes to work in this area of grinding.

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