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History of Numbers and Fractions and Arithmetic Calculations in the Tamil Region: Some Observations

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Abstract

The Tamil inscriptions of the medieval period provide a very detailed account on the land measures in minute fractions. In the Medieval period, the villages were completely measured and documented, especially in the fertile Kāviri delta and other cultivable areas, for accurately assessing tax. The land measurements are discussed in terms of various units of measures, especially miniscule fractions, and the calculations of taxable and tax-free areas are also specified in some of the inscriptions. This paper presents a preliminary account of the fractions, numbers and arithmetic calculations found in the Medieval Tamil inscriptions.

Keywords: Arithmetic Calculation, History of Science, Land Administration, Medieval Measurements, Tamil Fractions

1. Introduction

In the study of history of science, the development and use of concepts related numbers, fractions and arithmetic calculations form an important component. There is a lot scope to research on the history of science and traditional knowledge systems in India, and when we look at this subject objectively, without any emotional attachment and pride, there is much to learn from these traditional concepts of science, which can be very useful for academic understanding as well as for imparting cognitive skills among the youngsters as part of the contemporary education. In the area of history of numbers, arithmetic calculations and mathematics in India, much research has been done [1, 6], and in this paper, I present a preliminary survey of the numbers, fractions and arithmetic calculations that were used in Tamil region in the historical period.

Simple arithmetic calculations were used for day-to-day accounting and for the assessment of land revenue during the medieval period. The Tamil inscriptions, literature and palm-leaf manuscripts serve as important sources for understanding the fractions, numbers and arithmetic calculations. Specific symbols or markers were used to identify these fractions, apart from specific words. The land measurement, weight and volume measurements of various materials donated to the temple are also listed out in minute detail, in the inscriptions. In the medieval period, there were accountants who calculated the land area and assessed them for tax, and maintained detailed accounts of the lands and taxes to be collected.

The use of numbers or counting, perhaps began in the prehistoric period, when people calculated the number of people, fruits and other countable elements that were essential for their day-to-day activities. Nowadays people tend to visualize the figures such as 10, 100, 200 and 1000 as complete; but, the perception of these decimal based numbers as complete entities is nothing, but our own imagination. The number of body parts such as head, eyes, fingers that humans have perhaps helped in counting numbers in the very early stages of history. In the Indian tradition, the earliest reference to numbers comes from the *Védic* texts; many other early Indian texts have information on large numbers (e.g. *Vālmīki Rāmāyana* mentions about very high numbers [6, 11] (e.g. 10^{57}). Symbols for

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numerals are also found in the Asokan inscriptions, and also in the Nānéghat inscription [6]. It appears that the decimal system might have developed in India from the Harappan times [6]. Different systems were used in India for denoting numbers in the historical period. In the *Bhūta Samkya* system, numbers were identified by objects or ideas. The *Kaṭapayāti* system seems to have been developed by Vararuci, an astronomer and mathematician from Kérala, belonging to the medieval period. In this method, different letters are assigned numerical values, and words or *slōkās* are formed; for example, "nanajnanapragalbhah" in this system means the figure of 43,20,000 [11].

2. Numbers in Tamil

Different symbols were used in Tamil Nadu for denoting numbers. The Brahmi inscriptions have evidence for the use of numbers, according to Iravatham Mahadevan [9]. At Azhakankulam (Alakankulam), the Early Historic site, numerals are found on pottery. However, the numbers, fractions and arithmetic calculations are found more frequently in the inscriptions from the medieval times. The medieval inscriptions give the numbers as figures and sometimes also as text, perhaps to make sure no error appears in their reading and interpretation. The Tamil work of *Kaṇakkatikāram* by Kāri Nāyanār is an important medieval work on mathematics [8]. This tradition of using Tamil numerals has continued to this day, although in the limited circles of Tamil studies and among Tamil scholars during the modern (colonial) period, when the so called Arabic numerals were introduced. As a result of the change in the use of numerals during the modern period, nowadays many people cannot understand the traditional Tamil numerals.

The traditional Tamil numerals of the contemporary period very much resemble the Tamil letters/characters that have specific phonetic value. For example, the Tamil letter அ (a) stands for the number 8 and ௷ stands for quarter (1/4). Kālamégappulavar, a famous poet of the modern period, made a pun, using the symbols of letters "8" and "1/4." In a poem, he mentions about "8 1/4 (eṭṭékāl) laṭcanamé" which means "avalatcanam," (=ugly).

The word "eṇ' in Tamil denotes numeral in a general sense and it also denotes the number eight. Perhaps, eight was seen as a complete or the largest number in the Tamil tradition. In Tamil system, the numbers 1 to 8 are mentioned as individual numbers, while 9 is mentioned as one less than 10. Earlier the term *tondu* was used to refer to 9. The term "onpatu" for nine, actually means one less than 10. Similarly, the numbers 90 and 900 are also mentioned in relation to 100 and 1000, respectively. Based on these features, it can be argued that eight-based numeral system was probably used in the Tamil region in the early period. However, this proposition needs to be investigated further.

The manner in which numbers are mentioned in Tamil has a pattern much different from Sanskrit. "Eleven" is written in Tamil as 'ten plus one.' However, in Sanskrit, "one" is mentioned first and then ten, e.g. *ékādasi*, 11. The French language has a system of writing numbers, which is similar to Tamil.

3. Tamil Numerals and their Symbols

Tamil numerals and symbols are mentioned in the Tamil Inscriptions and they are also found in the palm-leaf manuscripts. In the inscriptions, the numbers are mentioned in a specific pattern. There are separate symbols for 1 to 10, 100 and 1000, and these symbols are combined to covey a specific figure. There is no place value system here. The number 88 is conveyed with the symbols of "8,10,8," which means the first two figures have to be multiplied and the last figure has to be added. The number 800 was written as 8, 100= then it has to be read as 800. For example, *Kali* year 4820 would be written as 4, 1000; 8, 100; 2, 10 = 4820 (without any punctuation in between). The numbers are placed as they spelled in Tamil language, twenty-four would be spelled as "irup attu nāṅgu"= 2,10,4. Tamil inscriptions and manuscripts have the various symbols used for numbers, fractions and various land, weight and volume measures. Recently, Tamil Virtual Academy (TVA) under the control of Government of Tamil Nadu has created a document in register of the Unicode Consortium (UC) [22] (Table 1) for creating standard symbols for digital records.

4. Fractions

The knowledge of fraction is generally traced to the *Védic* period [21]; but there are chances for their use in the Harappan culture, since the Harappans were involved in extensive commercial activities. Fractions were frequently used in the medieval inscriptions of Tamil Nadu. The word *arai* in Tamil refers to half and interestingly, *arai* means waist part of the humans. The term *kāl* refers to leg in Tamil and perhaps, the term derived from the length of one portion of human leg. *Mukkāl* refers to ¾ and it written as three quarters. A few of the smaller fractions such as *araikkāl* (1/8), *araiyéaraikkāl* (1/16) are referred to as *mākāṇi* or *vīsam*. *M*ā refers to 1/20, *kāṇi* refers to 1/80. *Muṅtiri* refers to the fraction of 1/320. However, Subrahmanian [20] lists

Table 1. Numerals, fractions and their symbols

Figures in Words	Numerals	Multiplication of	Symbol 1	Symbol 2 TVA2014
Āyiram	1000			
Nūṛu	100			
Pattu	10			
Onpatu	9			
Eṭṭu	8			
ézhu	7			
Āṛu	6			
aintu	5			
Nāṅgu	4			
Mūnṛu	3			
Iraṇdu	2			
Onṛu	1			
Mukkāl	¾			
Arai	ō			
Kāl	¼			
Nālumā	1/5	4 x 1/20		
Araikkāl	1/8	¼ x ō		
Irumā	1/10	2 x 1/20		
Mākani, vīsam	1/16	(1/20+1/80=5/80=1/16)		
Mummākāṇi	3/16	=3/16		
Mummā	3/20	3 x 1/20		

Mā	1/20				
Araivīsam	1/32				
Araimā	1/40				
Kālvīsam	1/64				
Kāṇi	1/80	1/20 x ¼			
Mukkālvīsam	3/64				
Mukkāṇi	3/80				
Araikāṇi	1/160	1/320 x ō			
Muṅtiri	1/320				
Araikāṇimuṅtiri	3/320	1/320x1/160			
Kīzh arai	1/640	1/320x1/2			(Bharathiyar n.d 201-204)
Kīzh mukkāl	3/1280	1/320 (*muṅtiri*) x 3/4 (*mukkāl*)			(Bharathiyar n.d 201-204)
Kīzh kāl	1/1280	1/320 (*muṅtiri*) x 1/4 (*kāl*)			(Bharathiyar n.d 201-204)
Kīzh nālumā	1/1600	1/320x4/2			(Bharathiyar n.d 201-204)
Kīzh araikkāl	1/2560	1/320 (*muṅtiri*) x 1/8 (*araikkāl*)			(Bharathiyar n.d 201-204)
Kīzh irumā	1/3200	1/320 (*muṅtiri*) x 2/20 (irumā)			(Bharathiyar n.d 201-204)
Kīzh mummā	3/6400	1/320 (*muṅtiri*) x 3/20 (*mummā*)			(Bharathiyar n.d 201-204)
Kīzh vīsam	1/5120	1/320 (*muṅtiri*) x1/16 (*vīsam*)			(Bharathiyar n.d 201-204)
Kīzh orumā	1/6400	1/320 (*muṅtiri*) x1/20 (*mukkāṇi*)			(Bharathiyar n.d 201-204)
Kīzh araimā	1/12800	1/320 (*muṅtiri*) x 1/40 (1/2 *mā*)			(Bharathiyar n.d 201-204)
Kīzh mukkāṇi	3/25600	1/320 (*muṅtiri*) x3/80 (mukkāṇi)			(Bharathiyar n.d 201-204)
Kīzh kāṇi	1/25600	1/320 (*muṅtiri*) x1/80 (*kāṇi*)			(Bharathiyar n.d 201-204)
Kīzh araikkāṇi	1/51200	1/320 (*muṅtiri*) x1/160 (*araikkāṇi*)			(Bharathiyar n.d 201-204)
Kīzh muṅtiri	1/102400	1/320 (*muṅtiri*) x 1/320 (muṅtiri)			(Bharathiyar n.d 201-204)
Immi -1 (*Immimuṅtiri*)	1/1075200	1/102400 (*kīzh muṅtiri*) x 1/10.5			(Bharathiyar n.d 201-204)

Immi -2	1/2150400	1/102400 (*kīzh muṅtiri*) x 1/21	Half of *immi* 1
Nunmuṅtiri	1/3225600	*Immi* 1 x 1/3	
Aticāram	1/1838400	1/320 x 1/5745	(Bharathiyar n.d 201-204)
Mummi	1/23654400	1/102400 x 1/231	11 times smaller than *immi* 2
Cinnam	1/32256000	*Immi* 1 x 1/10	
Kīzh kīzh muṅtiri	1/32768000	1/320x1/320x1/320	
Aṇu	1/165580800	1/102400 x 1/1617	Seven times smaller than *mummi*
Guṇam	1/1490227200	1/102400 x 1/14553	9 times smaller than *aṇu*
Pandam	1/7451136000	1/102400 x 1/72765	Five times smaller than *Guṇam*
Pāgam	1/44706816000	1/102400 x 1/436590	6 times smaller than *Pandam*
Kīzh kīzh kīzh muṅtiri	1/10485760000	1/320x1/320x1/320x1/320	
Vintam	1/312947712000	1/102400 x 1/3056130	7 times smaller than *Pāgam*
Nāgavintam	1/5320111104000	1/102400 x 1/51954210	17 times smaller than *viṅdham*
Sintai	1/74481555456000	1/102400 x 1/727358940	14 times *nāgaviṅdham*
Katirmunai	1/489631109120000	1/102400 x 1/14547178800	Muthukumar 2014
Kuralvalaippidi	1/9585244364800000		Muthukumar 2014
Veḷḷam	1/575114661888000000		Muthukumar 2014
Nuṇmaṇal	1/57511466188800000000		Muthukumar 2014
Térttugal	1/23238245302272000000000		Muthukumar 2014

the value of a kāṇi as 1/64 and *muṅtiri* as 1/256, and perhaps these fractions were based on a different calculation system, and probably a *mā* had a value of 1/16 in this system. It might have been an early system or it was not widely used in Tamil Nadu and disappeared in the early medieval period. A similar term, *kani* used in Karnataka has a value of 1/64, according to Jagadish and Hegde [5]. In Andhra Pradesh too the term denotes the fraction of 1/64.

Other minute fractions below *muṅtiri* (1/320) were mentioned as *kīzh*, which means below *muṅtiri*. *Kīzharai* means *muṅtiri* multiplied by *arai* (1/320 x ō= 1/640), and *kīzh muṅtiri* means 1/320 x 1/320 = 1/102400. Some of these fractions were represented by different symbols (Table 1). Fractions such as *immi muṅtiri* and *nuṇmuṅtiri* are found in *Kaṇakkatikāram* [8]. The Tanjāvūr Brihadhīswara temple inscriptions have references to minute fractions and the term *kīzh* has been referred to for such fractions below 1/320. A unit of the last series *kīzh kīzh kīzh muṅtiri* x 1/2 is used in the Tanjāvūr temple inscription refers to the fraction of 1/5242,8800000 of a *véli*, according to Venkayya [23]. Another inscription mentions about the fraction of 1/320x1/320x1/320x1/320x3/4x1/20 [7]. Subbarayalu [17, 18] has argued that such small fractions mentioned in the inscriptions for land areas resulted due to the reduction (*madakku*) of the area based on several parameters.

There are a lot of variations in the use of terms to denote the minute fractions and more detailed work is necessary on the original manuscripts, i.e. the primary sources. Many of the publications on numbers and fractions, available on the Internet, which were obviously collected from the early publications, list different figures for the Tamil words referring to fractions [2, 12, 15]. In the work *Iniya Tamizh Ilakkanam* by Cuddhananta Bharathiyar, *kīzh araikkāni* is listed as 1/512000, perhaps this figure appears due to typographical error; but, the correct figure is 51200, since it is 1/320 x 1/160 [3].

5. Large Numbers

Very large numbers were known to Indians. The ancient Indian texts have references to numbers as large as 10^{145} [4]. The large number of āmbal is mentoined in the

Sangam Tamil texts. Although the text does not mention about the exact nature of the number, from other sources we know that it is a very large numer (=10^{21}). Terms such as *tamarai*, āmbal, *veḷḷam* are found in the Sangam texts and the *Paripātal* lists *neytal*, *kuvaḷai*, *āmbal*, saṅgam, *kamalam* and *veḷḷam* (*Paripātal* 2: 12-15). The term kōti also occurs in *Puṛanānūṛu* (18: 5-6; 202:7). Several works such as *Tolkāppiyam* a nd *Kambarāmāyanam* mention about large numbers [2, 15]. It appears that several terms were used for the numbers and there are variations in the defintion of numbers. For example, the term *Mahapadma* was used in ancient India for 10^{12} and 10^{34} [4].

6. Technical Terms

Several technical terms related to measurement of linear, area, volume and weights in the inscriptions. A few of the important terms are described as shown in Table 2.

6.1 K½l or Danda

Kōl or *Danda* refers to the measurement rods. They are found in various names in the inscriptions, and often their length is mentioned in terms of their size in *cān* or span, cubit or *muzham* or foot or *aṭi*. These measurement rods were used for building and temple constructions and also for land measurements. Different types of rods such as 16 span, 8 span, 22 span rods were used for measuring lands [14]. The span was defined as consisting of 12 *aṅgulam* or *viral*. Twenty-four *aṅgulam* or *viral* units formed a *muzham* or cubit. Sixteen-span rod was one of the commonly used measurement rods and the measurement rods of the medieval period are marked in the temples and on rock surfaces (Selvakumar 2014, 2015). Perhaps, each village had its own measurement rod and hence they were conveyed as "*ivvurpaṭi*," (according the measurement rod system of this village) when the land was converted into the scheme of measurement of particular village.

Table 2. Large Numbers mentioned in Tamil/Indian system

Number	Formula	Name	As in Bhagavathy 2003/ Shanmugan 2006
10	10^{1}		*Pattu*
100	10^{2}		*Nuṛu*
1000	10^{3}		*Āyiram*
10,000	10^{4}		*Pattāyiram*
1,00000	10^{5}		*Nurāyiram*
10,00000	10^{6}		*Pattulatcham*
10000000	10^{7}		Kōti
100000000	10^{8}	*Pattukōti*	*Aṛputam* [4]
1000000000	10^{9}	*Tolluṇ*	*Nigaraṛputam*
10000000000	10^{10}		*Kumbam*
100000000000	10^{11}	*Aṛputam*	*Kanam*/ Kanagam
1000000000000	10^{12}	*Itiyam*	Kaṛpam
10000000000000	10^{13}	*Nigaraṛputam*	*Nikaṛpam*
100000000000000	10^{14}		*Patumam*
1000000000000000	10^{15}	*Nelai*/*Karvam*	*Sangam*
10000000000000000	10^{16}		*Vellam*/*Samutiram*
100000000000000000	10^{17}	*Sangam* [4]	*Anniyam*/*Antiyam*
1000000000000000000	10^{18}	*Ilanji*	*Mattiyam*/Artam
10000000000000000000	10^{19}	*Artam*	*Parartam*
100000000000000000000	10^{20}	*Veḷḷam*	Puriyam
1000000000000000000000	10^{21}	*Āmbal*/Puiyam	*Brammakaṛpam* (*kōti kōti kōti= mukkōti*)
100000000000000000000000	10^{25}	Mukkōti	
10000000000000000000000000000000000000000000000000000000000000000	10^{57}		*Mahāyugam* [11]

6.2 Kuzhi

Kuzhi is the Tamil term for pit and it has been used in the sense of area. An area covered by one *kōl* by one was called a *kuzhi*. It was commonly used as the basic measurement unit for area. One hundred *kuzhi* units formed one *mā* unit. 100 or 128 or even higher number of *kuzhi* units (128 and 256) formed one *mā,* in some instances, and twenty *mā* units formed one *Véli*, the major area unit of measurement for land. This unit of measurement is still used in many parts of Tamil Nadu; more particularly in the Kāveri delta region. There is another term *peruṅkuzhi* found in the inscription perhaps refers to larger unit of *kuzhi* area.

6.3 Madakku (Reduction)

Madakku is a method of reduction of the area from one unit to another unit [7, 17–19]. Most probably, when an area measured by a smaller scale unit was converted on the basis of a larger scale, the process was called *madakku*. It was a kind of reduction process, in which a large area can be reduced to a small area.

6.4 Virivu (Expansion)

Virivu refers to expansion of a smaller area into a larger area on the basis of smaller measurement rod.

7. Arithmetic Calculation in Inscriptions

The inscriptions mention about the lands that had to be taxed and the area that had to be exempted from tax, such as temple lands, settlement areas cremation ground. Here two samples of arithmetic calculations from the inscriptions are analysed.

7.1 *Nallūr* Inscription

NallÂr in Pāpanāsam taluk of Tanjavur district has a Siva shrine called Kalyāna sundaréswarar temple. An inscription, issued during the 3rd regnal year of Rājarāja III (1218 CE) found in this temple [10] (Marxiya Gandhi and Ramachandran 2004:138, TNAD No 32/1995) mentions about the area of the land that was exchanged for another piece of land meant for the passage for carrying the dead body, which created a dispute in the village.

The area is mentioned as 2 *kōl,* in north-south, by 15 ¾ +3/20 (*patinañjé mukkālé mīnṛu mā*) on the east-west, and the total area was 31 ¾+1/20 *peruṅkuzhi* or square *kuzhi,* in the inscription.

When we multiply the above numbers, we get

(15 x 2) = 30 + (2 x ¾)= 1.5 + (2 x 3/20)= 6/20 = 31 ¾ + 1/20

Or 2 x (63/4+3/20)

2 x 315+3/20

2 x 318/20= 636/20=31+16/20=31+3/4+1/20

The term *perunkuzhi* mentioned here, perhaps meant a larger measurement rod which was employed for measurement.

For the above mentioned piece of land, another land was given in exchange and its size was

8 ¼ kol by 3 ¾ 2/20 kol = 33/4 x (15/4 + 2/20) = 75+2/20= 77/20
33/4x 77/20= 2541/80= 31 +61/80=31 +3/4+1/80

Actually the land given exchange for the original piece of land was slightly less and it is only 31 *mukkāle kāṇi*; however, it is mentioned as 31 ¾ + 1 *mā* (1/20). Perhaps, they intentionally ignored the small variation in the land.

7.2 Another Inscription from Nallūr

Another inscription from the same temple issued in the 30th year of Rājarāja III talks about the same issue and mentions about another land area (ARE 31/1995).

Here a piece of land is mentioned as 2 *kōl* by 28 *kōl* = 56 *kuzhi.*

28 x 2 = 56

Here another tem *uriya kuzhi* is mentioned.

The area of 87.5 *kuzhi* is mentioned as *uriya kuzhi for 56 kuzhi*, which means an equivalent unit in another scheme of measurement. We can deduce the approximate variation in the expansion of the measurement rod here. The unit of 2 *kōl* in one scheme becomes 2.5 *kōl* in the new scheme and therefore 28 *kōl* units become 35 *kōl* units; therefore, 25 percent increase per unit.

2.5 *kōl* by 35 *kōl* = 87.5 *kuzhi.*

If the original measurement rod measured 16 spans, the new one would be 20 spans or if the original specimen was 8 spans or feet in length, the new rod would be 10 spans or feet. This case proves that the land measurement was done different scales and for the conversion of the land from one measurement system to another, they just used arithmetic calculation.

7.3 Madakku: Reduction of Area

In the medieval period, several measurement rods were used and sometimes the areas were reduced for taxing

purpose using simple calculations and they are mentioned as *tarmiṭṭu madakki* [17–19].

An inscription from Vaidyanāta Temple at Tirumazhavādi [16] issued in the 26th year of Rājadhirāja mentions about the land calculation and its conversion. The two measurement units are compared here; however, it may not be accurate, and could be only assumption, since the definition of *véli*, *mā* and the size of the measurement rods varied greatly. Hence if we do not know all the parameters, proper comparison is not possible. However, an attempt is made here assuming the *véli* units in the two schemes discussed here are same.

A. The original size of the land is mentioned as:

iraṇdéi mukkālé orumā varaikkāṇi muṅtirikaik kīzh mukkāl

2 ¾, 1 *mā*, *ō kāṇi*, *muṅtirikaik kīzh mukkāl*

2 ¾ *véli* + 1 *mā*=

2 ¾ x 20=55 +1 = 56 *mā* +

ō kāṇi (1/160) *muṅtirikaik kīzh mukkāl* (1/320+¾x1/320)

1/160+1/320+3/1280

56 *mā* + 1/160+1/320+3/1280

56/20+1/160++1/320+3/1280

3584+8+4+3/1280 = 3599/1280=

2¾=3520+ 79/1280

B. After Reduction

The land that originally had 3599/1280 was reduced (*taramiṭṭu madakki*)

After reduction, it became,

Onpatumā araikkāni muṅtirikaik kīzh mukkāle mummāvarai araikkāṇi

9 *mā* + 1/160+ 1/320+1/320(3/4+ 3.5/20+ 1/160)

1/320+1/320(3/4+ 3.5/20+ 1/160) =

1/320+1/320x (3/4x3/20+1/40+1/160)

1/320+1/320 x (120+24+4+1/160)= 149/160

1/320+1/320x149/160

=1/320+149/51200

=160+149/51200=309/51200

9 mā+1/160+309/51200

9/20+1/160+309/51200

23040+320+309/51200=23669/51200

C. Comparing A and B

For comparison, A can be converted into

= 3599/1280x40/40 = 143960/51200

A) 143960/51200 and B) 23669/51200

Now it is clear that after reduction, A becomes 6.0822 times smaller, assuming the size of the *véli* is same; however, they must have been different, since the ratio of 6.0822 does not seem to conform to the traditional fraction units, and hence, more research is necessary.

D. Calculation of Half of an area

Another calculation in the above mentioned inscription is about the half of the reduced area (see, B After Reduction), which is called "*sempāti*."

The original area is:

Onpatumā araikkāṇi muṅtirikaik kizh mukkāle mumāvarai araikkāṇi

9 *mā* + 1/160+ 1/320 (3/4+ 3.5/20+ 1/160)

According to the inscription, half of the above is

4.5 *mā*+ *muṅtirikaik kīzh mukkāle nāṅngumākkāṇi muṅtirikai*

In the above case, the reduction by half of 9 *mā* and *araikkāṇi* is clear, and the result is 4.5 *mā*+ 1/320.

Now let us check, if they have reduced *muṅtirikaik kīzh mukkāle mummāvarai araikkāṇi* properly into two halves.

muṅtirikaik kīzh mukkāle mummāvarai araikkani=

1/320+1/320x (3/4x3/20+1/40+1/160)

1/320+1/320 x (120+24+4+1/160)= 149/160

1/320+1/320x149/160

1/320+149/51200

160+149/51200

=309/51200

Half of 309/51200, according to the inscription is

=*kīzh mukkāle naṅgumākkāṇi muṅtirikai*.

1/320x (3/4+4/20+1/80+1/320)

1/320 x (240+64+4+1/320)

1/320x 309/320

= 309/102400

The result of the above division is perfect, which proves the accuracy of the calculation of fractions during the medieval period, i.e. from the Chola times.

8. Discussion

Reference to large numbers is found in the Sangam texts. The Sangam Chola king Karikālan is said to have given 16,00,000 coins to KatiyalÂr urutirankaṇṇanār, the poet who composed the text *Pattinappālai*. The inscriptions provide the details of fractions and their calculation very minutely. The conception of large numbers are more notional, one can reach large numbers just by repeating a large number, for example, *kōti kōti kōti kōti*. However,

the story found in the work *Lalitavistāra* about Buddha's learning related to mathematics does prove that the large numbers were taught to students [13]. There are many theories on the origin of the place value system and one suggests that it came up in India [6, 13]; but, these questions are not easy to address, as borrowing of ideas and movement of people and knowledge were happening in the early historic period, although historians sometimes tend to underestimate such contacts and interaction. However, more critical study is required on these aspects; but it would not be a surprise if this idea had evolved in India, considering the variety of developments in the medieval period.

The medieval measurement units such as *mā* and *véli* units were not uniform in size and their area varied, according to the size of the measurement rod. Hence, the area units based on one measurement system cannot be directly compared to another system, without going into the various parameters involved.

From about tenth century CE, we do notice very fine, minute arithmetic calculations in the inscriptions. The terms such as *madakku* and *virivu* reveal the various operations related to land assessment. The medieval records of land and taxes were properly maintained in order to collect and manage tax revenue systematically. When they converted the land areas, which were based on smaller scale (measurement rod) to a larger scale, minute fractions resulted. These fractions and numerals could be used to improve the cognitive skills of the students in contemporary education. Detailed research is needed for understanding the emergence of numbers and fractions in the Tamil region.

9. References

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