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Charge exchange cross-sections in high-energy proton-deuterium collisions

Y N Tiwari
Department of Physics
North-Eastern Hill University, Shillong-793022, Meghalaya (India)
yntiwari@gmail.com

Abstract

Charge exchange integral cross-sections of protons colliding with deuterium atoms have been calculated from the energy range 5keV to 5MeV. The differential cross-sections have been calculated in the angle ranging from 0-4 mrad. We have employed the Coulomb-projected Born approximation to obtain differential as well as integral cross-section for the system $H^{+}+D(1s)$ under study.

Key words: Coulomb-projected Born approximation

Introduction

Deuterium plays a very important role in addressing many fundamental questions in astrophysics. The deuterium abundance is a key constraint for models of big-bang nucleosynthesis. Primordial D (H) measurements provide the most sensitive probe of the baryon-to-photon density ratio ¦ . This, in combination with the cosmic microwave background measurement of the photon density, can be used to determine the cosmological baryon density (Savin (2002), Burles and Tytler (1998), Lemoine *et al.,* (1999), Tytler *et al.,* (2000)). The process $H^{+}+D(1s)\rightarrow H\left(1s\right)+D^{+}$ is very important in the early universe (Galli and Palla, 1998).

The charge transfer in the $H^{+}+D(1s)$ system is profoundly more challenging than that of the $H^{+}+H(1s)$, because the difference in the nuclear mass, which affects the symmetry present in H_2^{+} under the exchange of nuclei. The symmetry breaking term in HD^{+} (anhydride deuteron) couples the nuclear and electronic degrees of freedom and produces the splitting of the adiabatic potential energy curves as the nuclei separate. Hunter and Kuriyan (1977), and Bates and McCarroll (1958), studied the deuteron-hydrogen system quantum mechanically in very low energy range. Newman et al (1982) measured the absolute charge transfer cross-sections for the process $H^{+}+D(1s)$ only in the energy range from 0.1-150eV. Well *et al.,* (2001), have also investigated the charge transfer cross-sections for the same system in adiabatic region in slow collisions. Wells *et al.,*(2003), have also measured the cross-section for the system $H^{+}+D(1s)$ in very slow collisions and they claim good agreement of their coupled channel calculations. It was investigated that the charge transfer cross-sections in slow collisions using hyperspherical close coupling approximation in the energy range from 20 $eV-2keV$. Wells et al (2003), have studied very slow $H^{+}+D(1s)$ collisions using the ground-state dissociation of HD^{+}. We have investigated the deuteron-hydrogen charge transfer cross-sections in the framework of the Coulomb-projected Born approximation proposed by Geltman (1971), in the energy range from 5keV-5MeV. Throughout this paper atomic units ($e=m=\hbar=4®£_0$) have been used except for the integral cross-sections which are in the units of $®a_0^2$.

Theory

We have considered the charge transfer reaction

$$D^{+}+H\left(1s\right)\rightarrow D\left(1s\right)+H^{+} \tag{1}$$

The differential cross-sections for the process (1) is,

$$\frac{d±}{d\Omega}=\frac{«_i«_f}{4®^2}\frac{k_f}{k_i}\left|T_{if}\right|^2 \tag{2}$$

Where,

$$«_i=\frac{m_P\left(m_T+1\right)}{m_P+m_T+1} \text{ and } «_f=\frac{m_T\left(m_P+1\right)}{m_P+m_T+1}$$

Where again, m_P = mass of projectile ion and m_T = mass of target atom

Fig.1. Differential cross-sections for energies 50and 100keV

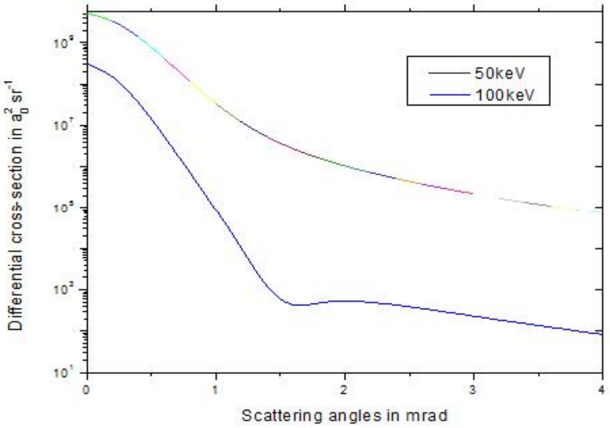


Fig.2. Differential cross-sections for energies 300 and 400 keV

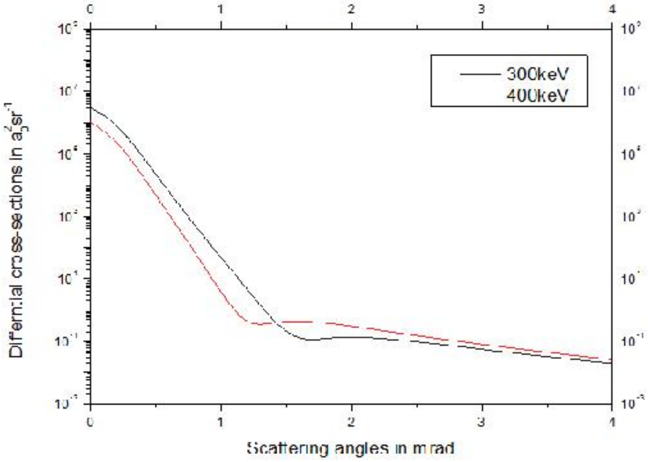
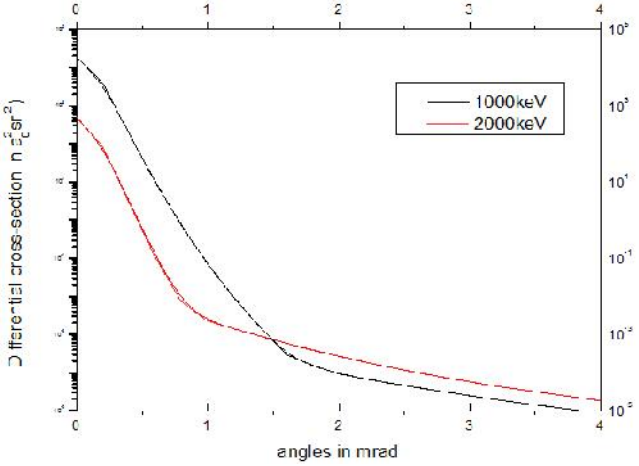


Fig.3. Differential cross-sections for energies 1000 and 2000 keV



The transition matrix in CPB approximation T_{if}^{CPBA} from an initial state i to a final state f in the CB approximation for the process (1) are given by

$$T_{i\,f} = \int \Phi_f^* V_f^* \Phi_i d\vec{r} d\vec{R}_f \tag{3}$$

Where Φ_i and Φ_f are the wave functions for the process (1) in the initial and final channels respectively and are given by

$$\Phi_i = \text{¤}_i\left(\vec{r}\,'\right) \cdot {}_i\left(\vec{R}_i\right) \tag{4}$$

and

$$\Phi_f = \text{¤}_f\left(\vec{r}\right) \cdot {}_f\left(\vec{R}\right) \tag{5}$$

Where again,

$\text{¤}_i\left(\vec{r}\,'\right) = \dfrac{1}{\sqrt{\text{®}}} e^{-S r_i}$ = wave function of the H-atom in its ground state in the initial channel $\left(S = 1\right)$

$\Phi_f\left(\vec{r}\right) = \dfrac{1}{\sqrt{\text{®}}} e^{- \quad r}$ = wave function of the hydrogen atom in $1s$ state in the final channel $\left(\quad = 1\right)$

$\text{¨}_i\left(\vec{R}_i\right) = e^{i\vec{k}_i . \vec{R}_i}$ is a plane wave in the initial channel.

$\text{¨}_f\left(\vec{R}_f\right) = e^{-\text{®}\sim/2}\Gamma\left(1 - i\sim\right) e^{i\vec{k}_f . \vec{R}_f} {}_1F_1\left(i\sim; -ik_f R_f - i\vec{k}_f . \vec{R}_f\right)$ is Coulomb wave

Where $\vec{k}_i$ and $\vec{k}_f$ are the momentum vectors in the initial and final channels respectively.

$\sim = \dfrac{\text{«}_f}{\vec{k}_f}$ is the repulsive Coulomb parameter and ${}_1F_1\left(i\sim; -ik_f R_f - i\vec{k}_f . \vec{R}_f\right)$ is the confluent hyper-geometric function.

Now the transition matrix element given in equation (3) will be

$$T_{if}^{CPBA} = \int d\vec{r} d\vec{R}_f e^{i\left(\vec{k}_i . \vec{R}_i - \vec{k}_f . \vec{R}_f\right)} \frac{e^{-S r_i}}{\sqrt{\text{®}}} e^{-\text{®}\sim/2}\Gamma\left(1 + i\sim\right) {}_1F_1\left(i\sim; -ik_f R_f - i\vec{k}_f . \vec{R}_f\right)$$
$$\times \left(-\frac{1}{r_i}\right)\frac{1}{\sqrt{\text{®}}} e^{- \quad r} \tag{6}$$

Expressions for transition matrix given in equation (6) can be evaluated following Tiwari (2008),

The total cross-sections are obtained by integrating the expressions for differential cross-sections given by equation (2) as

$$\pm = 2® \int_0^{®} \frac{d\pm}{d\Omega} \sin{}^{-} d^{-} \qquad (7)$$

Numerical calculations were performed by using Gauss-Legendre quadrature formula by taking proper care of convergence.

Results and Discussions

Tab. 1. Integral cross-sections (in units of ® a_0^2) the number in the bracket shows that number raised to the power of 10.

Energy (keV)	Cross-sections (® a_0^2)	Energy (keV)	Cross-sections (® a_0^2)
5	2.219(0)	150	5.846(-2)
10	3.666(0)	250	2.130(-2)
15	3.570(0)	300	9.036(-3)
20	3.087(0)	400	4.285(-3)
25	2.571(0)	500	1.222(-3)
30	2.116(0)	600	4.349(-4)
40	1.431(0)	700	1.806(-4)
50	9.826(-1)	800	8.399(-5)
60	6.888(-1)	900	4.264(-5)
70	4.928(-1)	1000	1.336(-5)
80	3.593(-1)	2000	3.050(-7)
90	2.664(-1)	3000	3.067(-8)
100	2.007(-1)	4000	5.860(-9)
120	1.185(-1)	5000	1.604(-9)

The calculated total cross-section from energy range 5keV to 5MeV. It was observed that as the energy increases the cross-sections are decreasing as expected. Higher the energy lowers the integral cross sections. The cross-section are given in the units of ® a_0^2.The differential cross-sections are shown graphically from figures 1-3.In fig.1, the differential cross-sections for energies 50and 100keV are plotted against mrad to $a_0^2 sr^{-1}$. A dip has been observed at about 1.5 mrad for energy 100keV.In fig.2, the differential cross-sections for energies 300 and 400 keV are plotted against mrad and it has been observed the occurrence of two peaks one at 1.6 mrad for 300 keV curve and 1.2 mrad at 400 keV curve. Similarly, two other curves for energies 1000 and 2000 keV are just smooth curves showing slightly a tendency of peaks at 1 mrad for 1000 kEv and at 1.6mrad for 200 keV (fig.3). Unfortunately, we have no other data available either theoretically or experimentally at such high energies. Hence, we have not compared our findings with other data. However, we may hope that in future other results will be available for comparisons.

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