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Lasing without inversion in three level atoms and atomic coherence effects

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Abstract
We reviewed briefly about the concepts and physics behind lasing without inversion and atomic coherence effect. The discussion concentrates on the analysis of physical conditions under which a Λ-type three level system exhibits lasing without inversion. At the end of this review, a possible application of LWI was reported.

Keywords: LWI, Atomic Coherence Effect.

Introduction

Quantum optics and laser physics is entering into an exciting new eras in which we are in a position to realize new devices violating the usual dogmas in physics. There are in fact many examples where certain complex ideas of quantum mechanics used to explain phenomena like electromagnetically induced transparency (EIT) (Alzetta, 1976; Arimondo *et al.,* 1976; Harris *et al.,* 1991), new type of lasers which has the capacity to show lasing action without the need for population inversion (LWI) (Kocharovskaya *et al*., 1989; Scully *et al*., 1989), and production of light with greatly reduced noise (Winters *et al.,* 1990). In other words these developments are tantamount to overthrowing a generally accepted idea in physics.

Lasing without inversion could be useful in many areas of science and technology. One of the areas is the production of laser at extremely short wavelength, i.e, in the X-ray wavelengths. Production of coherent light in X-ray region is an extremely difficult task and usually requires very high power which is not easily available. X-ray lasers would have an enormous impact on the computer industry and in medical science. Similarly, reducing noise could lead to improving the precision of laser based measurement devices such as laser gyroscopes used in navigation systems. In the present paper we would like to review the concepts of Lasing without inversion in a particular atomic system involving three level atoms.

Laser with inversion

Before proceeding further in the discussion of lasing without inversion (LWI) it is worthwhile to give a brief explanation about the laser which we all know and which needs population inversion. The word LASER stands for Light Wave Amplification by Stimulated Emission of Radiation. We are all grateful to Einstein because the idea of stimulated emission was first given by him in the year 1917, when he derived the famous black body radiation of Planck with the help of the concept of stimulated emission. Einstein presumably did not know about laser and indeed it took long forty years to invent a practical devise known as laser. The first laser emitting a coherent beam of red light from a crystal of ruby was made by Maiman in the year 1960.

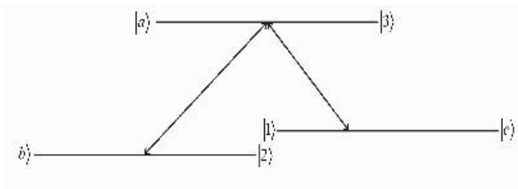
The phenomena which are primarily responsible for the interaction of radiation with matter in everyday life are absorption and emission (spontaneous emission). Light falling in matter is absorbed bearing the atoms in excited states which spontaneously emit radiation with a spread of frequency inversely proportional to the decay time. The situation is modified when the atomic decay times are sufficiently long and the radiation sufficiently strong. In that case the radiation may fall on excited states resulting in stimulating emission rather than absorption and this third process was required by Einstein to derive the Planck black body radiation law.

To make any laser device we require few basic components like a resonating structure or cavity consisting of two parallel dielectric mirrors, an active medium consisting of atoms, molecules or any system to create population inversion, and pumping scheme to excite the active medium.

Laser without inversion

This is generally true that to obtain laser action population inversion is needed. This means that there are more atoms in the excited state than in the ground state. It is necessary to understand the need of population inversion in a

Fig.1. Three level atomic system in the co-called Λ configuration interacting with two fields of frequencies υ_1 and υ_2.



clear manner. We have seen earlier that the possibility of stimulated emission has interesting applications in diverse fields of scientific investigations. If there is very intense and right radiation present, it will induce downward transition. The transition then adds to its energy $\hbar\omega$ to the available light energy, if there were some atoms available in the upper level. Now we can arrange to have by some non-thermal method a gas where the number of excited atoms is very much greater than the number in the ground state. This is non equilibrium situation and is not given by the usual Boltzman equation $N_2 = N_1 e^{-\hbar\mu/kT}$. In this case the light which has frequency corresponding to the energy difference $E_2 - E_1 = \hbar\mu$ will not be strongly absorbed. On the other hand it will induce emission from the upper level. So if we have a large number of atoms in the upper level there would be a certain sort of chain reaction in which the moment the atom begun to emit, more would be caused to emit. This is what is known as laser. Thus we find that the first requirement of laser action must be that the energy levels concerned are not in thermal equilibrium and that the upper of the two levels must be more populated than the lower.

Hence, population inversion is needed to overcome absorption from the lower level. But what if, we can arrange things (i.e. atoms) such that absorption is cancelled? Can we then have laser without inversion. The answer is yes. In order to see how it happens let us consider three level atoms known more appropriately as Λ system and V systems. It is worthwhile to note that many interesting phenomena in optics, such as quantum beats, the Hanle effect, and self induced transparency originate from atomic coherence and interference of radiative processes. Under special conditions coherent atomic transitions can cancel absorption. This result of atomic coherence is used in the concept of coherent trapping, electromagnetically induced transparency (EIT) and Lasing without inversion (LWI).

The situation of lasing without inversion can be realized, for instance, in a three level system, when two coherent atomic transitions interfere destructively, and hence, cancel absorption. To illustrate the concept of LWI, let us consider a three level Λ (pronounced as lambda) atomic system as shown in Fig.1.

The Λ-three level atomic is formed by the upper level $|a\rangle$ connected to the lower levels $|b\rangle$ and $|c\rangle$ via interaction with electromagnetic fields E_1 and E_2 respectively. Only the atomic transitions $|1\rangle$ to $|b\rangle$ and $|a\rangle$ to $|c\rangle$ are dipole allowed. The physical reason for canceling absorption in this system is the uncertainly in atomic transitions $|c\rangle - |a\rangle$ and $|b\rangle - |a\rangle$ which results in destructive interference between them. It should be noted that the separation between $|b\rangle$ and $|c\rangle$ is extremely small. Since both the transitions $|b\rangle - |a\rangle$ and $|c\rangle - |a\rangle$ are directed to the same atomic state $|a\rangle$, it is extremely difficult to locate along which path such a transition is made. The situation is similar to Young's double slit experiment where interference is a consequence of uncertainty in determining which of the two slits the photon passed (Cohen-Tannoudji, 1977).

Let us consider the probability of absorption in the three level system as shown in Fig.1. This absorption probability will be equal to the sum of the probability amplitude squared corresponding to $|b\rangle \rightarrow |a\rangle$ and $|c\rangle \rightarrow |a\rangle$ transitions. When there is a correlation between these probability amplitudes, it will lead to an interference term which under appropriate phase conditions makes the total absorption probability equal to zero. The emission probability is equal to the sum of the transition probabilities $|a\rangle \rightarrow |c\rangle$ and $|a\rangle \rightarrow |b\rangle$ and is independent of their mutual correlation. This is primarily due to the fact that the final states $|b\rangle$ and $|c\rangle$ are different, and therefore it is known exactly along which path the atom makes transition to the lower state, $|a\rangle \rightarrow |c\rangle$ or $|a\rangle \rightarrow |b\rangle$. Thus there is no uncertainty in the atomic transition routes and as a result there is no transition interference between these transitions. This shows that there is an asymmetry in up and down transitions which allows amplification in the three level Λ type atomic system with zero or minimum absorption laser. In the presence of several ways to demonstrate this we followed the method adopted by Scully (Scully *et al.,* 1997) in the following treatment. The approach adopted by them is semi classical where the electromagnetic field is considered classically but the atom is treated quantum mechanically. Fully quantum mechanical treatment is also given by Lee (Hai-Woong, 1995). The idea is to calculate the time dependent probability amplitudes for each level and then to demonstrate that the probability of a transition to the upper level can vanish for a particular initial condition but the transition probability to the lower level i.e., emission probability does not vanish. In rotating wave approximation (RWA) (Lamb *et al.,* 1974), the Hamiltonian for the system is given by

$$H = H_0 + H_1 \qquad \text{… … (1)}$$

Where, $H_0 = \hbar \mu_a |a\rangle\langle a| + \hbar \mu_b |b\rangle\langle b| + \hbar \mu_c |c\rangle\langle c|$ … … (2)

$$H_1 = -\frac{\hbar}{2}\left[\Omega_{R_1} e^{-i\phi_1} e^{-i\nu_1 t} |a\rangle\langle b| + \Omega_{R_2} e^{-i\phi_2} e^{-i\nu_2 t} |a\rangle\langle c|\right] + H.C. \text{… … (3)}$$

where $\Omega_{R_1} e^{-i\phi_1}$ and $\Omega_{R_2} e^{-i\phi_2}$ are the complex Rabi frequencies associated with the coupling of the field modes of frequencies υ1 and υ2 with the atomic transitions $|a\rangle - |b\rangle$ and $|a\rangle - |c\rangle$ respectively.

and
$$\left.\begin{aligned} \Omega_{R_1} e^{-i\phi_1} &= \wp_{ba} E_1/\hbar \\ \Omega_{R_2} e^{-i\phi_2} &= \wp_{ba} E_2/\hbar \end{aligned}\right\} \text{… … (4)}$$

The symbol $\wp$ stands for dipole moment matrix element.

$$\wp_{ba} = e\langle b|r|a\rangle \; ; \; \wp_{ca} = e\langle c|r|a\rangle \text{… … (5)}$$

The wave function of this three-level atomic system is

$$|\psi(t)\rangle = C_a(t)|a\rangle + C_b(t)|b\rangle + C_c(t)|c\rangle \text{… … (6)}$$

The equations of motion for the probability amplitudes $C_a(t)$, $C_b(t)$ and $C_c(t)$ can be derived from the Schrödinger equation

$$i\hbar|\dot{\psi}(t)\rangle = H|\psi(t)\rangle \text{… … (7)}$$

We have

$$\dot{C}_a = \frac{i}{2}\left(\Omega_{R1} e^{-i\phi_1} c_b + \Omega_{R2} e^{-i\phi_2} c_c\right) \text{… … (8)}$$

$$\dot{C}_b = \frac{i}{2}\Omega_{R1} e^{-i\phi_1} c_a \qquad \text{… … (9)}$$

$$\dot{C}_c = \frac{i}{2}\Omega_{R2} e^{-i\phi_2} c_c \text{… … (10)}$$

It is assumed that the fields are at resonant with the $|a\rangle \to |b\rangle$ and the $|a\rangle \to |c\rangle$ transitions respectively, i.e., $\mu_{ab} = \nu_1$ and $\mu_{ac} = \nu_2$.

We now assume that the initial atomic state is prepared with the superposition of the two lower levels $|c\rangle$ and $|c\rangle$

$$|\psi(0)\rangle = Cos(\theta/2)|b\rangle + Sin(\theta/2)e^{-i\psi}|c\rangle \text{… … (11)}$$

Solutions of equations (8)–(1) subject to the initial condition (11) are given by

$$C_a(t) = \frac{i\sin(\Omega t/2)}{\Omega}\left[\Omega_{R1} e^{-i\phi_1}\cos\left(\frac{\theta}{2}\right) + \Omega_{R2} e^{-i(\phi_2+\psi)}\sin\left(\frac{\theta}{2}\right)\right] \text{… … (12)}$$

$$C_b(t) = \frac{i}{\Omega}\left\{\left[\Omega_{R2}^2 + \Omega_{R1}^2 \cos(\Omega t/2)\right]\right\}\cos\left(\frac{\theta}{2}\right)$$

$$-2\Omega_{R1}\Omega_{R2}e^{i(\text{¤}_1-\text{¤}_2-\cdot)}\sin^2(\Omega t/4)\sin\left(\frac{-}{2}\right) \text{ … … (13)}$$

$$C_c(t)=\frac{1}{2}\Big\{-2\Omega_{R1}\Omega_{R2}e^{-(\text{¤}_1-\text{¤}_2)}\sin^2(\Omega t/4)\cos\left(\frac{-}{2}\right)$$

$$+\left[\Omega_{R1}^2+\Omega_{R2}^2\cos\left(\frac{\Omega t}{2}\right)\right]e^{-i\cdot}\ \sin\left(\frac{-}{2}\right) \text{ … … (14)}$$

with $\quad \Omega=\left(\Omega_{R1}^2+\Omega_{R2}^2\right)^{1/2}$

An interesting phenomenon in which a coherent superposition of atomic state is responsible for a novel effect is coherent trapping. If an atom is prepared in a coherent superposition of states it is possible to cancel absorption or emission under certain conditions. These atoms are then effectively transparent to the incident field even in the presence of resonance transitions. It is seen that coherent trapping occurs for

$$\Omega_{R1}=\Omega_{R2},\ -=\frac{\text{®}}{2},\ \text{¤}_1-\text{¤}_2-\cdot\ =\mp\Pi \text{ … … (15)}$$

Under these conditions

$$C_a(t)=0 \text{ … … (16a)}$$

$$C_b(t)=1/\sqrt{2} \text{ … … (16b)}$$

$$C_c(t)=\frac{1}{\sqrt{2}}e^{-i\cdot} \text{ … … (16c)}$$

Consider first the case in which the population is initially equally distributed with a fixed phase between the two lower states $|b\rangle$ and $|c\rangle$

$$C_a(0)=0,\quad C_b(0)=\frac{1}{\sqrt{2}}\quad \text{and}\quad C_c(0)=\frac{1}{\sqrt{2}}e^{-\cdot} \text{ … … (17)}$$

This is a particular case of the initial condition (11) with

$$-=\frac{\Pi}{2} \text{ … … (18)}$$

Hence from the solution of equations (12)–(13), it follows that, to the lowest order,

$$C_a(t)\ \square\ i\frac{t}{2\sqrt{2}}\left[\Omega_{R1}e^{-\text{¤}_1}+\Omega_{R2}e^{-i(\text{¤}_2+\cdot)}\right] \text{ … … (19)}$$

In this equation the first and the second terms of the sum represent the probability amplitudes corresponding to the transitions $|b\rangle-|a\rangle$ and $|c\rangle-|a\rangle$, respectively.

When $\Omega_{R1}=\Omega_{R2}=\Omega_R$ … … (20)

Equation (19) becomes

$$C_a(t)=i\frac{t}{2\sqrt{2}}\Omega_R\left[e^{-i\text{¤}_1}+e^{-i(\text{¤}_2+\cdot)}\right] \text{ … … (21)}$$

Therefore, we have

$$|C_a(t)|^2=\frac{t^2\Omega_R^2}{4}\left[1+\cos(\text{¤}_1-\text{¤}_2-\Psi)\right] \text{ … … (22)}$$

This means that the absorption is cancelled $\left(|C_a(t)|^2=0\right)$ of

$$\text{¤}_1-\text{¤}_2-\cdot\ =\pm\Pi \qquad \text{ … … (23)}$$

This is the condition for coherent trapping. The atomic system will stay at low energy levels $|c\rangle$ and $|b\rangle$ at all times for these specific phase conditions. Since there are no transitions to higher energy levels the system will have no absorption.

Fig.2. The ground state is a double. The atoms are initially prepared in a coherent superposition of states

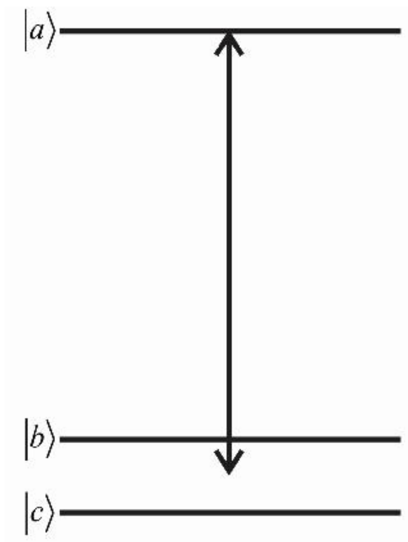
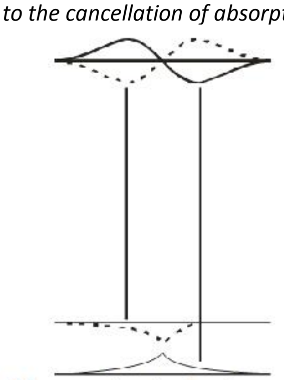


Fig.3. Two ground state wave functions with opposite sign lead to cancellation of the excited state wave functions and therefore to the cancellation of absorption.



Now consider if the system is capable of emission. Suppose that initially the population is in the upper state i.e.

$$C_a(0)=1;\quad C_b(0)=0,\quad C_c(0)=0 \quad \text{… … (24)}$$

The solution of equations (8)–(10) may be found as

$$C_a(t)=\cos\left(\frac{\Omega t}{2}\right) \quad \text{… … (25)}$$

$$C_b(t)=i\frac{\Omega_{R1}^*}{\Omega}\sin\left(\frac{\Omega t}{2}\right) \quad \text{… … (26)}$$

$$C_c(t)=i\frac{\Omega_{R2}^*}{\Omega}\sin\left(\frac{\Omega t}{2}\right) \quad \text{… … (27)}$$

Assuming $\left(\Omega_{R1}^2+\Omega_{R2}^2\right)^{1/2} t = \Omega t \ll 1$

One gets approximately

$$C_b(t)\cong i\frac{\Omega_{R1}^*}{2}t \quad \text{… … (28a)}$$

$$C_c(t)\cong i\frac{\Omega_{R2}^*}{2}t \quad \text{… … (28b)}$$

The emission probability is the sum of the squared probability amplitudes associated with the atomic states $|b\rangle$ and $|c\rangle$, that is,

$$P_{emission}=\left|C_b(t)\right|^2+\left|C_c(t)\right|^2=\frac{\Omega^2 t^2}{4} \quad \text{… … (29)}$$

One can note that the emission probability is independent of the relative phase between atomic states $|b\rangle$ and $|c\rangle$ and is always positive. Comparing the probability of emission with the absorption probability one can also notice that in emission the probability amplitudes are squared and only then summed. However, for the probability of absorption, first the probability amplitudes are summed and only then squared. This is why the absorption probability is mathematically dependent on the relative phase between the atomic transitions.

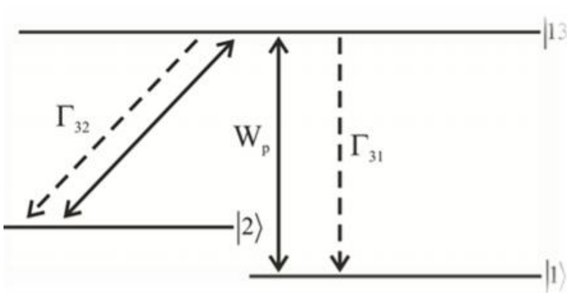
As can be recognized from equation (29) the emission probability is always larger than zero for $t\rangle 0$. Thus if the atomic system is prepared such that the phase conditions (18), (20) and (23) for cancelation of absorption are fulfilled, there will be net gain even in the absence of population inversion.

The three level Λ-atomic system is not the only configuration where lasing without inversion is possible for many differed configurations for three and four level atomic system (Kocharovskaya, 1999; Zevrov, 1999), and for some of them, was proved experimentally (Fry *et al.,* 1993; Veer *et al.,* 1999).

To illustrate how the subtle effects of atomic coherence and quantum interference can change the conventional belief of laser operation consider the case of a three atom where the lower level is a double as shown in Fig.2. In Fig.3 we depict the atom in a specific superposition of the two lower states in which the wave function for the atom in level |b> is "negative" and the wave function in level |c> is "positive".

Now we observe that when we shine right laser radiation in this atom, then the lower positive and negative wave functions generate two opposite contributions to the electric wave function in the excited state, as the sum wave function cancels and henceforth we can see the zero likelihood of exciting the atom occurs. In other words, we have no absorption when the atom starts life in the initial coherent superposition. This is the basic physics involved in the cancellation of absorption in any atomic system. One approach has already been discussed earlier in the case of a Λ system.

Fig.4 .Three level Λ atomic system for Lasing without inversion any state bases



In most of the laser schemes although there is no population inversion in bare-state basic (the eigenstate basis of the isolated atomic system), there is a population inversion in the dressed states (the eigenstate bases of the coupled–atom field system) (Karayajazyk, 1999). Thus the question of inversion and non-inversion in these atomic systems depends on the selected state basis. This type of situation can give some skepticism about the reality of inversionless amplification because true noninversion should be independent of the state basis. Several schemes were reported (Zhu *et al.,* 1992-93) for atomic systems where lasing without inversion is possible. One of these systems was initially demonstrated by Imamoglu and his coworkers (Fig.4)

This system can be considered as a three level Λ atomic scheme. The level $|3\rangle$ is pumped incoherently from both the lower states $|2\rangle$ and $|a\rangle$. A coherent field interacts with the transition $|3\rangle-|2\rangle$. The laser transition is $|a\rangle-|3\rangle$. On this transition the probe field is amplified. To work out the condition of the amplification without inversion for this system in any atomic state, it is worthwhile to use a density matrix approach and find a solution of the master equation

$$\frac{\partial p}{\partial t}=\frac{1}{i\hbar}[H,P]+L_{relax}[P] \quad \text{… … (30)}$$

and obtain the equation of motion for atomic density matrix in a frame relating at the probe frequency. Considering the steady state solution of these equations one can find stimulate emission and absorption rates. Assuming resonance for coupling and probe fields, i.e.

$$\sqcup W_{21}=W_2-W_e-W_p-W_1$$
$$=\sqcup W_{31}$$
$$=W_3-W_p-W_1$$
$$=0$$

it is possible to derive the necessary condition for amplification without population inversion in any atomic state basis. This condition is

$$\frac{\Gamma_{32}}{\yen_{21}}>\frac{\Gamma_{31}}{R_{13}}\frac{\Omega_{23}^2+(\Gamma_{32}+\Re_{23})\yen_{23}}{\Omega_{23}^2} \quad \text{… … (31)}$$

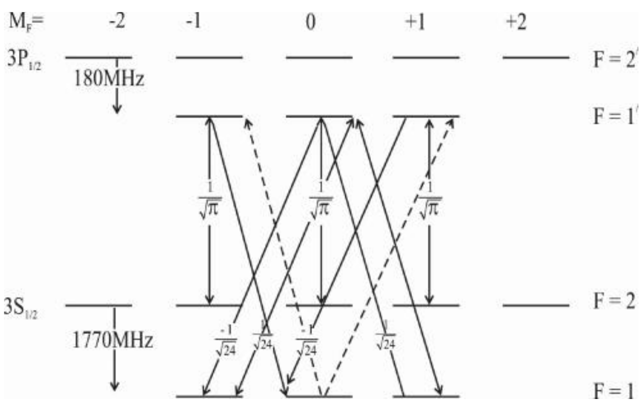
Where decay rates

$$\yen_{21}=R_{23}+R_{13};\ \yen_{32}=\Gamma_{23}+2R_{23}+R_{13}+\Gamma_{31}$$

Γ_{ij} is the spontaneous emission rate from $|i\rangle$ to $|j\rangle$; Γ_{ij} is the pumping rate; Ω_{ij} is the Rabi frequency. The first inequality is the condition for the net gain, and the second one is the requirement of no population inversion. The atomic system will satisfy the condition (31) if the spontaneous decay rate from state $|3\rangle$ to state $|2\rangle$ exceeds that from $|3\rangle$ to $|1\rangle$, i.e. $\Gamma_{32}\rangle\Gamma_{31}$ and the average number of thermal photons per mode in the $|1\rangle-|3\rangle$ transition exceeds that in the $|2\rangle-|3\rangle$ transition i.e.

In this connection we would like to point out that the three level method of population inversion was initially proposed by Basov (1955) who suggested it for application in a molecular beam apparatus. Bloembergen (1956) subsequently suggested that the method would be readily applicable to diamagnetic solids containing weak concentrations of paramagnetic ions and presented a theoretical treatment of the three level method. The salient features of the three level method are at thermal equilibrium the populations decrease with increasing energy of the level, and therefore the assembly will absorb energy from an incident radiation field. It has also supposed that two

Fig.5.Relevant energy levels for sodium atom indicating the nature like Λ type atom



radiation fields are incident on the assembly, one very strong one, with frequency near the resonance υ_{31} and a very weak one with frequency near υ_{32}. If the field υ_{31} is sufficiently strong and intense the (1, 3) transition may be saturated, with the result that the population of atoms in level 1 becomes approximately equal to that of level 3, that is,

$$|_{1} \cong |_{3} \cong \frac{1}{2}\left(|_{1}^{e} + |_{2}^{e}\right)$$

Assuming that the saturation process does not appreciate the disturbance of the system in level 2, it is obvious that a condition may be realized in which $|_{1}\rangle |_{1}^{e}$, and in that case the system will be in an emission state relative to the field frequency υ_{32}. Bloembergen's three level scheme which was worked out five decades ago also carry the essential features of LWI.

We have seen that in this system two fields are allowed to be incident on an assembly, an intense radiation or strong field at resonance with frequency υ_{31} (frequency separating the ground level and the highest excited level) and a weak field at frequency υ_{32}. In the scheme of LWI also fields are allowed to be incident on the three level atom in the Λ configuration. It is, however, interesting to note that the reference to Bloembergen's work related to three level laser is not found in any of the work of LWI.

Experimental demonstration of LWI

To show how LWI can be realized experimentally it is worthwhile to follow the procedure adopted by Padmabandu *et al.* (1996). In this experiment an atomic beam of sodium D_1 line was used as the active medium. By using a weak probe laser, it was first demonstrated with complete transparency and then lasing without inversion. Next, a cavity was installed and aligned. The probe was then blocked and it was found that laser started spontaneously from vacuum fluctuation.

The energy levels for Sodium atom used for the experimental demonstration of LWI are shown in Fig.5. The $2\rightarrow1^{/}$ transition were used for driving the field. The laser transition selected for the purpose was $1\rightarrow1^{/}$, since it had the slowest decay rate which was required by the gain condition given by equation (31), i.e., the radioactive decay on driving transition (Γ_{32}) must be faster than on the lasing transition (Γ_{31}). The incoherent pumping field was on the same $1—1^{/}$ transition.

Having described briefly the experimental demonstration of LWI, it is worthwhile to give a brief summary of what have been described earlier. Possible applications of LWI in various fields are also indicated. We may indicate here that the experiment of Hanle (1924) performed many year ago during the early part of twentieth century provides one of the clearest demonstration of atomic coherence effects.

Summary

In lasing without inversion, the essential concept is the cancellation of absorption by atomic coherence and interference. The phenomenon is also the key to the understanding of electromagnetically induced transparency. The concept of lasing without inversion has been discussed using a semi classical treatment. Lasing without inversion provides a new class of quantum generators having many properties different from the conventional lasers with inversion, and deserving much attention. They are also having potential for new applications. For instance, they could provide coherent radiation in cases when procedure of obtaining population inversion fails. In principle they could work on extremely short (X-rays, UV) wavelengths. The lasing threshold does not depend solely on the efficiency of an incoherent pumping but could be decreased by a strong coherent drive. These lasers (LWI) could also be tunable. X-ray lasers could have an enormous impact on the computer industry and on medical science. One approach to realize a γ-ray laser can be LWI. Kocharovskaya *et al.* (1999) suggested the way in which lasing without inversion could give a solution to this problem.

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