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Probabilistic Analysis of Ultimate Strength of Ferrocement Elements in Axial Tension

K Balaji Rao[a]* and Prakash Desayi[b]

[a]CSIR-Structural Engineering Research Centre, Taramani, Chennai 600 113, India
[b]Department of Civil Engineering, Indian Institute of Science, Bangalore 560 012, India

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This paper reports the results of probabilistic analysis of ultimate load, P_u, of normal and lightweight ferrocement elements under axial tension. The lightweight ferrocement element was realised by replacing the sand in cement mortar by blast furnace slag by 20%, 40%, 60%, 80% and 100%. In probabilistic analysis, the diameter and ultimate tensile strength of mesh wires and the modelling error associated with the prediction equation of ultimate load are treated as random variables. From the analysis of results of probabilistic analyses it has been found that the ultimate load follows a normal distribution at 5% significance level and also the bounds (mean±1.64*standard deviation) enclose the experimental scatter and hence the characteristic strength can be used for the design of ferrocement members against ultimate limit state.

Keywords: Ferrocement, Wire mesh, Mortar, Blast furnace slag, Ultimate strength, Probabilistic analysis, Characteristic strength

1 Introduction

Ferrocement is a composite in which brittle cement mortar matrix is reinforced with aligned, ductile fibres. There are number of practical applications of ferrocement for structural and architectural purposes. Construction of ferrocement water tanks, storage bins, roofing and walling elements are popular[1-3]. Being thin elements, they provide flexibility in fabrication and construction. One of the important design considerations for such elements is first-crack strength to improve their strength and durability. This paper focuses on this design aspect for normal and lightweight ferrocement elements within the probabilistic framework leading to designs based on LRFD format[2].

Number of investigators have made attempts in the past, to understand the strength and behaviour of ferrocement in tension. These include : Naaman and Shah[4], Desayi and Jacob[5], Johnston and Mattar[6], Huq and Pama[7], Somayaji and Naaman[8], Desayi and Reddy[9], Reddy[10], Al-Noury and Huq[11], Chen and Zhao[12]. In 1982 ACI Committee 549[1] presented a state-of-the-art report on ferrocement which provides information on the mechanical properties, performance and application ferrocement. Some of the general conclusions drawn from the experimental and analytical studies carried out by the above investigators are : (i) the first crack strength is a function of specific surface area of mesh wires, (ii) the spacing and width of cracks are affected by the specific surface (volume fraction) of mesh wires in the loading direction, slip modulus, ultimate bond strength, tensile strength of mortar and modulus of elasticity of steel, (iii) presence of mesh imparts ductility and strength to mortar, (iv) the presence of transverse wires affects the strength and stiffness of ferrocement, (v) the ultimate tensile strength of ferrocement is same as the tension that mesh wires in the loading direction can carry at ultimate, (vi) orientation of the reinforcement has marked effect on absolute strength, and on relative efficiency of various reinforcing systems, (vii) using equivalent aligned fibre concept modulus of elasticity of ferrocement can be determined, (viii) the ultimate strength of ferrocement with only small amount of reinforcing mesh is similar to that of the mortar alone.

Desayi and Reddy[9], and Reddy[10] tested normal weight and lightweight, streamlined tension specimens. A total of 216 specimens of which 36 were of normal weight and 180 of lightweight were tested in tension. The principal variables considered in the study were type of mesh (two types mesh wires used were 4/20 and 6/22), number of layers of mesh (viz. two, four, six, eight and ten) and percentage replacement of sand by blast furnace slag (viz. 0%, 20%, 40%, 60%, 80% and 100%). During testing first

*Corresponding author (E-mail: balajiserc@gmail.com)

crack load, ultimate load, crack pattern and crackwidths were recorded. For a given set of variables considered, three nominally similar specimens were tested. They found that first crack and ultimate strengths, of these specimens vary. To explain these variations probabilistic analysis has to be carried out.

An analytical, statistical model to predict the tensile properties of the fibre-reinforced concrete was developed by Naaman et al[13]. The experimental post cracking strengths were compared with those predicted using the statistical model, and it was noted that the trends predicted by the model were also observed in the experimental results. This model applies for prediction of tensile behaviour of concrete members reinforced with randomly oriented fibres.

Li and co-researchers[14], using fracture mechanics approach, have carried out investigations on fracture of brittle cementitious matrix reinforced with either discrete or continuous fibres. They include the fibre-matrix interaction for prediction of stress-strain relationship in tension. The stress-strain model proposed by them take in to account the interfacial elastic bond, and elastic mis-match between fibres and matrix. However, they seem to have not carried out probabilistic analysis of fibre reinforced matrices.

Recent research efforts have been directed towards development of ferrocement using green mortar which makes use of industrial by-products such as blast furnace slag, fly ash and silica fume as replacement of sand and/or cement. Another area of research is application of AI techniques for prediction of strength and behaviour of ferrocement elements using appropriately created database (viz. Gandhomi et al[15], Hanif et al[16], Naderpour et al[17]). In the following, probabilistic studies on tensile strength of composites are presented.

Gucer and Gurland[18] proposed a statistical model, using chain of bundles concept, for fracture of composites. According to them failure occurs when the weakest cross-section cannot sustain an increase in applied load.

A statistical analysis of material strength was proposed by Zweben and Rosen[19,20]. The theory developed attempts to bridge the gap between the microscopic and continuum approaches through fracture mechanics. The occurrence of the first multiple break for continuous fibre composites was suggested as criteria for fracture propagation.

The statistical aspects of the fracture of composite materials were addressed by Argon[21]. He found that the statistical variations in composite strength under static tensile load depend on the variations in strengths of fibres reinforcing the matrix, an observation similar to the one made by Zweben and Rosen[19,20]. The statistical aspects of fracture discussed in this paper were mainly applicable for composites made of ductile matrix reinforced with brittle fibres.

A careful and detailed review of past statistical analysis of the strength of fibrous material was presented by Harlow and Phoenix[22,23]. It was felt that Weibull distribution[24] could be used to describe the strength distribution of fibrous materials.

Harlow[25] presented a statistical model for the failure of composite materials. Using this model, the probability distribution of tensile strength of composites can be studied analytically. The results of the study implied that Weibull distribution can be used confidently to represent the composite strength, although the analytical results was not a Weibull distribution.

Chamis[26] presented a simulation based approach for the design of composite elements made of ductile matrix and brittle fibres. One of the important conclusions drawn from this study is that the variables that exhibit dominant sensitivity at low probabilities of failure need to controlled to assure high reliability while those which exhibit high sensitivities at high probabilities of failure must controlled for effective proof testing.

Probabilistic reliability analysis of aerospace composite components is still an active area of research[27]. Recent research has been directed towards developing stochastic models based on fiber breakage and matrix creep for the stress-failure rupture of unidirectional continuous fiber composites (viz. Englebrecht-Wiggans and Phoenix[28] and references therein).

Ferrocement elements made of the same grade of mortar and reinforced with the same number of layers of mesh, show variations in the first-crack strength and ultimate strength[9]. These variations are due partly to the variations in dimensions of member, diameter of mesh wires, strengths of mortar and mesh wires. Thus, the first-crack and ultimate strengths of ferrocement elements are random quantities. From the review of relevant literature presented above it is noted that the probabilistic models developed for composites assume elastic or elastic-plastic matrix reinforced with brittle continuous or discontinuous fibres. However, ferrocement elements, as already mentioned, is a composite, in which the brittle mortar matrix is reinforced with ductile, aligned, small diameter steel wires (fibres). Scant literature available dealing with statistical aspects of behaviour of ferrocement elements in direct tension[29]. Desayi and Balaji Rao[30] have presented the results of probabilistic analysis of first-crack strength of ferrocement elements subjected to axial tension. In this paper the results of probabilistic analyses of ultimate strength of normal and lightweight ferrocement elements are presented. In order to compare the results of probabilistic analyses with the experimental results, the information about the latter is obtained from the investigations reported by Desayi and Reddy[9]. Hence, the brief details of experimental work of Desayi and Reddy[9] are presented first. Then the details of probabilistic analyses of ultimate strengths of normal and light weight ferrocement elements in direct tension are presented. The results of probabilistic analyses are discussed and finally summary and conclusions are presented.

2 Materials and Methods

2.1 Tension Tests on Normal and Lightweight Ferrocement[9]

Desayi and Reddy[9] in their investigation tested a total of 216 ferrocement specimens in axial tension. These specimens belonged to six groups, A to F, with 36 specimens in each group. While the group A specimens were of normal ferrocement, groups B to F were of lightweight ferrocement. The percentage of sand replaced by blast furnace slag was varied from group to group, namely, 0% for group A, 20% for group B, 40% for group C, 60% for group D, 80% for group E and 100% for group F. The slag passed sieve size 2.36 mm and its fineness modulus was 3.45. In a given group, six specimens were of plain mortar and the remaining thirty specimens were reinforced with two types of square woven galvanised iron wire meshes, designated nominally as 4/20 and 6/22. The number of layers of mesh wires of a given type in each group varied from nil to tens layers at intervals of two layers. The diameter and ultimate strength of wires in the meshes slightly varied from group to group and their actual values are presented in Table 1.

The test specimens used had a unform thickness of 30 mm. Its width was 50 mm constant over its central length of 300 mm and was increased uniformly to 95 mm at each end in a length of 100 mm and the increase was ‘stream lined’. For holding the specimens, friction grips working on lazy tongs principle were used. The shape of the specimens used and the grips adopted ensured the application of axial force and failure of specimens in the central test section of 300 mm long, 50 mm wide x 30 mm thick cross-section.

The ferrocement elements were tested in tension, in a universal testing machine of Indian Institute of Science, Bangalore[10]. Suitable brass frames were attached to the specimens and tensile deformation was measured by three LVDTs and one dial gauge fixed between brass frames around the specimen. The load was applied in steps of 500 N and strain readings were taken. During testing, first-crack load, ultimate load and crack pattern were noted. To determine the density of mortar, 100 mm cubes were weighed before testing for compressive strength. The strength details of mortar are presented in Table 1. The experimental cracking stress and ultimate strength of ferrocement elements of groups A to F are determined. Since in this paper the focus is on ultimate strength, the experimental ultimate loads of the elements are presented in Tables 2 and 3. In these tables, the average ultimate load is obtained by taking average of the respective values of the three nominally similar specimens. Ferrocement specimens reinforced with 2 layers of mesh are not considered in this investigation since they did not behave properly.

From the tests conducted on normal and lightweight ferrocement elements, Desayi and Reddy[9] found that the ultimate load, P_u, of the specimen in

Table 1 — Strength details of mortar and mesh wires (Desayi and Reddy, 1985)

Group	Percentage replacement of sand by slag	Cube strength (N/mm^2)	Density (kN/m^3)	Tensile strength (N/mm^2)	Diameter (mm)		Proof stress (N/mm^2)	
					4/20	6/22	4/20	6/22
A	0	37.64	21.65	3.28	0.881	0.611	463.0	459.0
B	20	33.06	20.47	2.96	0.840	0.680	418.1	335.5
C	40	28.48	19.29	2.62	0.855	0.620	455.8	393.0
D	60	23.90	19.29	2.27	0.845	0.620	419.0	429.1
E	80	19.32	18.35	1.93	0.845	0.645	386.1	389.3
F	100	14.74	17.65	1.51	0.845	0.645	386.1	389.3

Table 2 — Experimental ultimate loads* of ferrocement elements reinforced with 4/20 mesh

No. of mesh layers	Group No. of mesh wires	A			B			C			D			E			F		
		Min.	Ave.	Max.	Min.	Ave.	Max.	Min.	Ave.	Max.	Min.	Ave.	Max.	Min.	Ave.	Max.	Min.	Ave.	Max.
4	28	7.475	7.475	7.475	7.974	8.175	8.472	8.472	8.472	8.472	7.475	7.641	7.974	6.977	7.309	7.475	6.479	7.475	8.970
6	42	10.465	11.791	12.459	11.462	11.628	11.960	11.462	12.625	13.954	10.964	11.462	12.459	9.469	10.465	11.067	9.967	9.967	9.967
8	56	12.957	13.954	14.452	14.452	15.283	15.947	14.950	15.947	16.944	14.452	14.785	14.950	12.957	13.954	14.452	11.960	12.957	13.954
10	70	15.449	17.941	19.436	18.937	19.602	20.931	16.944	17.276	17.941	17.442	18.107	18.937	17.941	18.439	18.937	15.947	16.778	17.942

* - load units kN

Table 3 — Experimental ultimate loads * of ferrocement elements reinforced with 6/22 mesh

No. of mesh layers	Group No. of mesh wires	A			B			C			D			E			F		
		Min.	Ave.	Max.	Min.	Ave.	Max.	Min.	Ave.	Max.	Min.	Ave.	Max.	Min.	Ave.	Max.	Min.	Ave.	Max.
4	44	5.482	6.648	8.173	5.482	5.814	6.479	5.482	5.814	5.980	4.984	5.150	5.482	4.984	5.150	5.482	3.987	4.236	4.485
6	66	9.668	10.595	11.462	7.077	7.907	8.472	7.475	7.808	7.974	6.977	7.974	8.472	6.479	7.143	7.475	6.977	7.143	7.475
8	88	13.256	13.555	13.954	9.967	10.631	10.964	9.967	10.300	10.465	9.469	9.967	10.465	9.469	9.635	9.967	9.469	9.635	9.967
10	110	15.449	16.774	17.941	12.957	13.622	13.954	11.960	12.957	13.954	12.957	12.957	12.957	10.964	11.462	11.960	10.964	11.960	12.957

* - load units kN

Table 4 — Statistical properties of diameter of mesh wires of different groups (Desayi and Balaji Rao, 1988)

Group	Type of mesh	Number of specimens	Mean diameter (mm)	Standard deviation (mm)	Coefficient of variation	Skewness coefficient	Kurtosis
A1	4/20	214	0.816	0.056	0.068	-0.602	4.209
B1	6/22	121	0.654	0.024	0.037	+1.160	5.629
C1	4/20	100	0.584	0.047	0.080	-0.354	2.069

tension is equal to the ultimate load taken by all the mesh wires running in the loading direction. Thus,

$$P_u = n\frac{\pi}{4}\varphi^2\sigma_{su} \qquad \dots (1)$$

where n is the number of mesh wires running in the longitudinal direction, φ is the diameter of the mesh wire and σ_{su} is the ultimate strength of mesh wire. The experimental ultimate loads of the elements belonging to groups A to F are presented in Tables 2 and 3. In order to carry out probabilistic analysis of ultimate strength of ferrocement element, which is an aim of this paper, the statistical variations in diameter and ultimate strength of mesh wires are needed. Towards this mesh wires were tested at the Structural Engineering laboratory of Indian Institute of Science, Bangalore. Brief details of these tests are presented next. Though some of these results have been presented elsewhere, for the sake completeness of the paper they are presented here.

2.2 Tension Tests on Mesh Wires

To arrive at the distribution function of diameter and ultimate strength of mesh wires, required for the probabilistic analysis of ultimate load of ferrocement elements, a total of 435 wires were tested in direct tension. The mesh wires of a given gauge were cut from different rolls (obtained from different sources), and were divided into 3 different groups. Group A1 contained 214 wires of 4/20 mesh, group B1 contained 121 wires of 6/22 mesh, and group C1 contained 100 wires of 4/20 mesh. Though the wires of groups A1 and C1 were of 4/20 type, the wires belonging to group C1, were from different batch appeared stiffer and hence were separated. The mesh wires were tested in direct tension, on an Amsler testing machine of 400 lb capacity. Before testing, the diameter of each wire was measured at three different locations and the average diameter was taken as representative value for that wire.

2.3 Distribution of diameter of mesh wires

From the experimental data, the statistical properties, namely, mean, standard deviation, coefficient of variation and coefficients of skewness and kurtosis of the distribution of diameter are computed and are presented in the Table 4. It can be noted from this table t the mean and coefficient of variation of mesh wire diameter of groups A1, B1 and C1 are (0.816 mm, 0.068), (0.654 mm, 0.037) and (0.584 mm, 0.080), respectively. Histograms of diameters are drawn for wires belonging to the three groups and are shown in Fig. 1. Normal and lognormal distributions failed to satisfy the Chi-square test criterion. However, in the present investigation a normal distribution is assumed to describe the statistical variations in diameter of wires of a given group, since this distribution is commonly used to describe the statistical variations in dimensions[30,31].

2.4 Distribution of ultimate strength of mesh wires

After testing each wire in tension, the stress in wire at failure is calculated based on mean diameter of that wire. From the test data of ultimate strengths of mesh wires, statistical properties, namely, mean, standard deviation, coefficient of variation and coefficients of skewness and kurtosis of the distribution of ultimate strength are computed and are presented in the Table 5. Histograms of ultimate strengths are drawn for wires belonging to the three groups and are shown in Fig. 2. From a detailed investigation on the ultimate strength data, it has been noted that a two parameter Weibull distribution can be used to represent the statistical variations in ultimate strength at 5% significance level. The values of shape and scale parameters of Weibull distribution of strengths of mesh wires of different groups are presented in Table 6.

2.5 Probabilistic analysis of ultimate strength

Due to variations in diameter and tensile strength of mesh wires, as noted from Eq. (1), ultimate strength of ferrocement specimen is a random variable. In order to take in to account the approximations and assumptions made in the development of a model, a modelling error term, M, is included in the prediction equation. Thus, the Eq. (1) would take the form,

$$P_u = M.\left(n\frac{\pi}{4}\varphi^2\sigma_{su} \right) \quad \ldots (2)$$

In general, the function form of P_u can be written as,

$$P_u = f_1(M, \varphi, \sigma_{su}) \quad \ldots (3)$$

In Eq. (3) f_1 is a function. The study reported in this section aims: (i) to determine the statistical properties

Table 6 — Shape and scale parameters of Weibull distribution of ultimate strength of mesh wires

Sl. No	Group	Number of specimens	Shape parameter	Scale parameter (N/mm^2)	Correlation coefficient
1	A1	214	12.100	440.711	0.929
2	B1	121	15.950	452.000	0.990
3	C1	100	15.197	1633.376	0.916

Table 5 — Statistical properties of ultimate tensile strength of mesh wires of different groups (Desayi and Balaji Rao, 1988)

Group	Type of mesh	Number of specimens	Mean ultimate tensile strength (mm)	Standard deviation of ultimate tensile strength (mm)	Coefficient of variation	Skewness coefficient	Kurtosis
A1	4/20	214	422.128	42.213	0.100	0.805	3.279
B1	6/22	121	436.862	33.628	0.077	-0.679	4.027
C1	4/20	100	1578.55	120.46	0.076	1.849	15.226

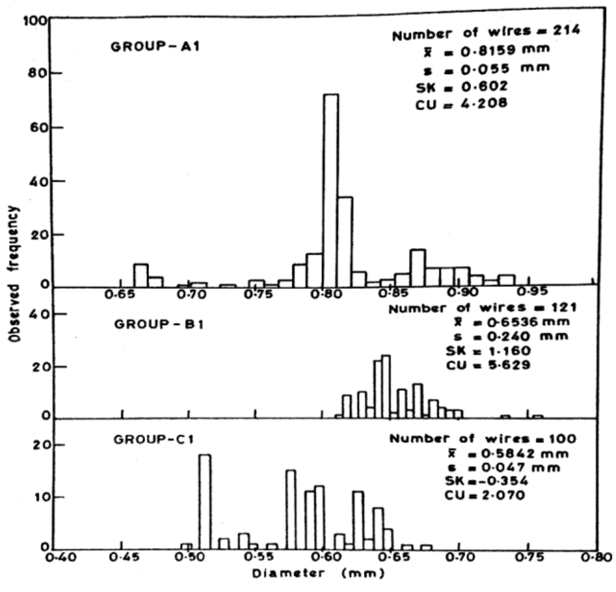

Fig. 1 — Histograms of Diameter of Mesh Wires belonging to Groups A1, B1 and C1.

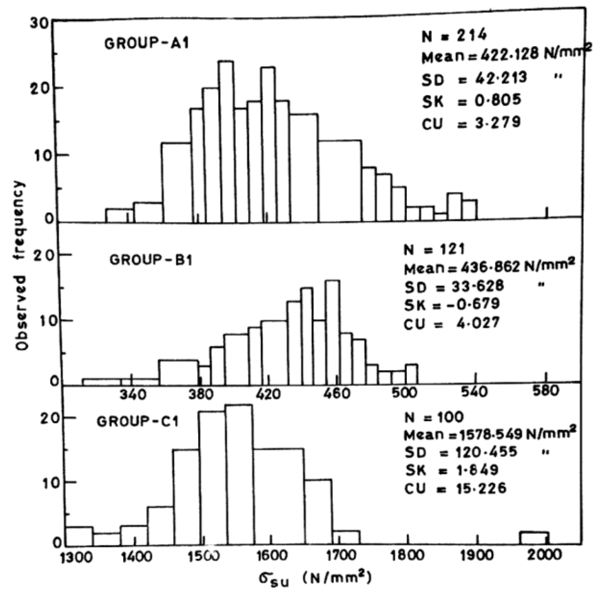

Fig. 2 — Histograms of Ultimate Tensile Strength of Mesh Wires belonging to Groups A1, B1 and C1.

of P_u of normal and lightweight ferrocement elements, and (ii) to propose the characteristic and design ultimate strength equations using the statistical properties obtained (at step (i)).

Determination of characteristic and design ultimate strength requires the probability distribution function of P_u. And, the distribution of P_u depends on distributions of basic variables (Eq. (2)). From the tension tests conducted on mesh wires it is found that the ultimate strength (σ_{su}) of mesh wires follows a two parameter Weibull distribution. Desayi and Reddy[9] used meshes of groups A1 and B1 as reinforcement of ferrocement elements. Hence, in the present investigation, for σ_{su}, two parameter Weibull distribution, with shape and scale parameters presented for groups A1 and B1 in Table 7, are used. The diameter of wires is assumed to follow a normal distribution[32,33].

To determine the probability distribution of P_u Monte Carlo simulation technique, involving 2000 simulation cycles, is used. The steps involved in the simulation are described below. Using two thousand simulation cycles, the two sided 95% confidence levels for mean and variance of P_u are $0.088S$ and $0.1242S^2$, respectively. Here S is the standard deviation of P_u determined from simulation. The specimens considered for probabilistic analysis are those tested by Desayi and Reddy[9].

2.6 Simulation Procedure:

a Two thousand random numbers are generated for each of the basic variables φ *and* σ_{su} of a given group and the modelling error (Table 7).

b Using the above random numbers of basic variables and using Eq. (2) random ultimate strengths of nominally similar specimens are obtained.

c The statistical properties, namely, mean $(\bar{X})$, standard deviation (s), skewness coefficient (SK) and kurtosis (CU) of P_u are computed, Also, the frequency distribution of ultimate strength is obtained.

d Steps 1) - 3) are repeated for different layers of given type of mesh and also for two types of meshes considered for a given group of elements.

e Repeat Step 4) for five different Groups A – F.

3 Results and discussion

Probabilistic analyses of tensile strength of normal and lightweight ferrocement elements have been

Table 7 — Basic variables together with their distributions considered in this study

Sl. No.	Quantity	Mean	Coefficient of variation	Distribution	Reference
1	Diameter of mesh wire				
	4/20 type	As specified in Table 1	0.068	Normal	Assumed
	6/22 type	As specified in Table 1	0.037	Normal	Assumed
2	Ultimate strength of mesh wire				
	4/20 type	Two parameter Weibull distribution; Shape parameter = 12.1; Scale parameter varied from group to group such that COV = 0.100; Mean = Proof stress as presented in Table 1			T
	6/22 type	Two parameter Weibull distribution; Shape parameter = 15.95; Scale parameter varied from group to group such that COV = 0.077; Mean = Proof stress as presented in Table 1			
3	Modelling error associated with prediction of Pu	1.0	0.100	Normal	Assumed based on relevant literature

T – From the tests conducted at IISc laboratory by the authors

Table 8 — Results of K-S test typically for specimens of Groups A and F reinforced with 4/20 and 6/22 meshes

Specimens	Absolute Maximum difference of cumulative distribution functions (D_{max}); hypothetical distribution is Normal distribution	Allowable value of D_{max} at 5% significance level (≃1.36/√n); n is the number of samples
4/20 – Group A	0.018	0.030
4/20 – Group F	0.019	0.030
6/22 – Group A	0.019	0.030
6/22 – Group F	0.021	0.030

carried out within the framework of Monte Carlo simulation. The results of probabilistic analyses for specimens reinforced with 4/20 and 6/22 meshes are presented in Tables 9 and 10, respectively. For each type of mesh reinforcement, the statistical properties are determined for various groups since the diameter and proof stress of mesh wires are slightly different. For a given group, the statistical properties are obtained for different layers of mesh reinforcement. From the results presented in Tables 9 and 10, the following points can be noted:

a For a given group, while the mean and standard deviation of ultimate load increases with increase in number of layers of mesh, the coefficient of variation (*COV*) and the coefficients of skewness (*SK*) and kurtosis (*CU*) of the distribution of ultimate load remains constant. This observation is expected since the variable '*n*' in Eq. (2) is deterministic and the statistical properties of the other random variables remain the same.

b The values of *COV*, *SK* and *CU* vary slightly from group to group due to the use of respective experimental values of diameter and proof stress of mesh wire as mean value of random variable. It may be noted that *COV* values of different groups are close to 0.20 for 4/20 mesh and 0.145 for 6/22 mesh. The skewness coefficients are positive for specimens reinforced with both 4/20 and 6/22 meshes indicating that the falling tails are longer than the raising tails of the distribution of P_U. The kurtosis values of the distributions are around 3.0 for all the specimens. These observations indicates that P_U may follow a normal distribution.

From the equation of the ultimate tensile strength of ferrocement element (Eq. 2), it is noted that the random ultimate strength is a summation of *n* independent and identically distributed random variables (iids); each random variable representing the distribution of strength of each mesh wire running in longitudinal/axial direction. The type of redundancy is active (Ang and Tang, 1984). Since the random variable P_U is summation of iids, the central limit theorem suggests that P_U may follow a normal

Table 9 — Statistical properties of ultimate loads of specimens reinforced with 4/20 mesh

No. of mesh layers	Statistical property	Group					
		A	B	C	D	E	F
4	Mean (N) ($\bar{X}$)	7965.117 (7902.795)	6518.464 (6487.639)	7384.866 (7327.477)	6651.81 (6579.234)	6131.914 (6062.631)	6096.06 (6062.631)
	SD (N) (s)	1593.225	1305.133	1434.244	1330.081	1215.491	1198.478
	COV	0.200 (0.196)	0.200 (0.196)	0.194 (0.196)	0.200 (0.196)	0.198 (0.196)	0.197 (0.196)
	SK	0.270	0.412	0.277	0.369	0.297	0.220
	CU	3.199	3.321	3.036	3.333	2.975	2.910
6	Mean (N) ($\bar{X}$)	11947.68 (11854.19)	9777.696 (9731.458)	11077.3 (10991.22)	9977.715 (9868.851)	9197.871 (9093.946)	9144.089 (9093.946)
	SD (N) (s)	2389.838	1957.699	2151.366	1995.121	1823.237	1797.718
	COV	0.200 (0.196)	0.200 (0.196)	0.194 (0.196)	0.200 (0.196)	0.198 (0.196)	0.197 (0.196)
	SK	0.270	0.412	0.277	0.369	0.297	0.220
	CU	3.199	3.321	3.036	3.333	2.975	2.910
8	Mean (N) ($\bar{X}$)	15930.23 (15805.59)	13036.93 (12975.28)	14769.73 (14654.95)	13303.62 (13158.47)	12263.83 (12125.26)	12192.12 (12125.26)
	SD (N/mm^2) (s)	3186.451	2610.265	2868.487	2660.162	2430.983	2396.957
	COV	0.200 (0.196)	0.200 (0.196)	0.194 (0.196)	0.200 (0.196)	0.198 (0.196)	0.197 (0.196)
	SK	0.270	0.412	0.277	0.369	0.297	0.220
	CU	3.199	3.321	3.036	3.333	2.975	2.910
10	Mean (N) ($\bar{X}$)	19912.79 (19756.99)	16296.16 (16219.10)	18462.16 (18318.69)	16629.52 (16448.09)	15329.79 (15156.58)	15240.15 (15156.58)
	SD (N) (s)	3983.064	3262.832	3585.609	3325.202	3038.729	2996.196
	COV	0.200 (0.196)	0.200 (0.196)	0.194 (0.196)	0.200 (0.196)	0.198 (0.196)	0.197 (0.196)
	SK	0.270	0.412	0.277	0.369	0.297	0.220
	CU	3.199	3.321	3.036	3.333	2.975	2.910

Note: Values within brackets are those estimated using FOA

Table 10 — Statistical properties of ultimate loads of specimens reinforced with 6/22 mesh

No. of mesh layers	Statistical property	Group					
		A	B	C	D	E	F
4	Mean (N) ($\bar{X}$)	5935.718 (5921.581)	5384.612 (5361.088)	5213.536 (5220.577)	5732.864 (5700.126)	5604.271 (5596.885)	5619.696 (5596.885)
	SD (N) (s)	861.7972	783.5782	772.7679	836.3348	814.08	826.0621
	COV	0.145 (0.146)	0.146 (0.146)	0.148 (0.146)	0.146 (0.146)	0.145 (0.146)	0.147 (0.146)
	SK	0.234	0.116	0.113	0.287	0.118	0.182
	CU	3.142	2.971	2.863	3.246	3.025	3.018
6	Mean (N) ($\bar{X}$)	8903.576 (8882.371)	8076.918 (8041.631)	7820.304 (7830.865)	8599.296 (8550.189)	8406.407 (8395.328)	8429.544 (8395.328)
	SD (N) (s)	1292.696	1175.367	1159.152	1254.502	1221.12	1239.093
	COV	0.145 (0.146)	0.146 (0.146)	0.148 (0.146)	0.146 (0.146)	0.145 (0.146)	0.147 (0.146)
	SK	0.234	0.116	0.113	0.287	0.118	0.182
	CU	3.142	2.971	2.863	3.246	3.025	3.018
8	Mean (N) ($\bar{X}$)	11871.44 (11843.16)	10769.22 (10722.18)	10427.07 (10441.15)	11465.73 (11400.25)	11208.54 (11193.77)	11239.39 (11193.77)
	SD (N/mm^2) (s)	1723.594	1567.156	1545.536	1672.67	1628.16	1652.124
	COV	0.145 (0.146)	0.146 (0.146)	0.148 (0.146)	0.146 (0.146)	0.145 (0.146)	0.147 (0.146)
	SK	0.234	0.116	0.113	0.287	0.118	0.182
	CU	3.142	2.971	2.863	3.246	3.025	3.018
10	Mean (N) ($\bar{X}$)	14839.29 (14803.95)	13461.53 (13402.72)	13033.84 (13051.44)	14332.16 (14250.31)	14010.68 (13992.21)	14049.24 (13992.21)
	SD (N) (s)	2154.493	1958.945	1931.92	2090.837	2035.2	2065.155
	COV	0.145 (0.146)	0.146 (0.146)	0.148 (0.146)	0.146 (0.146)	0.145 (0.146)	0.147 (0.146)
	SK	0.234	0.116	0.113	0.287	0.118	0.182
	CU	3.142	2.971	2.863	3.246	3.025	3.018

Note: Values within brackets are those estimated using FOA

distribution. In order to visualise the probability distribution of P_U histograms are plotted for the various groups of specimens reinforced both with 4/20 and 6/22 meshes. Typically, histograms of groups A and F specimens with 4 layers of reinforcement are shown in Figs. 3 – 6. From these figures it is observed that the distribution is approximately symmetrical about the mean value. This observation also suggests that P_U may follow a normal distribution. Both Chi-square and *K-S* tests have been performed to determine the goodness-of-fit of normal distribution for P_U. Only normal distribution is considered since there is a clear indication based on the discussion presented. The results of Chi-square and *K-S* tests indicated that P_U follows normal distribution at 5% significance level for specimens of all groups. The results of *K-S* test are presented in Table 8 typically for specimens of groups A and F reinforced with 4/20 and 6/22 meshes.

Since the experimental minimum, average and maximum ultimate load values of both normal and lightweight ferrocement elements are known (Tables 2 and 3), it is proposed to compare them

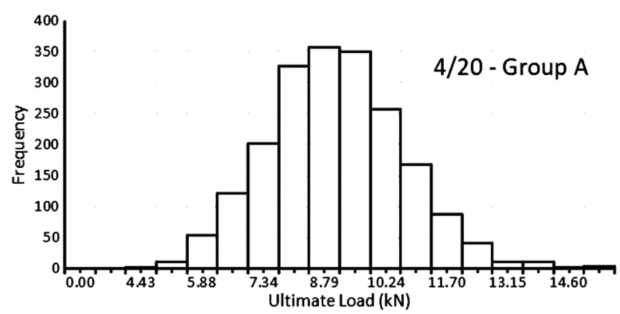

Fig. 3 — Histogram of Ultimate Load of specimen of group A reinforced with 4 layers of 4/20 mesh.

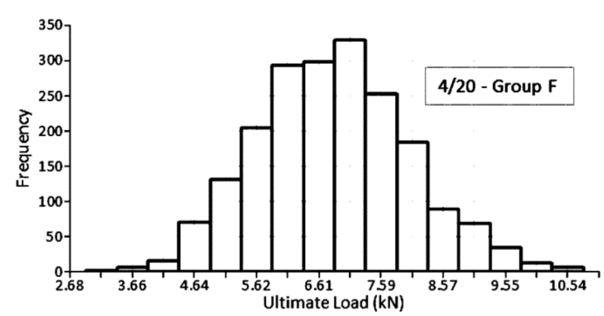

Fig. 4 — Histogram of Ultimate Load of specimen of group F reinforced with 4 layers of 4/20 mesh.

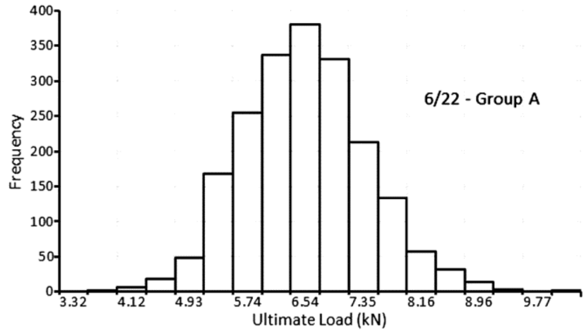

Fig. 5 — Histogram of Ultimate Load of specimen of group A reinforced with 4 layers of 6/22 mesh.

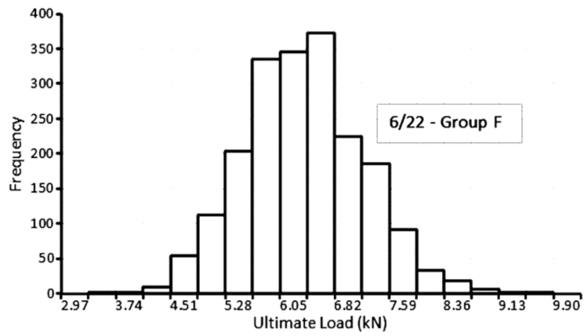

Fig. 6 — Histogram of Ultimate Load of specimen of group F reinforced with 4 layers of 6/22 mesh.

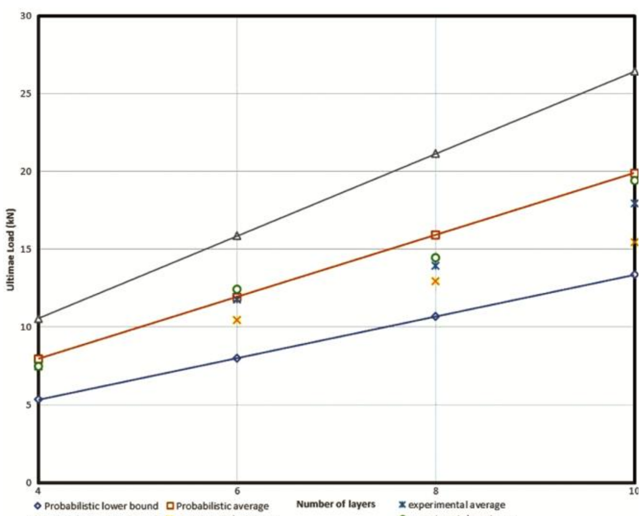

Fig. 7 — Comparison of experimental and probabilistic average and upper- and lower- bounds for Group A specimens reinforced with 4 layers of 4/20 mesh

with the values of $(\bar{X} - 1.64s), \bar{X}$ and $(\bar{X} + 1.64s)$, respectively. The values of $\bar{X}$ and s are the mean and standard deviation of ultimate load of ferrocement specimen, obtained from simulation. The results of probabilistic analyses are presented in Tables 9 and 10. The comparisons are shown, typically for group A and F specimens, in Figs. 7 - 10. Similar plots are

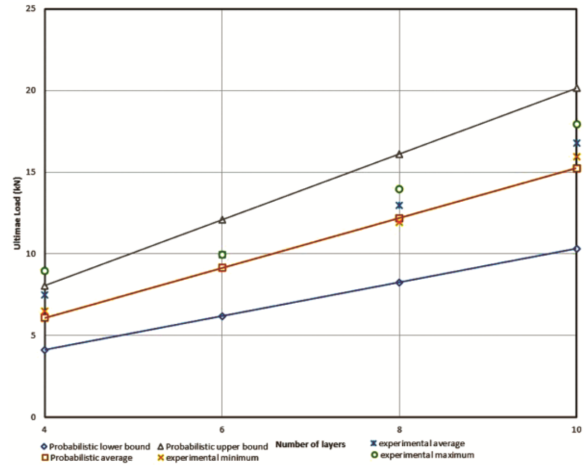

Fig. 8 — Comparison of experimental and probabilistic average and upper- and lower- bounds for Group F specimens reinforced with 4 layers of 4/20 mesh.

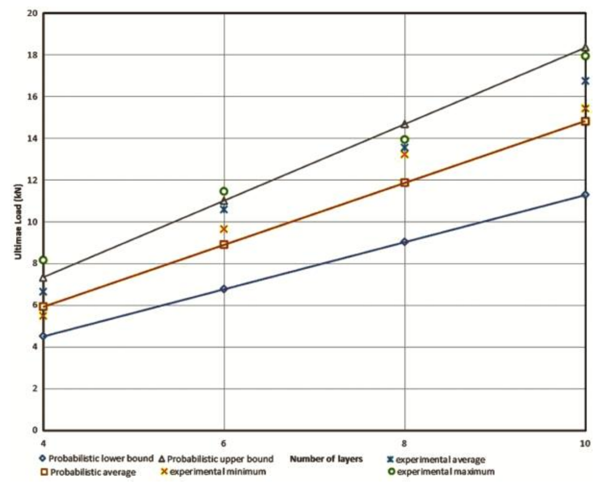

Fig. 9 — Comparison of experimental and probabilistic average and upper- and lower- bounds for Group A specimens reinforced with 4 layers of 6/22 mesh.

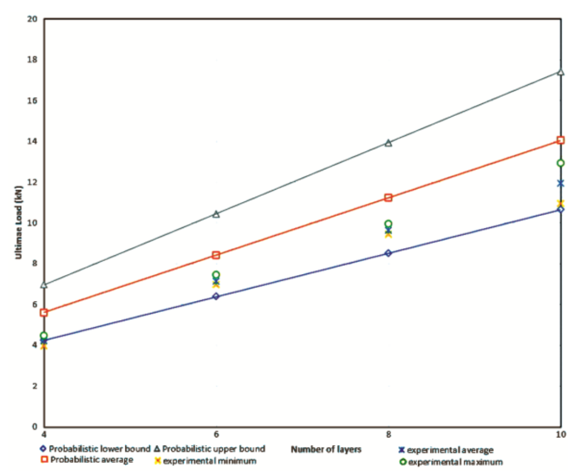

Fig. 10 — Comparison of experimental and probabilistic average and upper- and lower- bounds for Group F specimens reinforced with 4 layers of 6/22 mesh

made for other groups also. The ratios $(\bar{X}+1.64s)/(P_U)_{max}$, $\bar{X}/(P_U)_{ave}$ and $(\bar{X}-1.64s)/(P_U)_{min}$ are computed for specimens reinforced with 4/20 and 6/22 mesh wires of groups A to F and are presented in Tables 11 and 12. In Table 13, the mean and *COV* of the above ratios are presented separately for 24 specimens each reinforced with 4/20 and 6/22 meshes and also for all 48 specimen results put together. If the scatter in experimentally obtained ultimate loads of a given group is enclosed within $(\bar{X}\pm 1.64s)$, the ratio $(\bar{X}+1.64s)/(P_U)_{max}$ has to be greater than 1 and $(\bar{X}-1.64s)/(P_U)_{min}$ has to be less than 1. From the results presented in Tables 11 and 12, it can be noted that these conditions are satisfied in almost all the cases. Generally, it is of engineering interest to know whether the lower bound $(\bar{X}-1.64s)$ encloses the $(P_U)_{min}$ since the lower bound represents the

Table 13 — The mean and coefficient of variation of various ratios

Quantity	Mesh Type	Number of specimens	Mean	Coefficient of variation
$(\bar{X}-1.64s)/(P_U)_{min}$	4/20	24	0.653	0.123
$\bar{X}/(P_U)_{ave}$			0.915	0.104
$(\bar{X}+1.64s)/(P_U)_{max}$			1.156	0.116
$(\bar{X}-1.64s)/(P_U)_{min}$	6/22	24	0.852	0.116
$\bar{X}/(P_U)_{ave}$			1.054	0.121
$(\bar{X}+1.64s)/(P_U)_{max}$			1.239	0.135
$(\bar{X}-1.64s)/(P_U)_{min}$	4/20 and 6/22	24 +24 = 48 (all specimens)	0.753	0.178
$\bar{X}/(P_U)_{ave}$			0.985	0.134
$(\bar{X}+1.64s)/(P_U)_{max}$			1.198	0.130

Table 11 — Comparison of results of probabilistic analysis of ultimate load with experimental values for specimensreinforced with 4/20 mesh

No. of mesh layers	Quantity*	Group A	B	C	D	E	F
4	$(\bar{X}-1.64s)/(P_U)_{min}$	0.716	0.549	0.594	0.598	0.593	0.638
	$\bar{X}/(P_U)_{ave}$	1.066	0.797	0.872	0.871	0.839	0.816
	$(\bar{X}+1.64s)/(P_U)_{max}$	1.415	1.022	1.149	1.108	1.087	0.899
6	$(\bar{X}-1.64s)/(P_U)_{min}$	0.767	0.573	0.659	0.612	**0.656**	0.622
	$\bar{X}/(P_U)_{ave}$	1.013	0.841	0.877	0.871	0.879	0.917
	$(\bar{X}+1.64s)/(P_U)_{max}$	1.274	1.086	1.047	1.063	1.101	1.213
8	$(\bar{X}-1.64s)/(P_U)_{min}$	0.826	**0.606**	0.673	0.619	0.639	**0.691**
	$\bar{X}/(P_U)_{ave}$	1.142	0.853	0.926	0.900	0.879	0.941
	$(\bar{X}+1.64s)/(P_U)_{max}$	1.464	1.086	1.149	1.182	1.124	1.155
10	$(\bar{X}-1.64s)/(P_U)_{min}$	**0.866**	0.578	**0.743**	**0.641**	0.577	0.648
	$\bar{X}/(P_U)_{ave}$	1.110	0.831	1.069	0.918	0.831	0.908
	$(\bar{X}+1.64s)/(P_U)_{max}$	1.361	1.034	1.357	1.166	1.073	1.123

*- Numerator is obtained from simulation and the denominator is from experiment.

Table 12 — Comparison of results of probabilistic analysis of ultimate load with experimental values for specimens reinforced with 6/22 mesh

No. of mesh layers	Quantity*	Group A	B	C	D	E	F
4	$(\bar{X}-1.64s)/(P_U)_{min}$	**0.825**	0.748	0.720	0.875	0.857	**1.070**
	$\bar{X}/(P_U)_{ave}$	0.893	0.926	0.897	1.113	1.088	1.327
	$(\bar{X}+1.64s)/(P_U)_{max}$	0.899	1.029	1.084	1.296	1.266	1.555
6	$(\bar{X}-1.64s)/(P_U)_{min}$	0.702	**0.869**	0.792	**0.938**	**0.988**	0.917
	$\bar{X}/(P_U)_{ave}$	0.840	1.021	1.002	1.078	1.177	1.180
	$(\bar{X}+1.64s)/(P_U)_{max}$	0.962	1.181	1.219	1.258	1.393	1.400
8	$(\bar{X}-1.64s)/(P_U)_{min}$	0.682	0.823	0.792	0.921	0.902	0.901
	$\bar{X}/(P_U)_{ave}$	0.876	1.013	1.012	1.150	1.163	1.167
	$(\bar{X}+1.64s)/(P_U)_{max}$	1.053	1.217	1.239	1.358	1.392	1.400
10	$(\bar{X}-1.64s)/(P_U)_{min}$	0.732	0.791	**0.825**	0.841	0.973	0.972
	$\bar{X}/(P_U)_{ave}$	0.885	0.988	1.006	1.106	1.222	1.175
	$(\bar{X}+1.64s)/(P_U)_{max}$	1.024	1.195	1.161	1.371	1.451	1.346

*- Numerator is obtained from simulation and the denominator is from experiment.

characteristic value that can be used in the design. Keeping this in view, the maximum values of $(\bar{X} - 1.64s)/(P_U)_{min}$ have been underlined in Tables 11 and 12. It is noted from these tables that except in the case of one specimen of group F reinforced with 6/22 mesh, in all cases the ratio is less than 1. When the results of all 48 specimens are considered, as can be observed from Table 13, the mean and *COV* of the ratio $(\bar{X} - 1.64s)/(P_U)_{min}$ are 0.753 and 0.178, respectively. Since the mean value of the ratio is less than 1 the characteristic vale can be used for the design of ferrocement element against limit state of axial tension, an ultimate limit state. Also, it can be noted that the mean and *COV* of ratios $(\bar{X} + 1.64s)/(P_U)_{max}$, $\bar{X}/(P_U)_{ave}$ are (1.198, 0.130) and (0.985, 0.134), respectively. This shows that the results of probabilistic analyses agree satisfactorily the experimental maximum and average ultimate loads.

3.1 Determination of characteristic ultimate load

Knowing the ultimate load distribution as normal, the characteristic ultimate load of normal and lightweight ferrocement elements subjected to tension can be obtained by defining it as 5% fractile of P_U distribution. Thus, the characteristic ultimate load,

$P_U{}^*$, is given by,

$$P_U{}^* = (\bar{X} - 1.64s) \quad \ldots (4)$$

The comparison of $P_U{}^*$ with experimental minimum ultimate load are already discussed and the results are presented in Tables 11 – 13 and Figs. 7 - 10. While the $\bar{X}$ and s are obtained from simulation, In order to recommend Eq. (7) for the design simplified method need to be evolved. In this paper, a FOA method is proposed.

3.1.1 FOA

From Eq. (2), it is noted that the random variables are the modelling error, diameter, and tensile strength of mesh wire. The statistical properties of these random variables are presented in Table 7. Using the First Order Approximation of the Eq. (2) (or Eq. (3))[33,34], the mean and *COV* of the random variable P_U, for a given number of mesh wires, *n*, can be obtained from,

$$\langle P_u \rangle = \langle M \rangle . \left(n \frac{\pi}{4} \langle \varphi \rangle^2 \langle \sigma_{su} \rangle \right) \quad \ldots (5)$$

$$COV(P_u) = \sqrt{COV^2(M) + 4COV^2(\varphi) + COV^2(\sigma_{su})} \quad \ldots (6)$$

Where the quantities within ⟨.⟩ brackets represent the mean value of that particular random variable. In deriving Eqs. (5) and (6) it is assumed that the three random variables are statistically independent; a reasonable assumption. The mean and *COV* values of the random variable, estimated using *FOA* are presented in Tables 9 and 10, within brackets, for the purpose of comparison with the results of probabilistic analyses. It can be noted from the comparisons that the mean and *COV* of ultimate load, required to determine the characteristic ultimate load, can be estimated accurately using *FOA* and there is no need to carry out a detailed probabilistic analysis. This will encourage the designers to use the results of this study in the ferrocement against ultimate limit state of failure in axial tension.

4 Conclusion

Design of ferrocement elements such as water tanks and storage bins requires that it has to satisfy both first-crack and ultimate strength criteria. Due to variations in strength and dimensions of mortar and mesh wires both first-crack and ultimate strengths are random variables. From the review of literature on probabilistic analysis of strength and behaviour of composites in uni-axial tension, it has been found that several investigations have been carried out for aerospace composites. However, similar investigations for ferrocement members are scanty. The present authors have carried out probabilistic analysis of first-crack strength and ultimate strength of normal and lightweight ferrocement elements in axial tension. The results of the investigations related to first-crack strength have been presented in an earlier publication[30]. This paper presents the results of probabilistic analysis of ultimate tensile strength of ferrocement elements.

By treating the dimensions of the diameter of mesh wires, tensile strength mesh wires and the modelling error associated with ultimate strength prediction equation (Eq. (2)) as random variables (Table 7), probabilistic analyses of ultimate tensile strength of normal and lightweight ferrocement elements have been carried out and the results are presented in Tables 9 -13 and Figs. 3 - 10. Probabilistic analysis is carried out within the framework of Monte Carlo simulation involving 2000 simulation cycles. The equation used in simulation is Eq. (2). From the results of the analyses the following conclusions are drawn:

i. The ultimate tensile strength of ferrocement elements considered in this investigation follows a normal distribution at 5% significance level. This result is useful since most of design engineers are familiar with this distribution and requires that only two parameters namely, mean and standard deviation have to be estimated.

ii. The experimentally observed scatter in ultimate strength, for a given number of mesh wires, are enclosed within $(\bar{X}\pm1.64s)$ limits in almost all the cases. This suggests that $(\bar{X}-1.64s)$can be used in the design.

iii. In order to estimate the two parameters requires to compute $(\bar{X}-1.64s)$ easily, without resorting to simulation, a First Order Approximation (*FOA*) method is suggested in this paper (Eqs (5) and (6)). The mean and *COV* estimated using *FOA* method agree satisfactorily with the results of simulation and hence Eqs (5) and (6) can be for the determination of characteristic ultimate strength of ferrocement elements, in tension, for the design of ferrocement elements such as water tanks and storage bins[35,36].

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