FUZZY LOGIC: CERTAIN UNCERTAINTIES

Siddhartha Bhattacharyya
Lecturer, Department of Information Technology

1. Introduction:

Let us talk philosophy, no technology this time. Imagine yourself to be a poet philanthropic to nature and its beauty. Think yourself close to nature. Surely and certainly now you are in a position to appreciate the whims and whisks of nature in its true sense. A red rose will always seem red to a common man but it may or not be that same beautiful to every one. A poet in you can only aptly express the essence of nature. The nature is "*beautiful*", "*more beautiful*", "*most beautiful*" to you as well as "*wild*", "*more wild*", "*most wild*" to you at times. These different feelings about a red rose or about the nature cannot be predicted because it depends on your state of mind and my friend I have a question: "Can anyone measure mind?"

Strangely enough, these appraisals of nature, which can be made by some commonplace words, seem meaningless to a technologist. This is due to the fact that technologists think otherwise. They mean business. They mean quantity and measures.

But the sad part of the story is that these and a score of other complex human feelings cannot be quantified or measured. They are judged by their quality of ambiguity only. These qualities differ from man to man and this demarcation of human feelings is also not well defined. In this work-a-day world, each and every activity is complex and ambiguous in terms of its evaluation. There is always uncertainty in the form of ambiguity in every human feeling. In the words of **Lotfi Zadeh**, father of fuzzy logic and Professor, Systems Engineering, "*As the complexity of a system increases, our ability to make precise and yet significant statements about its behavior diminishes until a threshold is reached beyond which precision or significance (or relevance) become almost mutually exclusive characteristics*". However, problems encompassing complexity and ambiguity have been addressed subconsciously by humans since they could think and possess the power of reasoning.

As another example, this article may seem "good" if not "very good" if not "very very good" to you. But can you tell me as to how have you measured its quality (which is indicative of my own sense of art)? I am telling you that you cannot measure it because it is fuzzy.

All these uncertainties of human feelings in this real world as well those existing in the nature cannot be dealt with the conventional reasoning tools and logic since these tools rely solely on quantifying quantities. In fact these ambiguities pose a challenge to the technologists for proper explanation and clarification. Herein comes the mystic world of fuzzy logic to their rescue to explain these ambiguities. The power of fuzzy logic to tackle this situation lies in its ability to qualify qualities.

2. What is Fuzzy Logic?

The nature of uncertainty in a problem is a very important point that engineers should ponder prior to the selection of an appropriate method to express the uncertainty. Fuzzy sets and fuzzy logic provide a mathematical way to represent vagueness in humanistic systems. For example, suppose you are teaching your child to prepare tea for your guests and you want to give instructions about when to take the teapot out of the oven. You could say to take it out when the temperature inside the tea reaches 80^{0}C, or you could advise him to take it out when the water turns *dark brown.* Which instruction would you give? Most likely, you would use the second option. The first instruction is too precise to implement practically and in this case however, precision is not important. The vague term "*dark brown*" is more useful and can be acted upon even by your child. These linguistic terms are referred to as "fuzzy hedges" and fuzzy logic is centered on using these terms and other fuzzy data to tackle situations governed by chance or probability. However fuzzy logic and probability are not the same. This will be dealt with later on.

3. **Fuzzy Sets vs. Crisp Sets:**

Now let us talk mathematics. The conventional set theory deals with crisp sets, which contain explicitly different unique and well-defined elements from the universal set of discourse. A crisp set A of four natural numbers can be written as :

$$A = \{x_1, x_2, x_3, x_4\}$$

where x_1, x_2, x_3, x_4 etc. are the natural numbers. These elements are strictly contained in the set A. Any other element x_5, which is not an element of this set lies strictly outside the set as shown in Figure 1(a). From the figure it is clear that a crisp set has a sharp and well-defined set boundary.

A fuzzy set, however, contains elements, x_i which, are governed by some degree of containment (membership value $\mu(x_i)$, indicative of the uncertainty in containment) in the set. A fuzzy set $\underline{B}$ containing four elements x_1, x_2, x_3, x_4 can be represented in Zadeh's notation as:

$$\underline{B} = \left\{\frac{x_1}{\mu(x_1)} + \frac{x_2}{\mu(x_2)} + \frac{x_3}{\mu(x_3)} + \frac{x_4}{\mu(x_4)}\right\}$$

which can be interpreted as that the fuzzy set $\underline{B}$ contains four elements, out of which element x_1 is contained in the set with a membership of $\mu(x_1)$, x_2 is contained in the set with a membership of $\mu(x_2)$ and so on. This membership value of an element can be any value between 0 and 1 reflective of the uncertainty and ambiguity in the containment of the element in the set. Depending on the membership value, an element may be completely within the set or may be nearer to the set boundary or may be on the set boundary or may be outside the set boundary as well. To be precise, even if an element is outside the set boundary, for example, the element x_5, it cannot be an element of the crisp set A whereas it can be an element of the fuzzy set $\underline{B}$ with a very low membership $\mu(x_5)$. Thus, a fuzzy set has an imprecise and undefined set boundary as shown in Figure 1(b). When the membership values of all the elements in a fuzzy set become either 0 or 1, then the fuzzy set turns into a crisp set. Thus a crisp set is a fuzzy set with membership values of the elements as either 0 or 1, i.e. with no certainty (0) or with full certainty (1). Fuzzy sets are very useful in dealing with the very different uncertainties observed in nature.

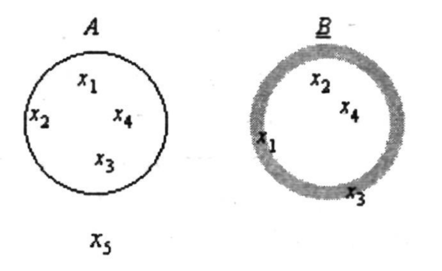

Figure 1(a) (b)

4. **Fuzzy Membership Function :**

All information contained in a fuzzy set is described by a membership value which is defined by a non-linear function of the memberships versus the range of values of the information containing fuzzy data elements. Different types of membership functions depending on the nature of the fuzzy data are in existence. A membership function is depicted in Figure 2.

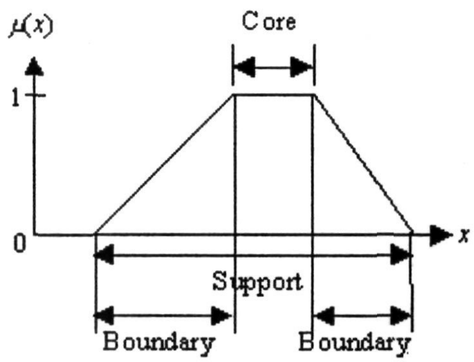

Figure 2

The *core* of a membership function for some fuzzy set is defined as that region of the universe that is characterized by full and complete membership of 1 in the set.

The *support* of a membership function is defined as that region of the universe that is characterized by non-zero membership. The *boundaries* of a membership function are defined as those regions of the universe containing elements that have non-zero but not complete membership. The maximum membership value of all the elements of a fuzzy set is referred to as the *height* of the fuzzy set. A fuzzy set with *height* =1 is referred to as a **normal fuzzy set** whereas that with a non-unity height is referred to as a **subnormal fuzzy set**. A *convex* fuzzy set is one whose membership values are either strictly monotonically increasing or strictly monotonically decreasing or either strictly monotonically increasing then strictly monotonically decreasing with increasing values for elements in the universe.

5. **Fuzzy Rule Base:**

The most powerful form of conveying information that humans possess in a situation is natural language. Since these information are often quite vague, imprecise, ambiguous and fuzzy, describing this form of communication with the help of fuzzy sets is a very useful proposition. The language can be broken down into fundamental terms referred to as "atomic terms". Some examples are "*slow*", '*medium*", "*young*", "*beautiful*" etc. These atomic terms can be connected together to form composite terms e.g. "*very slow speed*", "*medium height male*", "*young woman*", "*fairly beautiful picture*" etc. The adjectives or adverbs like "*very low*", "*more-or-less*", "*slightly*", "*almost*", "*roughly*", "*approximately*" etc. are also referred to as **"linguistic hedges"** as they act as modifiers or hedges to the natural language terms. If α is a fundamental atomic term, then:

$$\text{"Very"}\ \alpha = \alpha^2$$
$$\text{"Very Very"}\ \alpha = \alpha^4$$
$$\text{"Slightly"}\ \alpha = v\alpha$$

are the corresponding linguistic hedges. The first two expressions are referred to as *concentrations* while the third one is referred to as *dilation* of the atomic term α. Another hedge referred to as *intensification* is obtained by the combination of the *concentration* and *dilation* type of hedges.

These terms are used to formulate linguistic rules that can be best used to describe the fuzziness in human activity and knowledge as:

IF antecedent THEN consequent

It typically expresses an inference that if we know a fact (antecedent), then we can infer another fact (consequent). This type of representation is known as *shallow knowledge.* This can be substantiated by some examples like:

IF a tomato is red THEN it is ripe
IF x is red THEN y is cool ELSE y is hot

Fuzzy rule bases form the backbone of systems with nonlinear and fuzzy behaviors like that of humans. A set of such rules can be designed to explicitly describe system behaviour and status.

6. Fuzzy Control:

Modern control engineering is adept in dealing with linear systems. However, attaining at a stable nonlinear control system is a uphill task as the system response knows no bounds. Fuzzy rule based systems are efficient in simulating nonlinear systems such as experienced in human psychology and understanding without resorting to any mathematical or quantitative measures. Hence rule bases can be used to handle the nonlinear behaviour of complex nonlinear systems.

As an example consider a system which decides "tips to the bearer" in a restaurant. Before going into the intricacies of the required rule base, let us address the relevance of fuzzy logic to this system. In a restaurant, food is served to the guests. The quality of food, service of the bearer and the ambience of the restaurant are the guiding factors in deciding as to whether the guest decides to give "tips to the bearer". But none of these guiding factors are measurable. Can we quantify food quality or service or ambience? Hence, a properly designed rule base is the only way to reflect the mental state of affairs of the guests who walk in and take their food. Consider the human being in a guest. He/she will be impressed only by the goodness of all the three factors, he/she will be displeased with the badness of them and his/her impression will tell upon his/her willingness to pay "tips to the bearer". So such a system might be expressed by the following set of rules.

IF quality is good and service is good THEN tip is high

IF quality is medium THEN tip is moderate

IF quality is bad and service is bad THEN tip is low

Thus fuzzy logic is able to replicate human nature as regards to human reasoning.

7. Fuzzy Logic vs. Probability:

Fuzzy logic is no probability. A fuzzy membership value cannot be treated as a probability value. While probability deals with only two possible options about the existence of a variable/element in a set, fuzziness result in infinite options for the same. This can be enunciated by a simple example.

Consider a situation where you are given two glasses each concealed in boxes. One glass has a membership of 0.5 for having water in it (Fig. 3(a)), while the other is with a probability of 0.5 for having water in it (Fig. 3(b)). Now, can you tell as to which glass to choose if you are thirsty?

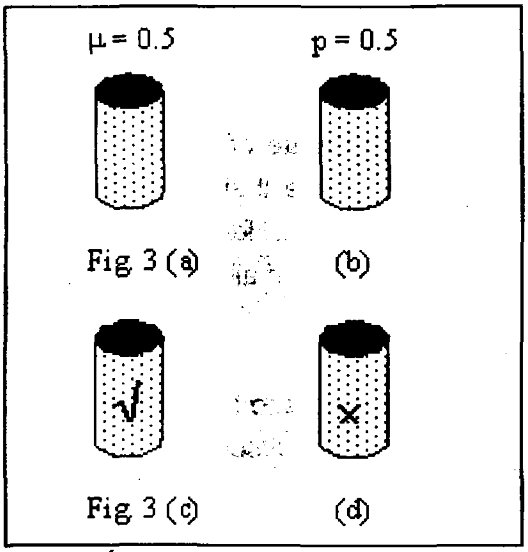

Fig 3 (a) (b)

Fig 3 (c) (d)

The answer is given in Fig.3(c) and 3(d). This can be attributed to the fact that while probability does not ensure presence of water in the glass, a membership value of 0.5 does not nullify the presence of water in the glass. The situation is similar to having a glass "half full" or "half empty" of water. This shows that fuzziness is different from probability.

8. Real Life Applications:

Fuzzy sets and fuzzy logic under the banner of "*soft computing*" find wide use in several modern fields of engineering and technology. Since fuzziness can certainly describe uncertainty, intelligent systems like robots heavily rely on fuzzy logic concepts. Moreover, image processing and pattern recognition tasks deal with voluminous amount of data. Proper interpretation of these data can be faithfully done with the help of fuzzy sets and fuzzy logic. Similar approaches are now a days taken for handling GIS data as well. Emerging technologies like real time embedded system design as well as image and web-mining applications are more and more becoming dependent on the power of reasoning that fuzzy logic possess.

9. Discussions:

In this article, I have tried to make you familiar with the jargons of fuzzy logic. The impact of fuzzy logic is so huge that it is not possible to touch upon all the facets in a single article. To be precise, fuzzy logic is involved with the softer (humanly) side of state of affairs rather than the harder (calculated) part.

10. Further Reading:

[1] L. A. Zadeh, "Fuzzy Sets", *Inform and Control*, vol. 8, pp. 338-353, 1965.

[2] T. J. Ross and T. Ross, *Fuzzy Logic With Engineering Applications*, McGraw Hill College Div, 1995.

[3] Elkan. C., "The paradoxical success of fuzzy logic", *IEEE Expert*, pp. 3-8.

[4] Bezdek. J., "Editorial: Fuzzy models: What are they, and why?", *IEEE Trans. Fuzzy Syst.*, vol.1, pp. 1-5.

Windows Tips and Tricks :

Can you move a window without your mouse? Just press Alt and SpaceBar keys to active the system menu of the nonmaximized active window, then press 'M' or 'm' and then press the cursor keys to move it. Isn't it simple? Send feedback to siddharthab2k@rediffmail.com or mail2arth@yahoo.com