Optimization In Mechanical Design

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In conventional design, majority of the parameters is selected as they appear for the fulfillment of the functional requirement with scant attention to minimum overall cost or maximum effectiveness. On the contrary, optimization approach emphasizes on the selection of parameters for maximization of overall effectiveness while considering the related factors and limitations simultaneously. An integrated approach is required for the optimum design. This article tries to introduce the concept of optimization in mechanical design through over-simplified examples.

1. Introduction

Mechanical Design is the assemblage of thoughts, generation of ideas, and application of scientific methods and practices put together to give a practical shape to a component or system for meeting a specific function. The conventional design emphasizes more on the fulfillment of the functional requirement through selection of parameters, often conflicting in nature, as they occur than the overall consideration for the cost or effectiveness. R.C. Johnson defines it as - the selection of materials and geometry, which satisfies specified and implied functional requirements while remaining within the confines of inherently unavoidable limitations.

2. Adequate Design:

In conventional design, we clearly state the functional requirements of the component to be designed along with the various limitations or constraints. We also identify the secondary effects, both desirable and undesirable, of the various design parameters on the component and its cost. Then we proceed on selecting the design parameters, mostly based on experience and practices, and attempt to satisfy the functional requirements while remaining within the confines of the identified constraints. While doing so, often we face conflicting situations, which are resolved through cut-and-try technique. Only a few parameters are selected through design formulations. This leads to an infinite number of possible solutions with varying degree of overall effectiveness. Any one of the possible design solutions, which satisfies the functional requirements within the confines of existing limitations, is an adequate design. It is characterized by cut-and-try technique using intuition and often, rule of thumb.

As a consequence of this approach of adequate design, we design a mechanical component for which the material, shape, production process, surface conditions and majority of the dimensions are selected from a large list to fulfill the main objective- functional requirement. We cannot ensure that the design is the best one. We may say that is an adequate design, which satisfies the functional requirement.

2.1 Secondary Effects of Adequate Design:

A mechanical component designed through conventional approach will result into some desirable and undesirable effects. The designer should generate a large number of alternatives, systematically identify the desirable effects and rank them in respect of primary function. Some of these desirable effects are-

- Power transmission capability
- Speed capability
- Momentary overload capability
- Useful length of life
- Reliability, maintainability, etc.

As a result of design, the component will have some undesirable effects also. Though we wish them at zero level, they do appear at higher levels in any practical design. Some of the undesirable effects are-

- Stresses
- Vibrations, noise
- Space occupancy
- Weight
- Cost

The degree of significance of the above effects depends on the particular application. When weight is the most significant undesirable effects in aircraft component design, cost may be the most significant undesirable effects in design of plant and machinery.We try to reduce the level of undesirable effects but cannot totally remove them. In other words, there exists a tolerable limit for the undesirable effects. The same is true for desirable effects. We have to identify the tolerable limits for both the desirable and undesirable effects very carefully and consciously.

3. Optimum Design:

The design alternative, which maximizes the most significance desirable effect or minimizes the most significant undesirable effect, is the optimum design. In fact, in real design situation, optimum design attempts to maximize the multiple objectives applying Operation Research techniques. In general, the least cost alternative, which satisfies the functional requirement, is the optimum design. Here, it is very important to specify accurately what is needed, the range of acceptance or rejection levels. Any thing more than the acceptable level will make the component costlier. We shall explain the approach with examples.

3.1 Example-1: Design of a Tray

It is required to design a tray capable of holding a specified volume, *V*, such that the liquid has a specified depth, *H*, and anti-spillover depth, *h*. The tray is to be manufactured in large numbers.

3.1.1 Adequate Design Solution:

Material: The material is selected on the basis of its compatibility with the liquid to be held and the manufacturing process. When more than one material are suitable for the purpose, we often select a commonly available or used material without thoroughly evaluating all the alternative materials and production process. Let us select steel sheet as the material for the tray and deep drawing as the production process.

Geometrical Shape: The tray could take any possible shape; but the total cost of manufacture per unit of tray, including the tooling cost will determine the shape. In conventional design, we feel comfortable in choosing a commonly used shape without investigating the other possible shapes. In this case, let us select the rectangular shape for the tray as shown in Fig.1.

Dimensions: Since a specified volume *V* of liquid is to be held in the tray with a constant spill-over depth of h above the liquid level, the various dimensions will be related as-

$V = blH$ .... (1)

where b = width and l = length of tray.

(Assuming wall thickness, T is small compared to b and l

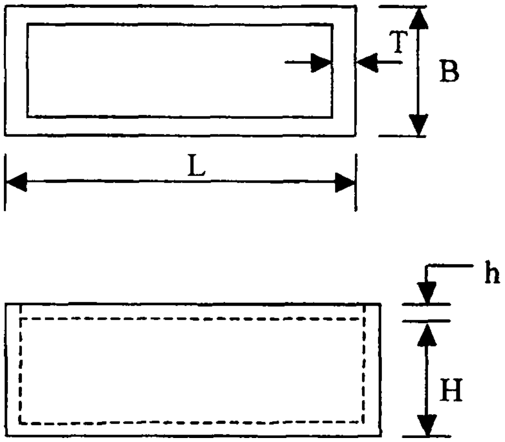

Fig. 1.

By selecting an arbitrary value of *b*, the value of *l* can be found out from equation-1. The wall thickness *T* is calculated on the basis of liquid pressure of the tray and strength of material used. But here it is selected arbitrarily, based on our past experience. This leads to an infinite number of solutions, which would be entirely satisfactory but may not be most economic.

3.1.2 Optimum Design Solution:

Here, the most significant (implied) undesirable effect is - Cost. Hence, the explicit objective would be to design the tray on the basis of minimizing the total cost per tray.

We can describe the total cost of tray (per unit basis) as -

$$C = Co + C_t + C_l + C_m$$

where C = cost of a tray

C_o = overhead cost;

C_t = tooling cost;

C_l = labor cost;

C_m = material cost.

(All on unit tray basis)

This may be called a Preliminary Design Equation (PDE) .

Since deep drawing process of production is used, overhead, tooling and labor costs are independent of reasonable geometrical shape. Hence, optimum design will be determined by only material cost, C_m. Let us investigate the effect of various shapes on the total cost.

For traditional rectangular shape of tray,

$$C_m = c(b\,l + 2\,b[H + h] + 2\,l[H + h])T \ldots \quad (2)$$

where c = cost of unit volume of material.

Replacing l from the above equation by using relation-1, we have –

$$C_m = c\,(V/H + 2b[H+h] + 2V[H+h]/Hb\,)T.. \quad (3)$$

Equation-3 may be called a developed Primary Design Equation as the optimum design can be extracted explicitly from this equation.

Differentiating C_m with respect to b and setting equal to zero, we have-

$$\delta (C_m)/\,\delta (b)=cT(2[H+h]-2V[H+h]/Hb^2)= 0$$

Therefore, $b_{opt} = \sqrt{(V/H)}$

Putting the value of b_{opt} in equation-3, we get-

$$(C_m)_{opt} = c\,[\,(V/H\,)+ 4\,(H+ h)\ \sqrt{(V/H\,)}\,]\,T \ \ldots(4)$$

Restrictions:

In practice, the shelf on which the tray will be placed, has a limited depth, b_{max}. If $b_{max} >= b_{opt}$, the optimum solution remains unchanged. If not, the optimum design will change as shown in Fig.2. The length of the tray will increase to l_1 to satisfy the volume restriction in equation-1 and consequently the cost will increase to C_{m1} as shown in Fig.3.

If there is also a space restriction on the length of tray, the optimum solution will change when $l_{max} < l_{opt}$.

- But, if $(b_{max}\ l_{max}) < V/H$, then there exists incompatible specification.

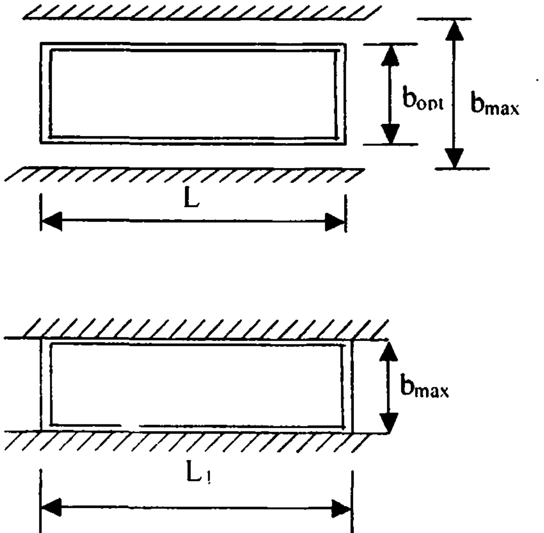

Fig.2

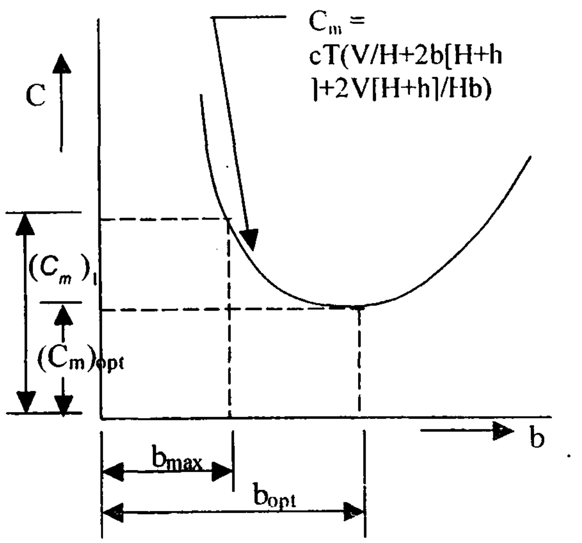

Fig.3

If a circular tray of diameter D is considered, then-

$$C_m = c\,(\pi D^2/4 + \pi D[H + h]\,)\,T \ \ldots\ldots(5)$$

and

$$V = (\pi D^2/4\,)\,H \ \ldots\ldots\ (6)$$

From equations-5 & 6, we get –

$$C_m = c\,[\,(V/H\,) + 2(H+ h)\ \sqrt{(\pi V/H\,)}\,]\,T$$

$$= c\,[\,(V/H\,) + 3.54(H+ h)\ \sqrt{(V/H\,)}]\,T \ \ldots\ldots(7)$$

Comparison of equations-4 & 7 indicates that circular tray will result in lower cost.

This economy in cost was made possible by extending our limits of traditional thoughts of selection of shape of the tray.

Space restriction, as discussed for rectangular tray, may also be applicable for circular tray.

3.2 Example-2 : Design of a Tensile Bar :

Design a tensile bar of specified length, L to transmit a constant force, P. The bar will be produced in large quantity. Select the section and material for minimum cost.

3.2.1 Primary Design Equation (PDE) :

The tensile bar under the constant force P is shown in Fig.4.

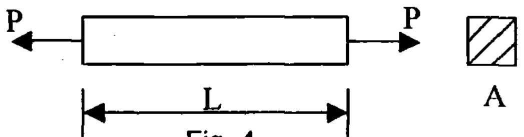

Fig. 4

The Primary Design Equation (PDE) may be written as - $C_m = c\,L\,A$

where C_m = cost of bar material.
c = cost of unit volume of bar material.
L = length of bar
A = cross sectional area of the bar.

Subsidiary Design Equation (SDE) :

From mechanics of deformed bodies, we know that - $\sigma = P/A$

where σ = normal stress in the bar, P = axial force, A = cross sectional area of the bar.

Let us assume that the application does not tolerate yielding of the bar. Then the stress is a significant implied undesirable effect.

Though there are no constraints or limit equations directly specified, permissible value of stress, σ is implied by the theory of failure. Hence limit equation (LE) is-

$$\sigma <= S/N$$

where S is the yield strength of the material, and N is the factor of safety based on occurrence of significant failure phenomenon (here, yield).

Based on the Subsidiary Design Equation, we can select the section of the bar, which will satisfy the functional requirement.

3.2.2 Optimum Design:

From the statement of the problem, the following may be observed:

P, L, N_y - are functional requirements of the bar
c, S_y - are material parameters of the bar
A - is geometrical parameters of the bar
σ, C_m are undesirable effects of the bar

Minimization of the cost of material could be the basis of Optimum Design.

Developed Primary Design Equation :

Developed Primary Design Equation can now be formed by combining PDE and SDE to get –

$$C_m = c\,L\,(P/\sigma) \quad ....(8)$$

When the maximum value of σ is taken as S/N, equation-8 takes the form-

$$C_m = (P\,L\,N)\,(c/S) \quad ....(9)$$

Here, $(P\,L\,N)$ = Functional requirement group
(c/S) = Material selection factor (MSF)

Material selection factor is very important in reducing the cost of the component designed. From a list of available feasible materials, MSF can be worked out and the material having the lowest MSF will be selected.

Though the factors P and L are given and hence independent, the value of N is chosen by the designer on the basis of load and other conditions. Once the material is selected, yield strength, and hence optimum cross -sectional area of the tensile bar can be determined by the relation-

$$A = (P\,N/S)$$

We rewrite the PDE –

$$C_m = c\,L\,A$$

Taking log on both sides,

$$\mathrm{Log}\,(C_m) = \log\,(c\,L) + \log\,(A) .....(10)$$

When drawn on log-log scale, equation-10 will be straight lines for different material. The lowest value of A and hence C_m will be restricted by yield strength of the material.

If there is also a limit equation for the minimum or maximum size of the cross-sectional area, the relations can be shown in graphical form. Use of such graphs help in selecting the optimum design parameters under various restrictions.

An automated optimum design algorithm named OPTIGO was developed by R.C.Johnson and used for very complicated mechanical elements.

4. Factor of Safety (FOS):

The concept of Factor of Safety (FOS) has long been present but the basis for selection remained vague. This lead to selection of different value by different designer even for the same component. FOS is often considered as 'factor of ignorance' or 'factor of carelessness'. This often leads to very high cost of the component. The subject is vital for any design, adequate or optimum, and hence a scientific study is required. The reasons for study of the subject are:-

a)To reduce cost of product.
b)To improve product reliability.
c)To take care of manufacturing errors.
d)To estimate FOS for original and critical design work objectively.

4.1 Actual Load (λ)

Actual Load (λ) on a component does not remain constant, it varies from cycle to cycle. The variation is often due to the functional requirement itself. For example, the connecting rod of a reciprocating engine is subjected to cyclic variation of load. Some time, it may be the external factors viz. varying working condition, work habits of operators, etc causes variation in the actual load. As a large number of factors cause the variation, the nature of variation is statistically distributed as a normal (Gaussian) distribution and shown in Fig.5, where λ' is the expected average or mean load with a variation $\pm \delta\lambda$.

The design engineer anticipates the mean load and the variation from past experience with a good degree of confidence.

4.2 Load Capability (*L*) :

The designer theoretically designs the component to withstand an anticipated load keeping in mind the failure phenomenon. For example, the load capability of the tensile bar is, $L = SA$. Though the designer designs the component for a fixed load capability, in practice, the load capability also changes from component to component.The reasons for such variation are inherent in the component.Uncontrollable significant and undesirable effects caused during manufacturing process affect the load capability. Tolerance in the composition of material, dimension, geometry, surface finish, etc causes the variation in the load capability with a mean load capability L' and a variation $\pm \delta L$. The variation is also normally distributed as shown in Fig.5.

From Fig.5, it may be observed that the mean load capability should be more than the mean actual load to prevent failure. In fact, factor of safety is the ratio of mean load capability to mean actual load, i.e.

$$N = \frac{L'}{\lambda'} \qquad \text{........(11)}$$

From the elementary knowledge of statistical distribution, chances of failure becomes about 50% if the means of the two normal distributions coincide, and the chances of failure decreases as mean load capability L' increases. The overlapping area between two distributions represents the risk of thumb, past practice, or random selection process as used in design. They should look into the problem with analytical mind and mathematical temper. Theoretically, the increase must be infinitely large to avoid any chance of failure. Such increase is practically and commercially uneconomic. Hence, it is advisable to accept very small chances of failure and reduce the cost. Generally, 0.25 - 5.0 % chances of failure or risk is accepted depending on application. Hence, we should first decide about the acceptable level of risk and decide on the factor of safety for the design purpose.

5. Conclusion

Optimization techniques are extensively used in almost all decision situations for ensuring the best results. Since mechanical design involves a large number of decision situations, simultaneous or sequential, it should be dealt with optimization techniques. Though there is not much difficulty in applying such techniques in mechanical design, the problem lies in changing our mind set. Young engineers should form the habit of questioning rule of freethinking. Then only it will be possible to introduce optimization in mechanical design successfully.

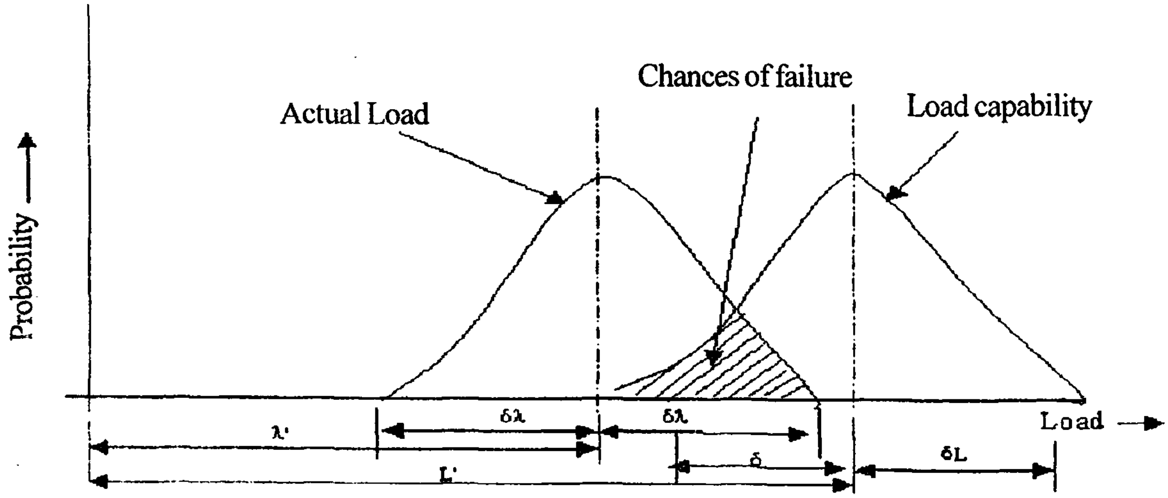

Fig. 5