Level 2 Fuzzy Relations in Database Modeling

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ABSTRACT

In this paper we introduce the concept of level 2 fuzzy relation. Level 2 fuzzy relations play very important role in level 2 fuzzy logic system. Level 2 fuzzy set is extension of ordinary fuzzy set in the sense, when domain elements of ordinary fuzzy sets are fuzzy; the resulting set is called level 2 fuzzy sets. Using same approach we have defined level 2 fuzzy relations, its operations and its role in database modelling.

Key words: Database Modeling Fuzzy relations, Fuzzy logic, Level2

Introduction

Level 2 fuzzy sets play a key role in humanization of machines. They can deal with vagueness as well as uncertainty in the data at a time. From definition of fuzzy relation if elements are replaced by fuzzy sets then resulting equation will work as definition of level 2 fuzzy relation.

This is one approach .The second approach to define level 2 fuzzy relation is to treat fuzzy relation as fuzzy subset of cartesian product of crisp sets as we do while defining fuzzy implication formula[5]. Then consider the level 2 fuzzy set whose elements are fuzzy relations. Without creating lot of mess of the language we directly move for the definitions of level 2 fuzzy relations in the next section.

Level 2 fuzzy relation with approach I (L2FRAI)

Suppose X and Y are crisp sets. M be collection of fuzzy sets on X and N be collection of fuzzy sets on Y. Then we form the product space of M and N. Note that only those fuzzy sets are considered in MxN whose outer layer membership in L2FRAI is non zero.

Definition 1: Level 2 fuzzy relation with approach I

Let X and Y be Crisp sets, and M and N be collection of fuzzy sets on X and Y respectively. Then the level 2 fuzzy relation with approach I is denoted by R and is defined as

$$\dot{R} = \{((A, B), \mu_R(A, B)) / A \in M, B \in N, \mu_R(A, B) > 0\} \quad \ldots\ldots\ldots.(1)$$

Where $\mu_R : M \times N \to [0,1]$ is the function

Instead of defining function from F(X) xF(Y) we have defined function on MxN as we wish to consider only those $_R$ (A, B) > 0 .

Example 1: Consider the statement “Age factor effects driving speed of the car.”

Let X=[0,100] and Y=[0,200].We define fuzzy sets A,B,C on X as follows. These three are fuzzy sets A=young person=TFN(x: 15, 25, 35)

B=Middle aged person=TFN (x: 30, 40, 50)

C= Old age person=TFN(x: 47, 60,100).

We define fuzzy sets M, N, O, and P on Y as follows. Speed of the car can be distributed in four fuzzy sets

M=low speed=TFN(x: 0, 30, 45)

N=medium speed=TFN(x: 40, 60, 80)

O=high speed=TFN(x: 75, 100, 120)

P= very high speed=TRAP(x: 110, 150, 200, 200)

Then the level 2 fuzzy relation is defined as follows,

age / speedofcar	M	N	O	P
A	0.1	0.6	0.8	1
B	0.5	1	0.3	0.2
C	0.9	0.5	0.1	0

Level 2 fuzzy relation with approach II (L2FRAII)

Definition2: Level 2 fuzzy relation with approach II

Let X and Y are crisp sets. S is collection of all fuzzy sets on XxY. Then level 2 fuzzy relation with approach II on XxY is denoted by and is defined as follows,

$$\ddot{R} = \{(R, \mu_{\ddot{R}}(R)) / R \text{ is fuzzy relation on } XxY\}$$

We consider only those relations in $\ddot{R}$ whose outer layer membership is nonzero.

$$\text{i.e. } \mu_{\ddot{R}}(R) > 0$$

Here $\mu_{\ddot{R}}$: F(X ,Y) $\rightarrow$ [0,1] is the function. In fact, we confine ourselves to

$$\mu_{\ddot{R}} : S \rightarrow [0,1]$$

Example 2: Consider the statement: “**x is real number possibly near to y**”

This can be modelled by using L2FRAII as follows.

Let R be fuzzy relation on XxY defined by x is near to y.

And consider the fuzzy relation defined on XxY as S: x is not near to y Then level 2 fuzzy relations with approach II is given as follows

Above statement can be interpreted as

1) x is near y with possibility of 0.9

2) x is not near to y with possibility 0.2

Hence the level 2 fuzzy relation with approach II corresponding to statement given above is

$$\ddot{R} = 0.9 / R + 0.2 / S$$

Relation between L2FRAI and L2FRAII

n L2FRAI, domain of outer layer membership function is F(X)xF(Y) while in L2FRAII, domain is F(XxY).Although cardinality of both F(X)xF(Y) and F(XxY) is same in continuous domain ,and there exists a mapping

$$\eta : F(X) \times F(Y) \to F(X \times Y) \text{ defined by}$$

$$\eta(A, B) = A \otimes B$$

Where A is fuzzy set on X and B is fuzzy set on Y. And A B is fuzzy set on XxY whose membership is given by fuzzy implication formula .That is

$$\mu_{A \otimes B}(x, y) = \min\{\mu_A(x), \mu_B(y)\}$$

Also there exists a function : $\sigma : F(X \times Y) \to F(X) \times F(Y)$ defined by

$$\sigma(R) = (R_x, R_Y)$$

Where R_x and R_y are projections of R on X and R on Y respectively.

One may get confused from the above discussion that F(XxY) is isomorphic to F(X)xF(Y) but this is in not true in the case of discretized domain. We will prove in the coming theorem.

Theorem 1: Show that F(XxY) and F(X)x F(Y) are not isomorphic to each other in discretized domain.

Proof: We will prove the result in discrete case as follows.

Let X be crisp set with m elements and Y be crisp set with n elements.

For time being , we discretize the co domain[0,1] into k points.(As done by Zadeh in proving that number of type 2 fuzzy sets and number of level 2 fuzzy sets on a domain are not same.)[5].

Then the number of sets in F(X) are k^m , similarly number of sets in F(Y) are k^n.

Then the total number of sets in F(X)xF(Y) are k^m x k^n. While total number of elements in F(XxY) are k^{mn}. This shows that both spaces are not isomorphic.

Role of L2FRAI and L2FRAII in data base modeling.

These two types of level 2 fuzzy relations handle different types of linguistic statements. The basic difference is that the first handles the relations between different types of fuzzy sets while the second deals with uncertainty of existence of the relations between the crisp objects. The compositions of L2FRAI are analogous to union and intersection of level 2 fuzzy sets and hence are not discussed in this paper

.In the next section we will discuss the compositions of L2FRAI on same product space and different product spaces.

Compositions of L2FRAI under same product space

Definition 3: Union of two L2FRAI under same product space.

Let X and Y be crisp sets. Let Mi and Ni ;i=1,2 denotes collections of fuzzy sets on Xand Y respectively.Let R_1 and R_2 be level 2 fuzzy relations with approach I on XxY respectively defined as follows

$$\ddot{R}_i = \{(A_i, B_i), \mu_{\ddot{R}_i}(A_i, B_i) / A_i \in M_i, B_i \in N_i\}; i = 1,2$$

$\mu_{\ddot{R}_i} : F(X) \times F(Y) \to [0,1]$ is the function .We restrict ourselves to

$$\mu_{\ddot{R}_i} : M \times N \to (0,1]$$

Then the union of two L2FRAI is again L2FRAI whose membership is given by following equation.

$$\ddot{R}_1 \cup \ddot{R}_2 = \int_{A \in M} \int_{B \in N} \mu_{\ddot{R}_1 \cup \ddot{R}_2}(A, B) \Big/ \left(\int_{x \in X} \mu_A(x)/x, \int_{y \in Y} \mu_A(y)/y \right)$$

Here

1) $A = A_1 \cup A_2; A_1 \in M_1, A_2 \in M_2$

2) $B = B_1 \cup B_2; B_1 \in N_1, B_2 \in N_2$

3) $$\mu_{\ddot{R}_1 \cup \ddot{R}_2}(A, B) = \max_{A = A_1 \cup A_2, B = B_1 \cup B_2} \{\mu_{\ddot{R}_1}(A_1, B_1) \cup \mu_{\ddot{R}_2}(A_2, B_2)\} \quad \ldots\ldots.(2)$$

Definition4: Intersection of two L2FRAI under same product space.

Let X and Y be crisp sets. Let Mi and Ni ;i=1,2 denotes collections of fuzzy sets on X and Y respectively. Let R_1 and R_2 be level 2 fuzzy relations with approach I on XxY respectively defined as follows

$$\ddot{R}_i = \{(A_i, B_i), \mu_{\ddot{R}_i}(A_i, B_i) / A_i \in M_i, B_i \in N_i\}; i = 1,2$$

$\mu_{\ddot{R}_i} : F(X) \times F(Y) \to [0,1]$ is the function .We restrict ourselves to

$$\mu_{\ddot{R}_i} : M \times N \to (0,1]$$

Then the intersection of two L2FRAI is again L2FRAI whose membership is given by following equation.

$$\ddot{R}_1 \cap \ddot{R}_2 = \int_{A\in M} \int_{B\in N} \mu_{\ddot{R}_1 \cap \ddot{R}_2}(A, B) \Big/ \left(\int_{x\in X} \mu_A(x)/x, \int_{y\in Y} \mu_A(y)/y \right)$$

Here

1) $A = A_1 \cap A_2; A_1 \in M_1, A_2 \in M_2$

2) $B = B_1 \cap B_2; B_1 \in N_1, B_2 \in N_2$

3) $\mu_{R_1 \cap R_2}(A, B) = \max_{A=A_1 \cap A_2, B=B_1 \cap B_2} \{\mu_{R_1}(A_1, B_1) \cap \mu_{R_2}(A_2, B_2)\}$

Composition of L2FRAI under different product space.

Definition 5: Composition of two L2FRAI under different product space.

Let X, Y and Z be crisp sets. Suppose A_i's,B_j's and C_k's are fuzzy sets on X,Y and Z respectively for i=1,2,…,m_x; j=1,2,3,…,m_y; k=1,2,…, m_z

We define L2FRAI R_{XxY} and R_{YxZ} by following equations then

$$R_{XxY} = \{(A_i, B_j), \mu_{R_{xxy}}(A_i, B_j)\}$$

$$R_{YxZ} = \{(B_j, C_k), \mu_{R_{yxz}}(B_j, C_k)\}$$

Then their composition is given by

$$R_{XxZ} = \{(A_i, C_k), \mu_{R_{xxz}}(A_i, C_k)\}$$

Where

$$\mu_{R_{xxz}}(A_i, C_k) = \max_{B_j}\{\min \mu_{R_{xxy}}(A_i, B_j), \mu_{R_{yxz}}(B_j, C_k)\} \quad \text{...(4)}$$

Seems simple and straight forward but there is one complication and that is Bj used in first L2FRAand second L2FRAI may be different. In that case, we have to make them identical.

It seems that composition of L2FRAI on different spaces seem to be less complicated as compared to composition of L2FRAI under same spaces. But this is not the case. Complications will increase as dimensions of model increases.

Composition of L2FRAII under same product space

We know that L2FRAII is again a level 2 fuzzy set. Hence composition of L2FRAII under same space is identical to union and intersection under the same space as defined by equation 2.10 and equation 2.11. The only difference is that A and B are replaced by R_1 and R_2

respectively and T is replaced by R.

Definition6: Union and intersection of two L2FRAII under same product space.

R_1 and R_2 be level 2 fuzzy relations with approach II defined on product space XxY as follows.

$$\ddot{R}_1 = \{(R, \mu_{\ddot{R}_1}(R)) / \mu_{\ddot{R}_1}(R) > 0\}$$

$$\ddot{R}_2 = \{(R', \mu_{\ddot{R}_2}(R')) / \mu_{\ddot{R}_2}(R') > 0\}$$

Then $\ddot{R}_1 \cup \ddot{R}_2$ is again a L2FRAII on XxY defined as follows

$$\ddot{R}_1 \cup \ddot{R}_2 = \{R \cup R', \mu_{\ddot{R}_1 \cup \ddot{R}_2}(R \cup R')\}$$

Where ...(5)

$$\mu_{\ddot{R}_1 \cup \ddot{R}_2}(R \cup R') = \max_{R,R'}\{\mu_{\ddot{R}_1}(R), \mu_{\ddot{R}_2}(R')\}$$

And intersection is given by

$$\ddot{R}_1 \cap \ddot{R}_2 = \{R \cap R', \mu_{\ddot{R}_1 \cap \ddot{R}_2}(R \cap R')\}$$

Where ...(6)

$$\mu_{\ddot{R}_1 \cap \ddot{R}_2}(R \cap R') = \min_{R,R'}\{\mu_{\ddot{R}_1}(R), \mu_{\ddot{R}_2}(R')\}$$

$$\ddot{R}_1 = \{(R, \mu_{\ddot{R}_1}(R)) / \mu_{\ddot{R}_1}(R) > 0\}$$

$$\ddot{R}_2 = \{(R', \mu_{\ddot{R}_2}(R')) / \mu_{\ddot{R}_2}(R') > 0\}$$

Composition of L2FRAII under different product spaces

Composition of L2FRAII under different spaces is much more complicated. If is R_1 L2FRAII over XxY and is L2FRAII over YxZ then their composition can be done R_2 in following way.

Here R is fuzzy relation on XxY and R' is fuzzy relation on YxZ. Here R R' denotes the max-min composition as done in chapter one.

Note that if R_1 has m elements and R_2 has n elements, then total number of compositions required to compute R_1 R_2 are mn.

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