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Research Article

ANALYTICAL, EXPERIMENTAL AND NUMERICAL STUDY ON FORMATION OF LAMINAR BOUNDARY LAYER OVER A FLAT PLATE USING AIR AS WORKING FLUID

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Abstract: The work presents in this paper is related to comparison of laminar boundary layer thickness of flow of air over a horizontal flat plate at different points among Analytical, Experimental and Numerical approaches.

Keywords: *Reynolds number, Wind tunnel, Staggered grid, Marker And Cell algorithm, Blasius equation, Shooting Technique.*

1. Introduction

When a real fluid flows past a solid wall, the fluid particles adhere to the boundary and condition of no slip occurs. This means that the velocity of fluid close to the boundary will be same as that of the boundary. If the boundary is stationary, the velocity of fluid at the boundary will be zero. Further away from the boundary, the velocity will be higher and as a result of this variation of velocity, the velocity gradient $\frac{du}{dy}$ will exist. The velocity of fluid increase from zero velocity on the stationary boundary to free-stream velocity (U_∞) of the fluid in the direction normal to the boundary. This variation of velocity from zero to free-stream velocity in the direction normal to the boundary takes place in a narrow region in the vicinity of solid boundary. This narrow region of the fluid is called boundary layer. [1][4]

Till today no direct analytical solving technique is available to solve the laminar boundary layer equations. so, at first Shooting technique was applied with proper boundary conditions in this particular case to find out most probable boundary layer thickness of air flow over a horizontal flat plate. Next, an experiment was done in low speed wind tunnel in Fluid Mechanics And Fluid Machinary Laboratory to find out the boundary layer thickness at the same points of the the same working fluid. At last taking the control volume of the top portion of the horizontal flat plate for a finite difference scheme using Staggered grid and Marker And Cell algorithm was applied with proper boundary conditions to solve Navier-Stokes equation to find out boundary layer thickness at the same experimentation points .In the all above cases Reynolds number value was taken as 5×10^5.

2. Analytical Approach

The Prandtl boundary layer equations in case under consideration are

$$u\frac{\delta u}{\delta y} + \nu\frac{\delta u}{\delta y} = \nu\frac{\delta^2 u}{\delta y^2}$$

$$\frac{\delta u}{\delta x} + \frac{\delta v}{\delta y} = 0$$

The boundary conditions are

At y=0, u=v=0

At y=δ, u = U_∞ where

u= x component of fluid velocity

u= y component of fluid velocity

U_∞ = Free steam velocity

By similarity analysis,

$$\frac{U}{U_\infty} = \left(\frac{y}{\sqrt{\frac{vx}{U\infty}}}\right) = F(\eta)$$

Where $\eta = \frac{y}{\delta}$ and $\delta = \frac{vx}{U_\infty}$

and δ = Boundary layer thickness

From this analysis an equation can be developed as

$$2f'''(\eta) + f(\eta)f''(\eta) = 0$$

This is known as Blasius Equation.

Where $F(\eta) = \int f(\eta)d\eta$

The modified boundary conditions are,

at $\eta = 0: \; f(\eta) = 0, \quad f'(\eta) = 0$

at $\eta = \infty: \quad f'(\eta) = 1$

Blasius equation is a third order nonlinear differential equation. Blasius obtained the solution of this equation in the form of series expansion through analytical techniques. Here a numerical technique to solve the aforesaid equation has been discussed.

It is to be observed that the equation for f does not contain x. Further the boundary conditions at x = 0 and y = ∞ merge into the condition $\eta \to \infty$, $\frac{u}{U_\infty}$ = $f' = 1.$

This is the key feature of the similarity solution.

The Blasius Equation can be written as three first order differential equations in the following way:

$f' = G$

$G' = H$

$H' = \frac{1}{2} f H$

Let one next consider the boundary conditions. The condition f (0) = 0 remains valid. Next the condition $f' = 0$ means that G (0) = 0. Finally $f'(\infty)$ = 1gives us G (∞) = 1. Note that the equations for *f* and *G* have initial values. However, the value for *H(0)* is not known. Hence, there is no usual initial-value problem. Nevertheless, this problem is handled as initial-value problem by choosing values of *H(0)* and solving by numerical methods f (η), G (η), and H (η). In general, the condition will not be satisfied for the function *G* arising from the numerical solution. Then choosing other initials values of H so that, eventually results are found. This method is called the *shooting technique*. This is solved by fourth order Runge-Kutta method.[2]

3. Experimental Approach

3.1 Procedure

1. Using the barometer to determine the density of the air flowing in the wind tunnel and thermometer in the laboratory to determine atmospheric temperature during experiment.
2.Using the Wind Tunnel Calibration to calibrate the flat plate test section in equal five parts and by generating plot of velocity(m/sec)versus distance of pitot tube(m) using the upstream pitot-static tube and Bernoulli's equation.

Applying governing equations, one gets;

$$\frac{\Delta z}{\Delta h} = \sin\theta$$

or, $\Delta z = \Delta h \sin \theta$

Now,

$\Delta h_{air} \times \rho_{air} = \Delta z \times \rho_w$

or,

$\Delta h_{air} \times \rho_{air} = \Delta h \sin \theta \times \rho_w$

Also,

$$V_{air} = \sqrt{2 \times g \times \Delta h_{air}}$$

or,

$$V_{air} = \sqrt{2 \times g \times \frac{\rho_w}{\rho_{air}} \times \Delta h \sin \theta}$$

Here the respective constant values are as follows:

$\rho_w = 1000$ kg / m^3

$\rho_{air} = 1.2$ kg/m^3

$\theta = 16.5°$ indination angle of manomeler

3. The tunnel is operated at different air speeds by using variac such as 50%, 60%, 70%, 80% and pressure measurements are taken on the flat plate at different positions.
4. At first at 60 % variac measures the pressure variation at same interval of the flat plate in different depth of Pitot tube. Thus, measure other data similarity at other intervals also.
5. The air speed is altered by changing variac and similarly, notes all data at other variac positions.

3.2 Numerical relations

1. The values of wind velocity are calculated at different intervals of variac positions by using above mentioned equations.
2. Also the values of Reynolds's number are calculated by using well known relationship between wind velocity (U_∞), distance of pitot tube over flat plate(x), kinematic viscosity(ν)

$v = 15.89 \times 10^{-6 m^2} /_s$ at T = 307K

$$Re_x = \frac{U_\infty x}{v}$$

3. Finally calculate boundary layer thickness (δ) when it is related with Pitot tube position as

$$\frac{\delta}{x} = \frac{5.0}{\sqrt{Re_x}}$$

4. Numerical Approach

As it has been seen, the major difficulty encountered during solution of compressible flow is the non-availability of any obvious equation for pressure. This difficulty can be resolved in the stream function-vorticity approach.[3]

A numerical technique for the solution of inviscid, incompressible flow is the relaxation technique. Inviscid, incompressible flow is governed by elliptic partial differential equations and the relaxation technique, which is essentially an iterative process, is a classical numerical method for solving elliptical problems. In contrast, viscous, incompressible flow is governed by the incompressible Navier-Stokes equations, which exhibit a mixed elliptic parabolic behavior. The purpose of the present section is to describe an iterative process called *pressure correction technique,* which has been found wide spread application in the numerical solution of incompressible Navier-Stokes equation.[3]

4.1 Numerical steps

Preparation of input file for coding:
For grid system,

$\Delta x = 0.00125$m

$\Delta y = 0.005$m

$\nu = 0.000009$ m^2/s

Running coordinate along x direction,

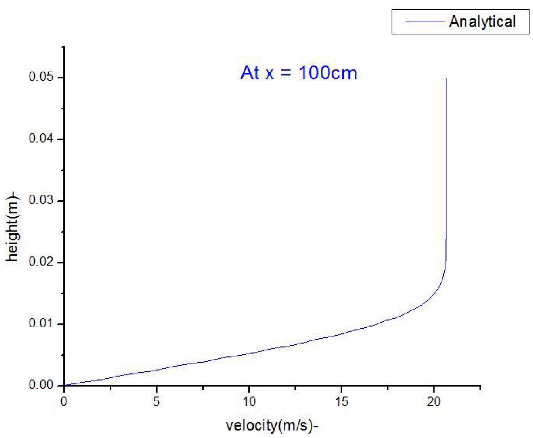
i= 1 to 801

Running coordinates along y direction, j=1 to 31

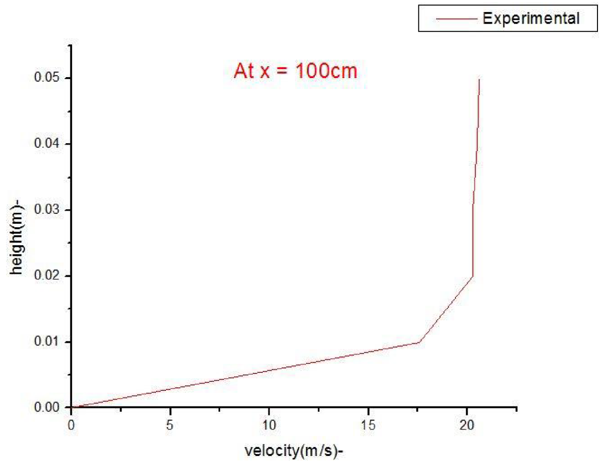
a. Creation of output files from velocity, profile, boundary layer measurements.
b. Initialization of velocity fields.
c. Implementation of boundary conditions.
d. Computation of velocity fields.
e. Writing outputs in velocity, profile, boundary layer respectively.

5. Results

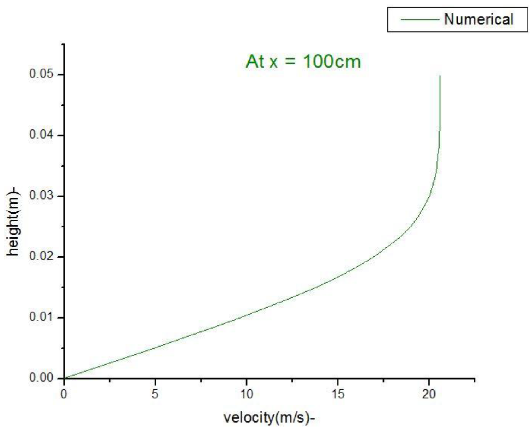
5.1 Graph by analytical approach



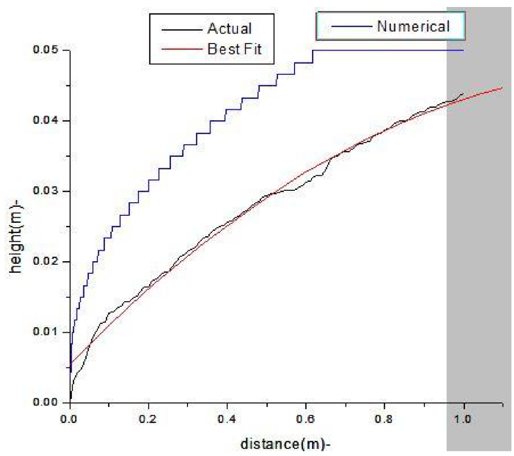
5.2 Graph by experimental approach



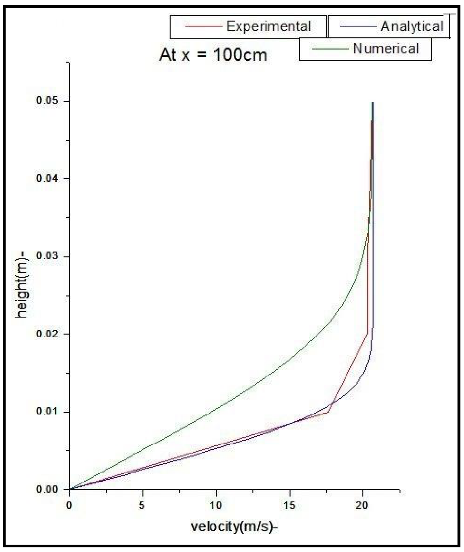
5.3 Graph by Numerical approach



5.4 Merged graph of velocity profile



5.5 Merged graph of boundary layer



6. Discussion

6.1 Analytical analysis

In the Shooting Technique the Blasius Equation over a Horizontal Flat Plate is solved by Tri Diagonal Matrix Algorithm (TDMA). Here the values are uniformly distributed along a curve hence a smooth curve is obtained. [3]

6.2 Experimental analysis

The differences of the experimental data from the analytical and numerical results are due to the occurrence of the following errors.

1. Jamming of the pitot tube tip.
2. Manual errors during the collection of the data from the manometer due to oscillation of the fluid.
3. Pressure drop in the wind tunnel due to the leakage of air.

6.3 Numerical analysis

In the numerical analysis it is not possible to achieve the exact solution due to some inevitable errors like truncation errors and discretization error etc.
The Truncation error (TE) is the difference between the Partial Differential Equation (PDE) and Finite Difference Equation (FDE).[3]

7. Conclusion

From the above three analysis (Numerical, Experimental and Analytical) a close relationship is found among the obtained graphs stated earlier. Firstly, comparing the experimental results with the analytical results it is seen that the experimental values are very close to the analytical (ideal) one. It is not possible to carry out the experiment everywhere by the wind tunnel. Hence, an alternative way by numerical approach by Computational Fluid Dynamics. By merging the graphs of numerical with analytical and experimental result, it is seen that the developed program can be accepted as an alternative way. Hence it is found out that the numerical coding serves the purpose of a Portable Wind Tunnel.[1][2]

8. REFERENCES

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