

# Numerical Analysis of Crack Orientation Detection Using Eddy Current Testing and Particle Swarm Optimization

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## Abstract

The crack orientation has a crucial role in the fracture parameters estimation. For this reason, we proposed an inverse approach to detect the orientation of crack within an axisymmetric model based on the particle swarm optimization algorithm. The numerical study of this approach was derived from a finite element model of eddy current testing. The obtained results prove that this algorithm can be a good alternative to the direct method to predict the crack orientation when it is challenging to find crack orientation by eddy current testing.

**Key words :** Particle swarm optimization, Crack orientation, Crack detection, Eddy current testing, Inverse problem, Finite element method.

## I. Introduction

Every inverse problem, either in engineering or economics, requires decision making based on optimization algorithms. Making a decision means deciding on a choice between several solutions. These algorithms drive our decision to the best one. An objective function or performance index describes the alternatives' effectiveness. The best alternative in optimization theory is given in terms of the objective function. Meta-heuristic algorithms inspired from nature solve optimization problems by imitating biological or physical procedures. They can be divided into three categories: evolution-based, physics-based, and swarm-based methods [1]. The laws of natural evolution motivate the development of an optimization algorithm based on evolution approaches.

Among these, we can mention the genetic algorithms [2], the evolution strategy [3] and the biogeography based optimizer [4]. The physics-based algorithms mimic the universe's physical laws. The most known algorithms in this category are: simulated annealing [4], central force optimization [5], gravitational local search [6] and galaxy based search algorithm [7]. The third category is nature inspired algorithms, that imitate animal behavioral patterns. It includes methods like: ant colony optimization [8], whale optimization algorithm [1], Harris hawks optimization [9], gray wolf optimization [10] and particle swarm optimization (PSO). The commonly used algorithm in this category is the PSO, proposed by Kennedy and Eberhart [11] and which we adopted, in our case, to solve the inverse problem of the crack orientation using the eddy current testing.

### Particle Swarm Optimization

The PSO method is a meta-heuristic approach that allows finding the optimum of a function in a reasonable processing time. The PSO simulates the social behavior of birds' swarms. To discover the optimum answer, the PSO utilizes a number of particles or candidate solutions that “fly” around the search space ( i.e. the optimal position). In the meantime, they're all tracing their tracks to find the optimum spot (best solution). It means that particles examine both their own best answers and the best solution the swarm has found.

PSO analyzes social behavior using a mathematical equation that allows particles to be controlled throughout their displacement process [12]. The displacement of a particle is governed by three components as shown in figure 1. Each of these components represents a section of the particle's velocity equation (2).

- The inertia component: the particle tends to follow its current direction of displacement.
- The cognitive component: the particle will gravitate to the best location it has previously passed through.
- The social component: the particle will be attracted to the optimal location determined by its neighbors.

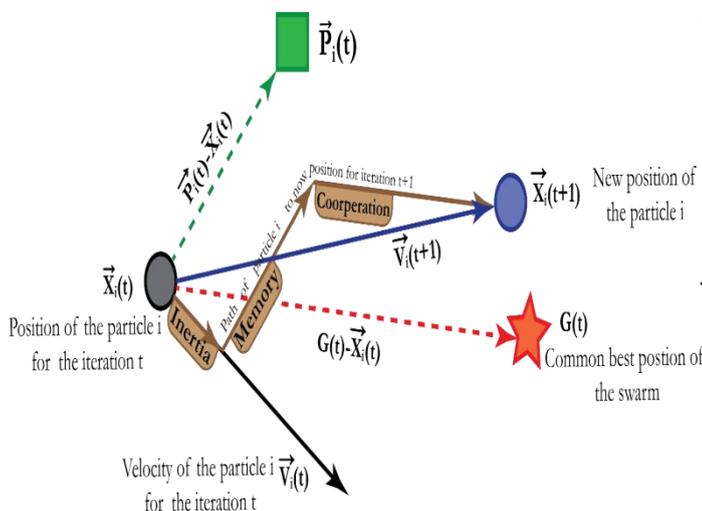


Fig. 1. Displacement and velocity of particle *i* towards the best solution

The displacement and the velocity of particle *i* between the iterations *t* and *t + 1* are given, respectively, by equations (1) and (2) :

$$\xi_i(\tau + 1) = \xi_i(\tau) + \zeta_i(\tau + 1) \quad (1)$$

$$\zeta_i(\tau + 1) = \omega \zeta_i(\tau) + X_1 \rho_1 (\pi \beta_i(\tau) - \xi_i(\tau)) + X_2 \rho_2 (\gamma \beta(\tau) - \xi_i(\tau)) \quad (2)$$

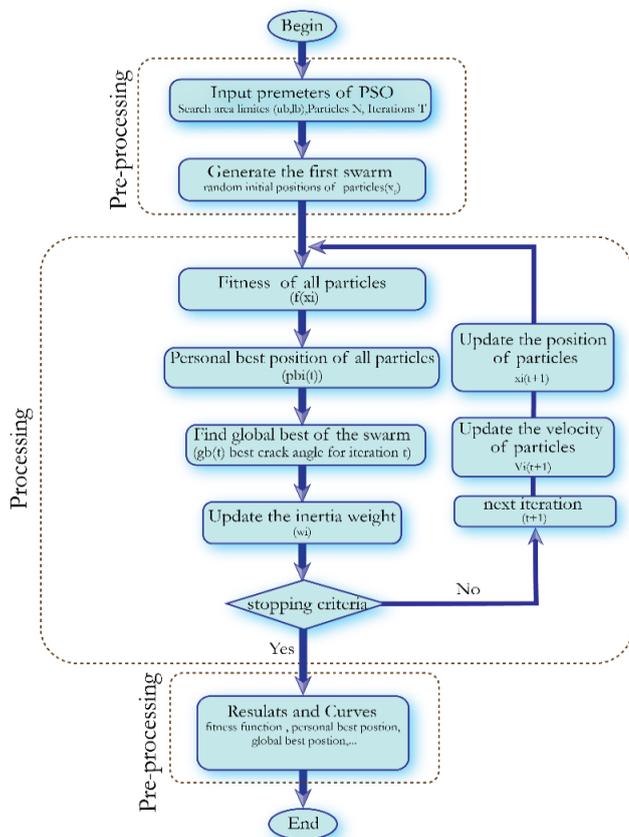
where  $\omega$  is the inertia weight,  $C_1$  the cognition learning factor,  $X_2$  the social learning factor,  $\pi \beta_i$  the personal best solution of the  $i^{\text{th}}$  particle,  $\gamma \beta$  the position of the global best-known solution of all the swarm.  $\rho_1$  and  $\rho_2$  are random numbers generated uniformly from the range of [0, 1].

Each particle's new velocity in the search space is composed of the follows:

1. The particle's original velocity or current motion  $\omega \zeta_i(\tau)$ .
2. The previous best position particle's given by the  $(X_1 \rho_1 (\pi \beta_i(t) - \xi_i(t)))$  and used to update the velocity towards the particle's optimal location.
3. The position of the best fitness value given by  $(X_2 \rho_2 (\gamma \beta_i(t) - \xi_i(t)))$  and used to adjust the velocity of all particles towards the global best position.

The particles move very quickly when the updated velocity has big values, leading to skipping or by passing the best solution. Thus, the particles' velocity should be limited to the range  $[-V_{\max}, V_{\max}]$ . The search region increases with the high values of  $V_{\max}$ , causing the particles to escape from the optimum solution and failing to converge to the optimal solution. However, the lower value of  $V_{\max}$  causes the particles to investigate within a tiny search region, which results in slower convergence. Thus a judicious choice of  $V_{\max}$  should be made. All particles' positions and velocity are modified repeatedly until a stopping criterion is reached [1,3,9]. The equation (1) gives the new locations of all particles by summing the velocity and the previous position of each particle. The PSO method uses current  $\xi_i$ ,  $\pi \beta_i$ ,  $\zeta_i$ , and  $\gamma \beta$  to search for better locations by continuously pushing the particles towards the best solution.

For more details, the flowchart in figure 2 presents the PSO procedure. Starting by the input parameters such as the iterations particles number and initiation of the first particle swarm, passing to the processing which contains the calculation of fitness and personal best solution of each particle and searching for the global best solution of the swarm, and for each iteration there is an updating of the positions and velocities of particles, until the stopping criterion is achieved by estimation of the best global solution and its fitness function which is in our problem the best crack angle and its relative error.



**Fig. 2.** Flowchart of particle swarm optimization

The PSO has many advantages, such as its simple implementation and efficiency as a global search algorithm. The only disadvantage of PSO is its slow convergence in the refined search stage, known as weak local search ability. However, since our problem at hand has no local optimum, we opt to use such an algorithm to solve the

inverse problem and, hence, find the best crack orientation.

## II. Problem Description and Governing equations of Electromagnetic

First, we briefly present the direct problem, which is the finite element modeling of the detection of crack orientation within the axisymmetric model using the non-destructive testing (eddy current), as shown in figure 3. This model is part of investigated pipeline as depicted in figure 4 using an absolute sensor (coil). [13,14]. The magnetodynamic equation for 3D problems in the case of a source is given by :

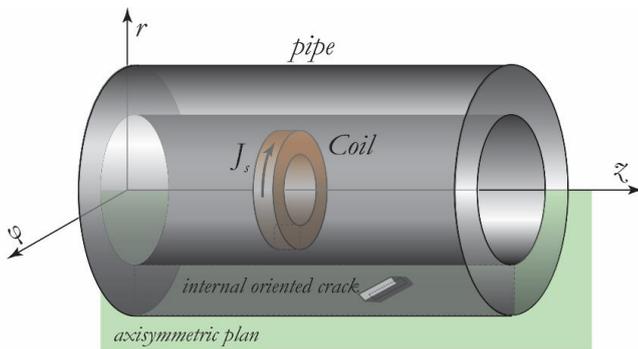
$$\vec{\nabla} \wedge (\vec{\nabla} \wedge \vec{A}) + j\omega\mu\sigma\vec{A} = \mu\vec{J}_s \quad (3)$$

Where:  $\vec{A}$  the magnetic potential vector,  $\sigma$  Electrical conductivity [ $\Omega^{-1}m$ ],  $\mu$  Relative magnetic permeability of the medium considered,  $\vec{J}_s$  vector of a source current density,  $j\omega = \frac{\partial}{\partial t}$ .

When the current is oriented in the direction, then  $\vec{A} = A_\phi [A \cdot m^{-1}]$ . As the vector  $\vec{A}$  coincides with its component  $A_\phi$ , its divergence is naturally zero ( $\vec{\nabla} \cdot \vec{A} = 0$ ). So the equation (1) which describes the magnetodynamic behaviour of an axisymmetric 2D problem (plan  $r, z$ ) in the case of the pipeline ( Fig. 3), becomes: [15]

$$\frac{\partial}{\partial z} \left( \frac{1}{r} \frac{\partial A^*}{\partial z} \right) + \frac{\partial}{\partial r} \left( \frac{1}{r} \frac{\partial A^*}{\partial r} \right) - j\omega\mu\sigma A^* = -\mu\vec{J}_s \quad (4)$$

$A^* = rA$  is the modified magnetic vector potential



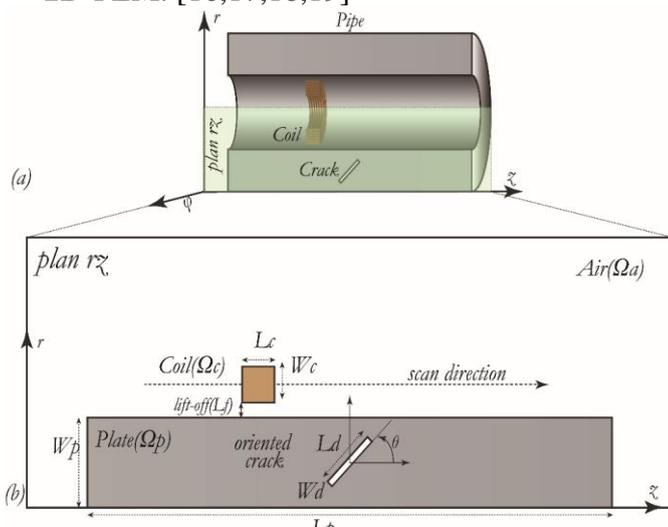
**Fig. 3.** pipeline under eddy current testing.

In the general case, an inductive sensor is characterized by its impedance given by :

$$Z = -\frac{j\omega}{I^2} \int_{\Omega(\text{coil})} A J_s d\Omega \quad (5)$$

### III. Numerical formulation of 2d sensing axisymmetric problem

In our study model, a pipeline with an internal and oriented crack under eddy current testing is shown in Fig.4.a. We have chosen the finite element method to simulate this process. Where this type of problem is considered as 2D axisymmetric, we take just one plan rz (Fig.4.b) to analyse the detection of the crack orientation by eddy current using 2D-FEM. [16,17,18,19]



**Fig. 4.** Axisymmetric model of the tested pipeline using absolute sensor.

The simulation which has been performed in this paper is based on the aforementioned

coil topology and the aluminiumplate specimen with an oriented internal crack ( $L_d = 4mm, W_d = 2mm$ ). The parameters of the plate and the coil are presented in Table .1 and Table .2, respectively.

An alternating current is used to ensure the excitation of the coil which is defined as follows:

$$I = I_0 \cos(2\pi ft) \quad (6)$$

to take into consideration the finite element approximation approach, we use the Galerkin method for equation 2, which consists of choosing the weighting functions equal to the interpolation functions  $\alpha_i$ . We end up with the following formulation:

$$\iint_{\Omega} \left( \frac{\partial \alpha_i}{\partial r} \frac{\partial A}{\partial r} + \frac{\partial \alpha_i}{\partial z} \frac{\partial A}{\partial z} \right) \frac{drdz}{r} + \iint_{\Omega} j\omega \sigma \mu \alpha_i A \frac{drdz}{r} = \iint_{\Omega} \mu \alpha_i J_s drdz \quad (7)$$

Where

$$A(r, z) = \sum_{j=1}^N (\alpha_j(r, z) A_j) \quad (8)$$

With  $A_j$  are nodal values,  $\alpha_j(r, z)$  shape functions for element e. after introducing equation (13) into (12) and integrating equation 13, we arrive at the following matrix formulation :

$$[M]\{A\} + j\omega[L]\{A\} = [F] \quad (9)$$

Where

$$M_{ij} = \iint_{\Omega} \frac{1}{\mu} \left( \frac{\partial \alpha_i}{\partial r} \frac{\partial \alpha_j}{\partial r} + \frac{\partial \alpha_i}{\partial z} \frac{\partial \alpha_j}{\partial z} \right) \frac{drdz}{r} \quad (10)$$

$$L_{ij} = \iint_{\Omega} \sigma \alpha_i \alpha_j \frac{drdz}{r} \quad (11)$$

$$F_i = \iint_{\Omega} \alpha_i J_s drdz \quad (12)$$

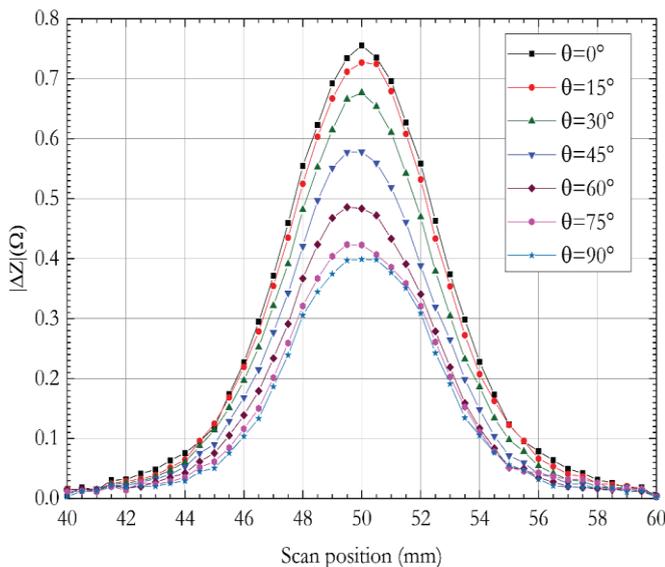
Where  $i = 1, \dots, N$  and  $j = 1, \dots, N$  number of the element nodes.

The resolution of the linear system (10) allows us to know the vector magnetic potential  $\{A\}$  in each node.

For the mashing, we use triangular elements with 3 nodes , and for refining of mesh based on geometries complexity and in

zones of concentration of magnetic field

The coil scans the plate along the z-axis in order to detect the position and the orientation of the crack with seven different angles, this detection is indicated by the normalized impedance signal of the coil as illustrated in figure 5.



**Fig. 5.** Normalized impedance amplitude in terms of crack orientation.

From this figure, we notice that the orientation angle of the crack has a significant effect on the impedance in terms of the peak value and its position in the scan range. This fact is our main and most important result. Indeed, the more the orientation angle increases, the more the impedance peak value decreases. Therefore, we can determine any crack orientation corresponding to the maximal value of the measured impedance of the sensor.

In case if there is no uniform relation between the crack orientation and the sensor impedance as shown in figure(5), we propose an alternative method to solve the inverse problem of crack orientation detection, which is the PSO algorithm to find the optimal crack angle corresponding to the chosen peak of the impedance from figure(5).

For example and during the scan process, the absolute sensor recorded the following maximal value of normalized impedance  $dZ_{exp} = 0.6364\Omega$ . According to the direct problem, treated previously, this value corresponds to the angle  $\theta_{exact} = 45^\circ$ . The question here is to find the best corresponding crack orientation angle  $\theta_{gb}$  using the particle swarm optimization with the following set of parameters:

- $N = 10$  the number of particles.
- $T = 10$  the number of iteration.
- $V_{max} = 6$  the particle' velocity maximal value.
- $\omega = [0.2, 0.9]$  the range of inertia weight.
- $C_1 = 2, C_2 = 2$  the cognition and social learning factor respectively.
- $lb = 0, ub = \pi/2$  the lower and the upper band of the search area for crack orientation angles, respectively.

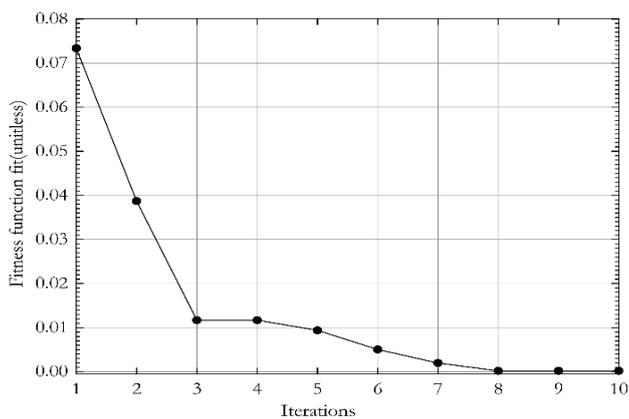
In this example, the fitness function is given by:

$$fit = \left| \frac{dZ_{est} - dZ_{exp}}{dZ_{exp}} \right| \quad (13)$$

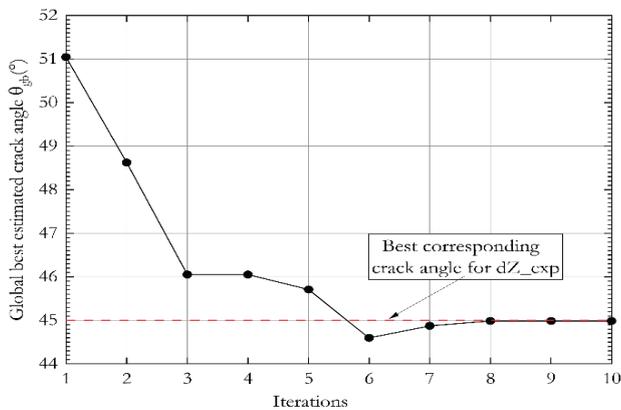
where  $dZ_{est}$  is the estimated normalized impedance using the PSO and  $dZ_{exp}$  is the numerical value of the normalized impedance which is calculated directly by eddy current testing.

#### IV. Results and interpretation

The figures [6a] and [6b] illustrate respectively the obtained global fitness and the best estimated crack angle using the aforementioned parameters.



a



b

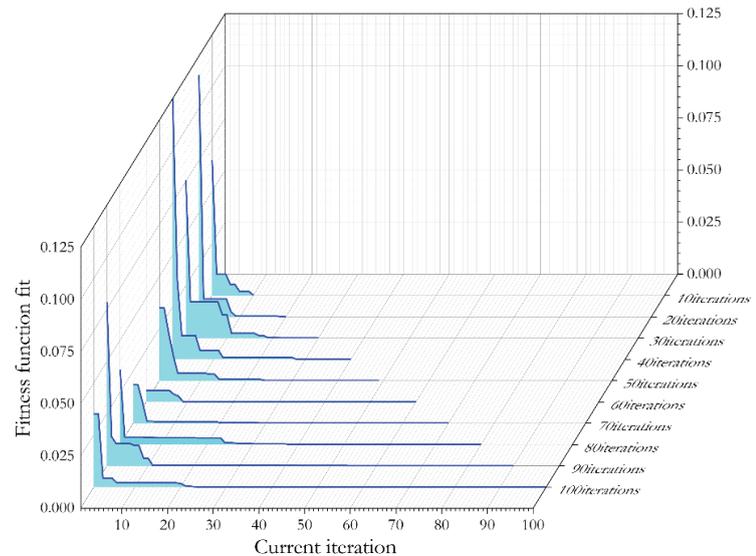
**Fig. 6.** Global result of the PSO for each iteration, **a)** fitness function, **b)** best estimated crack angle.

We remark that the global best solution decreases exponentially and converges rapidly to the exact value of the crack angle ( $\theta_{exact} = 45^\circ$ ), which is corresponding to the value of  $dZ_{exp}$  (measured experimentally) and thus, proves the effectiveness of this method to treat such type of problems.

In iteration 5 and 6, the value of the global best solution goes below the exact crack angle  $\theta_{exact} = 45^\circ$ . This is due to the fact that the investigated crack angle is in the middle of the search area  $[0, \pi/2]$  and due to the random nature of the searching which cause this anomaly. The computing time for 10 particles and 10 iteration is 1093s (18 minutes). It can be considered relatively large, but it is acceptable if we look at the size of the treated problem and the accuracy

of the obtained results.

Now we move on to the parametric study on the iteration and particle number. Figure (7) presents the fitness for a range of 10 to 100 iterations with a step of 10 iterations and a fixed number of particles equal to 10.

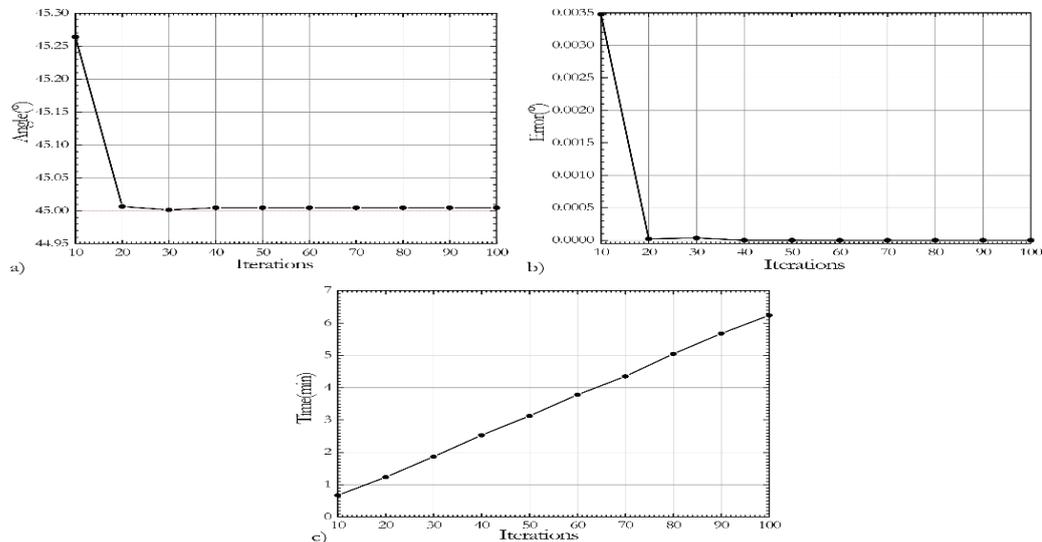


**Fig. 7.** Effect of the iteration number on the fitness function.

Firstly, the fitness continuously decreases whatever the iteration final number is but in a different pattern. That can be explained by the random nature of the particles' velocity.

Secondly, most elements converge to the minimal fitness value at 20 iterations. Consequently, 20 iterations are sufficient for this kind of problem to provide adequate accuracy of results.

Figure (8) illustrates the effect of the iteration number on the three critical factors, namely, the best solution (crack orientation) given in figure (8a), the relative error in estimating the crack angle shown in figure (8b), and the consumed computing time illustrated in figure (8c).



**Fig. 8.** Effect of the iteration number on the fitness function.

The interpretations of these results can be summarized as follows: From Figure 8, the PSO convergences, with stable dynamic, to the best angle, practically, attain that in iteration 20, a fact in accordance with the previous statements extracted from figure 7. Additionally, the computing time increases linearly with the iteration number, which is a logical fact, because the iterations are unrelated.

## V. Conclusion

To deal with the engineer’s difficulties in using the proposed direct approaches, especially for this present work, we propose an alternative method based on PSO to treat the inverse problem of the axisymmetric model by using non-destructive testing (eddy current). The proposed approach allows us to find and even predict the orientation of hidden cracks using measured values of the system impedance. We picked the impedance comprising the crack signing in order to establish a comparison between the direct

problem data and those acquired by the PSO in a reasonable Time. Additionally, the outcomes demonstrate its viability and reasonability. It is crucial to ensure that the use of AI will effectively replace or supplement the work of NDT operators.

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