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OPTIMAL SCHEDULING OF MULTIPLE DETERIORATED MACHINES FOR MULTIPLE JOBS

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Abstract: This paper is about proper scheduling of the jobs in deteriorated machines with theconsiderationthat machines have different processing time for same job depending upon the deterioration. At first a heuristic process is applied to determine the machine schedule to optimize the processing time for given jobs considering the deterioration of each machine. This enablesone to find proper order of jobs in each machine.Rate Modified Activity (RMA) is used to detect whether the next set of jobs can be processed by any machine or needs a rest to get full energy for the next job. After solving by above method, A.C.O. (Ant Colony Optimization)is introduced to find better alternative solutions. The main objective is to find a time interval by ACO, in which many alternative solutions of scheduling of jobs will be possible considering the deteriorated machines formulti machine multi job system. Another objectiveis to find scheduling so that penalties like tardiness andcan be minimized.

Keywords: Scheduling; Make span; Deterioration; RMA; A.C.O.**;** Ant Colony Optimization.

1. INTRODUCTION

For any production company it is eminent to find better scheduling of jobs so that makespan will beminimum. Due to excessive use of machine continuous breakdown occurs. These interruptions in work delay, the processing time. Proper measuresmust be taken before breakdown occurs to get jobs done in time. Therefore, scheduling of rest time for each machine also makes impact on production rate. In the last decade this mater became the prime interest. At present studies on scheduling relate many different aspects like makespan minimization, deterioration, deadlines, minimizing tardiness etc. the concept of scheduling of jobs in a deteriorated machine introduced by Browney and Yechiali [1].

The assumption about constant speed of a single machine or fixed process time in deteriorated machine [2-4] and multi machine[5,6] has been changed by rate –modifying activities (RMA)[7,8]. The RMA was first introduced by Lee and Leon [7]. The processing times of jobs vary depending on whether a job is scheduled before or after RMA.

Also Kubiak and Vendre [9] investigated the computational complexity of makespan under deterioration of machine. They presented a fully polynomial approximation scheme for a single machine scheduling problem to minimize makespan of deteriorating job. Cheng and Ding [9] studied with a single machine to minimize makespan within deadlines and increasing rates processing times. Bachman and Janiak [10] considered a single machine scheduling problem with minimizing maximum lateness under linear deterioration. Moreover, others showed that total weighted completion time is NP-hard for single machine scheduling in which the job processing times are decreasing time functions dependent on their start times.Y. He, M. Ji and T.C.E. Cheng [11] studied single machine scheduling problems involving repair and maintenance activities which they also used to call RMA. In 2005, they worked on a single machine to minimize makespan and total completion time of jobs. To minimize makespan, they presented a pseudo-polynomial time algorithm and a fully polynomial time approximation scheme (FPTAS). Lodree and Geiger [12] integrated time dependent processing times and RMA for assigning a single RMA to a position. After that RMA was used by Robert L. Bulfin [13], YucelOzturkoglu[14] and Emmett Lodree in finding the schedule of deterioration jobs on a single machine.

Here in this paper RMAis used for the multi machine. After finding the initial solution, authors used ACO to get alternative solutions. This also helps to find a time intervalin completion of jobs.

2. Ant Colony Optimization

The ant colony optimization algorithm (ACO) is a probabilistic technique for solving computational problems which can be reduced to finding good paths through graphs. This algorithm is based on the behavior of ants seeking a path between their colony and a source of food. Ant colonies are so good at finding the shortest path [15, 16] from one location to another that authors have developed an algorithm based on their behavior.

2.1. Ant System

Ant system is first ACO algorithm proposed in the literature. Its main characteristic is that, in each of iteration, pheromone values are updated by all the *m* ants that have built the solution in the iteration itself.

$$T_{ij}=(1-\rho)T_{ij}+\sum_{k=1}^{m}\Delta T^{k}_{ij}$$

T_{ij}= Pheromone associated with the edge joining cities *i* and *j*
ρ = Evaporation rate of pheromone
m = Number of ants
ΔT^{k}_{ij}= Quantity of pheromone laid on edge (i,j) by ant k

where,

$$\Delta T_{ij}^{k}=\begin{cases}\frac{Q}{L_k} & if\ ant\ k\ used\ edge\ (i,j) in\ its\ tour\\ 0 & elsewhere\end{cases}$$

Q = constant
L_k= length of the tour constructed by ant k

In the construction of a solution ants select the following city to be visited through a stochastic mechanism. When ant k is in city i and has so far constructed the partial solution S^p, the probability of going to city j is given by:

$$p_{ij}^{k}=\begin{cases}\frac{\tau_{ij}^{\alpha}\eta_{ij}^{\beta}}{\sum_{c_{ij}\in N(S^P)}\tau_{il}^{\alpha}\eta_{il}^{\beta}} & , \ if c_{ij}\in N(S^P)\\ 0 & , \ otherwise\end{cases}$$

$N(S^p)$ = Set of feasible components i.e. edge (i,l)
l = city not yet covered by the ant k
η_{ij} = 1 / d_{ij}
d_{ij} = distance between city i and j
α,β = Parameter that control the relative importance of the pheromone versus the heuristic information

3. MODEL FORMULATION

The main objective is to schedule 'n' jobs on 'm' parallel deteriorating machines to minimize the makespan. The machines are unrelated in a sense that Jobs processing times do not depend on the machine to which they are assigned.Machine speed and job completion time is dependent onthe deterioration rate of machines. This problem is common for any production company.

3.1. Assumptions and Notations

Let one consider n jobs as $J=\{J_1,J_2,J_3,\ldots,J_n\}$

α_i be the deterioration rate of jobs for m machines
b_i be the first m largest processing time within the unassigned jobs
sbe the set of jobs arranged in increasing order except the jobs in b_i
$m_{ij}$is position number j after RMA or initial position in ith machine
$J_{[k]}$ be the job position
$X_{i\{k\}}$ = 1 , if RMA takes place in kth position on ith machine otherwise zero
$P_{ij[k]}$is processing time job j in kth place in ith machine as $P_{ij[k]} = (1+\pm_i)^{k-1}P_{ij[1]}$
$C_{ij[k]}$is completion time of job in position k in ith machinex_{ijk}=1, if RMA is needed on ith machine after completing j th job otherwise zero
y_{ik}= 1, if an RMA is done before kth position on machine i, otherwise zero.

3.2. Mathematical Model of the Problem

Minimize Z = $\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{r} C_{ij[k]}$, i=1(1)m, j=1(1)n, k=1(1)r Sub to $C_{ij[1]}= \sum_{i=1}^{m}\sum_{j=1}^{n} P_{ij1}\ x_{ij1}$, completion time of first job in each machine

$C_{ij[r]} = C_{ij[r-1]}+\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{r} P_{ij[k]}\, x_{ijr-k}+\sum_{i=1}^{n} y_{ir}$, completion time in (r-1) positions plus processing time in rth position plus RMA times needed

$\sum_{i=1}^{m}\sum_{r=0}^{k-1} x_{ijr}$=1,j=1(1)n ,each job assigned to exactly one position

$\sum_{r=0}^{k-1}\sum_{j=1}^{n} x_{ijk}$=1, i=1(1)m , each position is scheduled for one job

$x_{kji} \le y_{i+1}$,k=2,3,….,r to determine the RMA position

$x_{ijk} \in \{0,1\}$, i,j=1(1)n k=0(1)r, binary condition

$y_{ik} \in \{0,1\}$, k=2(1)r, binary condition

$C_{ijk} \ge 0$, i=1(1)n, j=1(1)m, k=1(1)r,

non negativity constraints

4. PROBLEM MODELING BY HEURISTIC METHOD:SCHEDULE OF n DIFFERENT JOBS PROCESSED BY m PARALLEL MACHINES WITH LINEAR DETERIORATION

Since all the processing time are known so at first jobs having the maximum time assigned depending upon the calculation $Pi=p_i(1+\alpha_1)$ to m machines. Next the set of jobs will be assigned after checking the life of the machine. If processing time of the least job is less than the life of the machine, then jobs will be assigned to the machine, otherwise it will go for the recovery. The remaining jobs are arranged in ascending order of processing time by the rule $Pi =(1+\alpha_1)^{i-1}p_i$.

But one must consider the lifetime of the machine to get the idea that the job assigned to the machine could be completed without any break. If such break is needed then use the RMA and schedule the breaks for each machine. If RMA is done for any machine, then according to SPT (Shortest Processing Time) authors assign the job with the highest processing time after recovery.

4.1. Process

Let the authors discuss about the algorithm forarranging5 jobsin 2 machines.

STEP 1: Jobs are arranged in ascending order of processing times P_{ij}

For two machines five jobs processing time will be $P_{1j[1]} = p_{1j[1]}(1+\alpha_1)$ for j=1(1)5and $P_{2j[1]} = p_{2j[1]}(1+\alpha_2)$ for j=1(1)5, where α_1 and α_2 are the deteriorations of 1st and 2nd machines respectively.Assign the highest processing time job to the machine 1 & 2. If the highest processing job times become same then the second highest time, taking job from the second machine, assign it to the second machine. Let jobs be P_{13} and P_{21}.

STEP 2: Here K=2, Check $P_{13} > P_{21}$ then calculate $P_{2j[2]} = p_{2j[2]}(1+\alpha_2)^2$ then if the smallest processing time is greater than the life of the machine then use RMA followed by the biggest remaining job or process the smallest job, then go for RMA. Follow the same for the 1st machine.

STEP 3: Follow the same for k=3,4,…till the jobs completed.

4.2. Model in Tabular Form

	Machine I	Machine II	Machine III	……….	Machine N
Processing time	P_{1j}	P_{2j}	P_{3j}	………..	P_{nj}
Det	α_1	α_2	α_3	…………	α_n
Life of the machine	T_1	T_2	T_3	…………	T_n
First level of processing time with Det	$P_{1j}(1+\alpha_1)$	$P_{2j}(1+\alpha_2)$	$P_{3j}(1+\alpha_3)$	……………	$P_{nj}(1+\alpha_n)$
Selection of job	J_{1k} (max processing time)	J_{2k}(next max processing time)	J_{3k}	…………..	J_{nk}
Remaining life of the machine	$T_1 - P_{1j}$	$T_2 - P_{2j}$	$T_3 - P_{3j}$	……………	$T_n - P_{nj}$
Second level of processing time with Det	$P_{1j}(1+\alpha_1)^2$	$P_{2j}(1+\alpha_2)^2$	$P_{3j}(1+\alpha_3)^2$	……………	$P_{nj}(1+\alpha_n)^n$
RMA	← It is	calculated	depending	Upon remaining	time ←

4.3. Implementation of the Model with the Solution

Here is an example with 4 machines with the data given in Table-I. There be 10 jobs J_i,i=1(1)10. The actual processing times are given in Table-II.

Table-I

Machines	Machine life (in hrs)	Deterioration rate	Recovery time (in hrs)
Machine 1	5.5	0.2	1
Machine 2	8.5	0.4	1
Machine 3	7.5	0.3	1
Machine 4	6.5	0.6	1

Table-II

Jobs	J_1	J_2	J_3	J_4	J_5	J_6	J_7	J_8	J_9	J_{10}
Processing time	2	3.2	1.5	2.5	2.3	2.8	1.6	1.9	1.3	3.6

Table-III

	Machine 1	Machine 2	Machine 3	Machine 4
J_1	2.4	2.8	2.6	3.12
J_2	3.84	4.48	4.16	5.12
J_3	1.8	3.1	1.95	2.4
J_4	3	3.5	3.12	4
J_5	2.76	3.22	2.99	3.86
J_6	3.36	3.92	3.64	4.48
J_7	1.92	2.24	2.08	2.56
J_8	2.28	2.66	2.47	3.04
J_9	1.56	1.82	1.69	2.08
J_{10}	4.32	5.04	4.68	5.76

Life remaining for the machine

	Machine 1	Machine 2	Machine 3	Machine 4
Life	2.74	4.02	3.86	0.74

Table-IV

	Machine 1	Machine 2	Machine 3	Machine 4
J_1	2.88	3.92	3.38	5.12
J_3	2.16	2.94	2.535	3.84
J_4	3.6	4.9	4.225	6.4
J_7	2.304	3.136	2.704	4.096
J_8	2.736	3.724	3.211	4.864
J_9	1.872	2.548	2.197	3.328

TABLE-V

	Machine 1	Machine 2	Machine 3
J_3	2.592	4.116	3.2955
J_9	2.2464	3.5672	2.8561
Life	0.004	0.1	1.136

Machine-4 gone for the recovery and the remaining machines are working. Here the machine 4 is engaged for minimum1.3 (= 4-2.7) hours also the remaining machines need recovery time. The time of jobs are recalculated. The least job was on machine 3 and so authorsadd the recovery time and check whether it is less than the job processing of machine-4.

After finding the schedule of the jobs,authors get the initial matrix, then using the **A.C.O.** the optimality is checked and alternative better solutions are obtained.

4.4. Symbols and Notations for A.C.O.

- **m** = number of machine
- **n** = number of jobs
- **e** = evaporation coefficient.
- **alpha** = trace's effect
- **beta** = Heuristic information
- **t** = primary tracing.
- **x** = deteriorating rate of machine

- **ml**= Life of machine
- **mlo** = original machine life

4.5. Solution Procedure

This process finds minimum make span for multi machine under the observation of linear deterioration.

STEP 1 : Initialize all variables.

STEP 2 : Form Sight Matrix = 1/ Processing times

STEP 3 : Form Q matrix

A matrix (Q) that contains 'm' rows indicating machines and 'n' columns representing jobs used to show results.

$$Q = \begin{matrix} 3 & 6 & 9 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 10 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 5 & 8 & 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{matrix}$$

i.e. Job numbers 3, 6, 9 and 1 are assigned to the machine no 1
Job numbers 2 and 10 are given to the machine no 2
Job numbers 5, 8 and 4 are placedto the machine no 3
Job 7 is assigned to machine 4.

The sequence of jobs on the respective machine is also same as it is shown here.

4.6. The probability of Job i on machine j for ant k can be calculated as follows

$$p_{ij}^{k} = \begin{cases} \dfrac{\tau_{ij}^{\alpha}\eta_{ij}^{\beta}}{\sum_{c_{ij}\epsilon N(S^P)} \tau_{il}^{\alpha}\eta_{il}^{\beta}} & , \quad if\, c_{ij}\epsilon N(S^P) \\ 0 & , \quad otherwise \end{cases}$$

STEP 4 : To calculate makespan by considering deterioration.

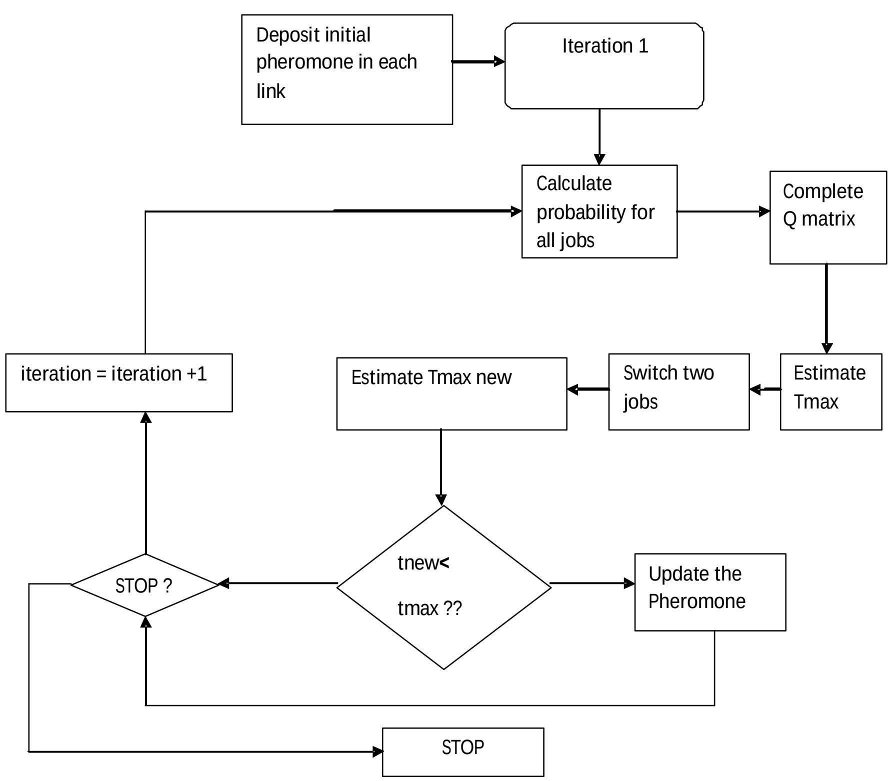
STEP 5 : To interchange any two jobs.

STEP 6 : To calculate new makespan by considering deterioration.

STEP 7 : If new make span is better than the previous one then update the pheromone

STEP 8 : To go for next iteration till stop criteria is reached.

FLOW CHART



5. RESULTS AND DISCUSSION

These results are for parameters-Evaporation rate= 0.05; alpha=3; beta=1; Primary tracing = 0.10 for 1000 iterations.

Iteration		Matrix of job scheduling				Matrix of time scheduling	Total time
1.	Machine 1	3	1	6	-	1.8000 2.8800 4.3600 =9.04	9.176
	Machine 2	9	7	5	-	1.8200 3.1360 4.2200 =9.176	
	Machine 3	10	4	-	-	4.6800 4.1200 0 =8.8	
	Machine 4	2	8	-	-	5.1200 4.0400 0 =9.16	
2.	Machine 1	9	7	10	-	1.5600 2.3040 5.3200 =9.184	9.184
	Machine 2	5	2	-		3.2200 5.4800 0 =8.7	
	Machine 3	4	3	8	-	3.1200 2.5350 3.4700 =9.125	
	Machine 4	6	1	-	-	4.4800 4.1200 0 =8.6	
3.	Machine 1	1	4	7	-	2.4000 4.0000 2.3040 =8.704	9.26
	Machine 2	5	10	-	-	3.2200 6.0400 0 =9.26	
	Machine 3	3	9	2	-	1.9500 2.1970 5.1600 =9.207	
	Machine 4	8	6	-	-	3.0400 5.4800 0 =8.52	
4.	Machine 1	7	5	4		1.9200 3.3120 4.0000 =9.232	9.344
	Machine 2	9	8	1		1.8200 3.7240 3.8000 =9.344	
	Machine 3	2	6	0		4.1600 4.6400 0 =8.8	
	Machine 4	10	3	0		5.7600 3.4000 0 =9.16	
5.	Machine 1	3	7	8	9	1.8000 2.3040 3.2800 1.8720 =9.256	9.32
	Machine 2	1	2	0	0	2.8000 5.4800 0 0 =8.28	
	Machine 3	6	10	0	0	3.6400 5.6800 0 0 =9.32	
	Machine 4	4	5	0	0	4.0000 4.8600 0 0 =8.86	
6.	Machine 1	2	8	9		3.8400 3.2800 1.8720 =8.992	9.294
	Machine 2	1	10	0		2.8000 6.0400 0 =8.84	
	Machine 3	6	7	3		3.6400 2.7040 2.9500 =9.294	
	Machine 4	4	5	0		4.0000 4.8600 0 =8.86	
7.	Machine 1	3	4	5		1.8000 3.6000 3.7600 =9.16	9.16
	Machine 2	10	1	0		5.0400 3.8000 0 =8.84	
	Machine 3	6	7	9		3.6400 2.7040 2.6900 =9.034	
	Machine 4	2	8	0		5.1200 4.0400 0 =9.16	
8.	Machine 1	7	9	10		1.9200 1.8720 5.3200 =9.112	9.161
	Machine 2	4	2	0		3.5000 5.4800 0 =8.98	
	Machine 3	5	8	3		2.9900 3.2110 2.9500 =9.161	
	Machine 4	1	6	0		3.1200 5.4800 0 =8.6	
9.	Machine 1	3	8	9	7	1.8000 2.7360 2.5600 2.3040 =9.4	9.4
	Machine 2	10	1	0	0	5.0400 3.8000 0 0 =8.84	
	Machine 3	2	6	0	0	4.1600 4.6400 0 0 =8.8	
	Machine 4	4	5	0	0	4.0000 4.8600 0 0 =8.86	

Here 9 results are discussed out of several results given by ACO. From the results, one can understand that these values lie within a particular limit. Moreover,by changing the parametric values,more suitable results can be obtained. Also, the alternative solutions give the opportunities to start with new jobs without hampering the time limit. In this process, due to randomly picking of two jobs for swapping, a number of solution sets evolves.

The values also show that the variation in makespan lies between 9.16 and 9.4 which seems to be very small for given jobs. Once the number of jobs increased with different sizes, this span will reduce further.

This algorithm provides more than one solutions which are nearly the same for makespan consideration. This gives an excellent opportunity to apply more constraints on this or make ita multi objective problem. Also there are more options to choose, which will result in the same makespan.

Also the objective of reducing the penalties will be obtained, as the makespan lies within a specific limit. Another facility is that one can choose any alternative schedule and get the work done in a stipulated time limit.

6. CONCLUSION AND FUTURE SCOPE

In this work, Ant Colony Optimization (ACO) is applied to solve the problem of scheduling of jobs in a multi machine multi job system having deteriorated machines for optimal selection of makespan when machines are arranged in parallel. This algorithm provides more than one solutions which are nearly the same for makespan consideration. Thus, Ant Colony Optimization can be effectively utilized in solving similar kind of problems.

This result is based on parallel machine. It can also be extended for a series of parallel machines. It is also possible to find a model for flow shop problems.

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