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Finite element evaluation of bearing capacity parameters for soils in the University of Agriculture, Makurdi

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Abstract

Samples of soil around the University of Agriculture, Makurdi were obtained and tested in the laboratory to obtain necessary parameters, such as Young's Modulus, Poisson's, ratio, unit weight of soil, angle of internal friction ϕ and cohesion c which would be used in the finite element model. Using the soil parameters obtained and iso-parametric formulation, the element stiffness matrix was derived which was used to model various shallow footing foundation parameters. The results obtained from the finite element analysis were compared with computed values using bearing capacity equations in the literature for the calculation of various parameters, which will be used to calculate safe bearing pressures for circular, square and rectangular foundation footings. The results closely agree with the calculated values. It was concluded that the finite element method is a numerical method that can be used to obtain bearing capacity parameters for use in the calculation of bearing capacities for shallow foundations for preliminary designs of such footings pending experimental verification of soil parameters used.

Key words: Soil; Agriculture; Finite element method; Internal friction.

Introduction

The bearing capacity of soil is an important parameter that facilitates rapid design of both shallow and deep foundations. Unfortunately, there is no easier way of determining it from the soil other than to carry out soil tests to obtain parameters that would be used in existing equations. Uncoordinated number of design equations that exist in the literature (Hansen and Christensen, 1969; Craig, 1978; Smith, 1980; Desai, 1981; Scott, 1974; Bowles, 1988) exacerbates the problem of designing shallow foundation footings. To date geotechnical engineers have not agreed on the use of one single unified equation for the design of shallow foundations. With the advent of computer and the finite element method, geotechnical researchers such as Zienkiewicz (1977), Desai (1981), Smith (1980) and Griffiths (1980a & 1980b) have used these tools in the modelling and design of shallow foundations.

Although attempt has been made to unify the calculation of bearing capacity of shallow foundations by the use of the finite element method, each problem solved by the method is unique in its own right, because each soil (site) has its peculiar foundation parameters. However, the advantage in using the method lies in the fact that it is a unified method for the solution of both shallow and deep foundation problems. Soil properties such as c, Young's modulus E, Poisson's ratio ν and angle of internal friction ϕ for the soil were obtained from (Gaadi and Sesugh, 2011). Thus, the present paper uses the unified method of solving shallow foundation problems for the unique foundation problem of the soils in the University of Agriculture, Makurdi site, which contains expansive clay (Black cotton soil) and renders solution of ordinary strip foundation footing helpless due to the presence of free swell condition of the soil.

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Equilibrium Equations

The usual curved thing shell iso-parametric element was used for the present investigation (Hansen and Christensen, 1969; Craig, 1978; Jiki, 2008). The shape of the element is shown in Fig.1. Using potential energy formulation and Rayleigh – Ritz process, the equilibrium equations were derived as follows:-

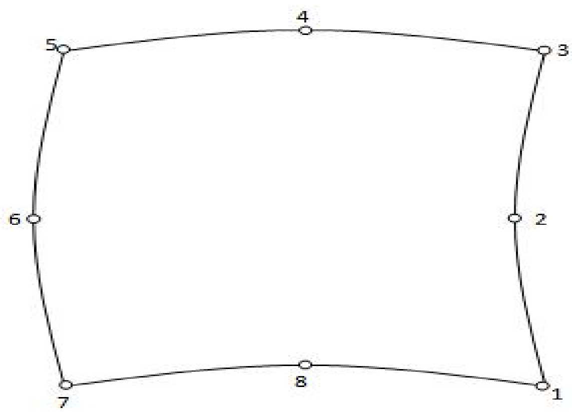

Fig.1. 8-node isoparametric element

The element total potential energy functional is given as:

$$\Pi^e = U^e - W^e \qquad (1)$$

where U^e is the strain energy of the element,

W^e is the external work done by the element during deformation.

The strain energy of the 8 node element is given as:

$$U^e = \frac{1}{2}\iiint_v \sigma_{ij}\varepsilon_{ij}\, dvol \qquad (2)$$

where σ_{ij} is direct stress

ε_{ij} is direct strain

Using iso-parametric formulation, the displacements are interpolated as:

$$u = \sum_{i=1}^{8} \bar{N}_i(\zeta,\eta)u_i \qquad (3)$$

$$v = \sum_{i=1}^{8} \bar{N}_i(\zeta,\eta)v_i \qquad (4)$$

The 8 node (curved) iso-parametric element is mapped into the normalised square space through the following transformations:

$$x = \sum_{i=1}^{8} \bar{N}_i(\zeta,\eta)x_i \qquad (5)$$

$$y = \sum_{i=1}^{8} \bar{N}_i(\zeta,\eta)y_i \qquad (6)$$

Where $\bar{N}_i$are the shape functions corresponding to node I with Cartesian coordinates (x_i, y_i) in *x, y* system of axis and non-dimensional coordinates (ζ_i, η_i)in the ζ, η coordinate system, where $\zeta_i, \eta_i = \pm 1$ for nodes and zero for mid nodes.

Then the shape functions are given as:

$$\bar{N}_i = \Big[(1-\zeta\zeta_i)(1-\eta\eta_i) - (1-\zeta^2)(1+\eta\eta_i) - (1-\eta^2)(1-\zeta\zeta_i)\frac{\zeta_i^2\eta_i^2}{4} + (1-\zeta^2)(1+\eta\eta_i)(1-\zeta_i^2)\eta_{\frac{i}{2}}^2 + (1-\eta^2)(1-\zeta\zeta_i)(1-\eta^2)\zeta_{\frac{i}{2}}^2\Big] \qquad (7)$$

where $i = 1, 2, \ldots, 8$

In terms of the shape function $\bar{N}_i$ the strain matrix $[B]$ is given as:

$$[B] = \begin{bmatrix} \frac{\partial \bar{N}_i}{\partial x} & 0 \\ 0 & \frac{\partial \bar{N}_i}{\partial y} \\ \frac{\partial \bar{N}_i}{\partial x} & \frac{\partial \bar{N}_i}{\partial y} \end{bmatrix} \qquad (8)$$

where

$$\begin{Bmatrix} \frac{\partial \bar{N}_i}{\partial x} \\ \frac{\partial \bar{N}_i}{\partial y} \end{Bmatrix} = [J]^{-1} \begin{Bmatrix} \frac{\partial \bar{N}_i}{\partial \zeta} \\ \frac{\partial \bar{N}_i}{\partial \eta} \end{Bmatrix} \qquad (9)$$

and $[J]$ is the Jacobian matrix given as

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$$[J] = \begin{bmatrix} \frac{\partial x}{\partial \zeta} & \frac{\partial y}{\partial \zeta} \\ \frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta} \end{bmatrix} \quad (10)$$

In terms of the strain matrix of equation (8) the element strain vector is given as:

$$\{\varepsilon\} = B\begin{Bmatrix} u_i \\ v_i \end{Bmatrix} \quad (11)$$

The element strain energy of equation (2) can also be written in terms of material matrix *(D)* as:

$$U^e = \frac{1}{2}\iiint_v \{\varepsilon_{ij}\}^T [D]\{\varepsilon_{ij}\} dvol \quad (12)$$

Substitution of equation (11) into equation (12) and assuming a unit thickness for the element gives:

$$U^e = \frac{1}{2}\iint_A [B]^T \begin{Bmatrix} u_i \\ v_i \end{Bmatrix}^T [D][B]\begin{Bmatrix} u_i \\ v_i \end{Bmatrix} dA \quad (13)$$

$$= \frac{1}{2}\iint_A [B]^T \{\delta\}^T [D][B]\{\delta\} dA \quad (14)$$

For the present investigation the element shown in Fig.2 is loaded with a uniform pressure q-over the element boundary between nodes. The external work done by the loads during displacement of the boundary is given as:

$$W^e = \int q\{\delta\}^T db \quad (15)$$

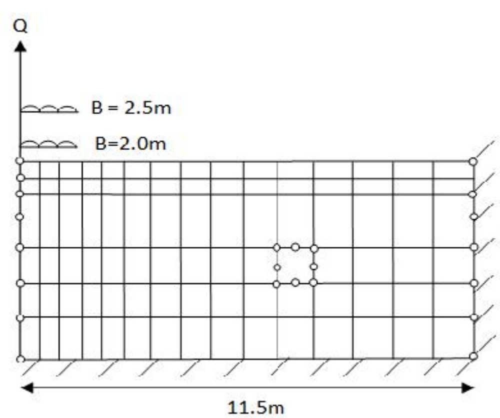

Fig.2. Finite Elements mesh for bearing capacity

Substitution of equations (14) and (15) into equation (1) gives the total potential energy of the element as:

$$\Pi^e = \frac{1}{2}\iint_A [B]^T \{\delta\}^T [D][B]\{\delta\} dA - \int_b q\{\delta\}^T db \quad (16)$$

$$= \frac{1}{2}\{\delta\}^T [K^e]\{\delta\} - Q^e\{\delta\} \quad (17)$$

where $[K^e]$is the element stiffness matrix and $Q^e$is the element boundary loads such that:

$$[K^e] = \int_{-1}^{1}\int_{-1}^{1} [B]^T [D][B] det[J] d\zeta d\eta \quad (18)$$

$$Q^e = 2q \quad (19)$$

In practice, the element stiffness matrix of equation (18) is obtained by performing a numerical integration using Gauss integration rule (Craig, 1978; Jiki, 2008).

The Rayleigh – Ritz process gives equilibrium equation as:

$$\frac{\partial \Pi^e}{\partial\{\delta\}} = [K^e]\{\delta\} - Q^e = 0 \quad (20)$$

After assembly of element stiffness matrices and applied loads into system stiffness matrix and system load vector, the system potential energy function is given as:

$$\Pi^s = \frac{1}{2}\{\Delta\}^T [K^s]\{\Delta\} - \{\Delta\}^T\{Q\} \quad (21)$$

where $\{\Delta\}$ is system node displacement vector

$[K^s]$is assembled system stiffness matrix

$\{Q\}$ is assembled system load vector.

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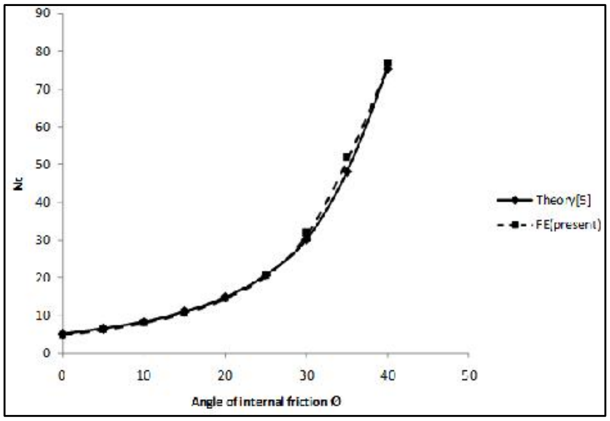

Fig.3. Nc for smooth strip footings

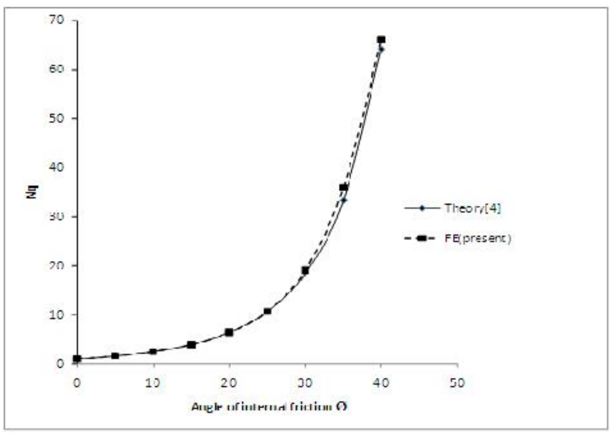

Fig.4. N_q for smooth strip footing

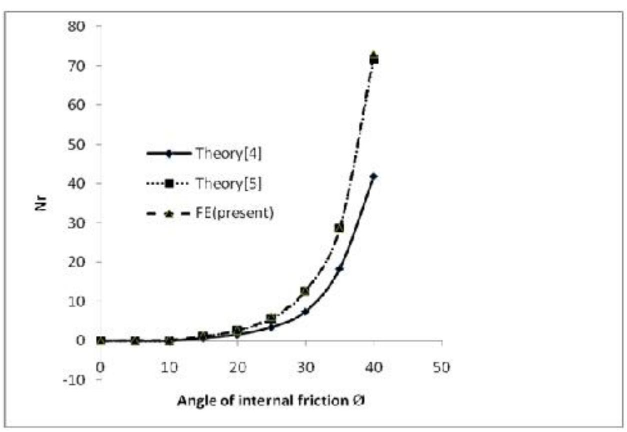

Fig.5. $N\gamma$ for smooth strip footing

Again application of Rayleigh – Ritz process leads to the system equilibrium equations as:

$$\frac{\partial \Pi^s}{\partial\{\Delta\}} = [K^s]\{\Delta\} - \{Q\} \qquad (22)$$

Equation (22) is solved for the nodal displacements, which lead to stresses as:

$$\{\sigma\} = [D]\{\varepsilon\} \qquad (23)$$

in which $\{\sigma\}$ is stress vector, $[D]$ is strain matrix and $\{\varepsilon\}$is strain vector.

Solution of system equilibrium equations

A typical finite element mesh is shown in Fig.2, for various footing breaths, B. The solutions to be captured in the present work are shown Figs.3-5 for smooth footings. The finite element solutions for the relevant parameters (N_C, N_q, N_γ) are compared in Figs.3-5 with relevant parametric equations presented herein considering smooth strip footings.

Solution for Smooth Footings

For a smooth footing on a weightless soil having angle of friction and cohesive strength but without surcharge load (q=0), Prandtl (Scott, 1974) has shown that the failure stress is:

$q_{f=}$ cNc (24)

where

$$N_c = \cot\phi\left[\left(\frac{1+\sin\phi}{1-\sin\phi}\right)\exp(\pi\tan\phi)-1\right] \qquad (25)$$

and for the same soil condition Sokolovsky (Smith, 1980) gives the value of the parameter N_c as:

$$N_c = \cot\phi\left[\tan^2\left(45+\frac{\phi}{2}\right)e^{\pi\tan\phi}-1\right] \qquad (26)$$

However, for a soil with surcharge q_0 but without weight $(\gamma = 0)$ Prandtl has proposed a solution of the form (Scott, 1974)

$$q_f = q_0 N_q \qquad (27)$$

Where

$$N_q = \left(\frac{1+\sin\phi}{1-\sin\phi}\right)\exp(\pi\tan\phi) \qquad (28)$$

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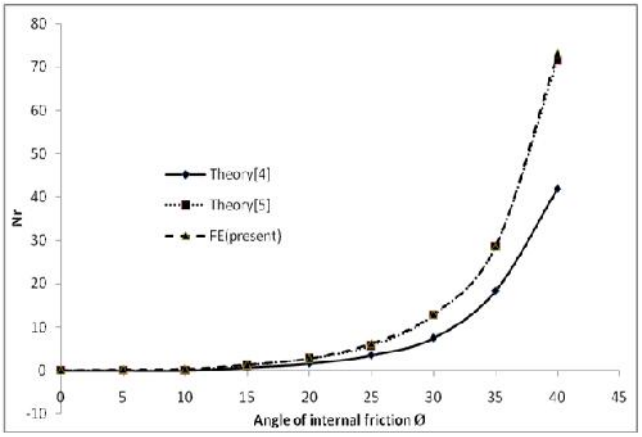

Fig.6. Nγ for Rough footing with C=0

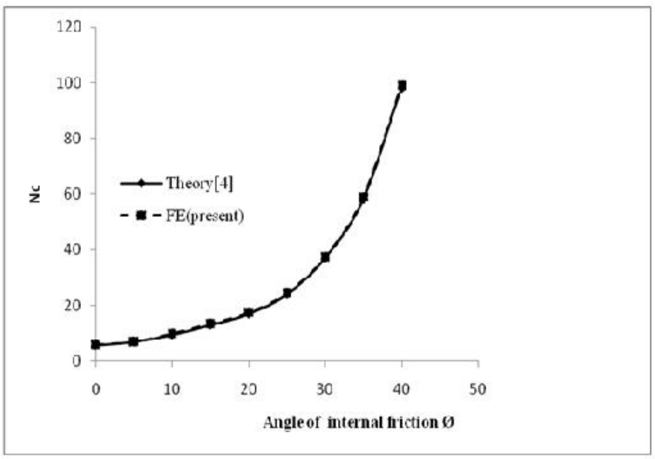

Fig .7. N_c for rough footing

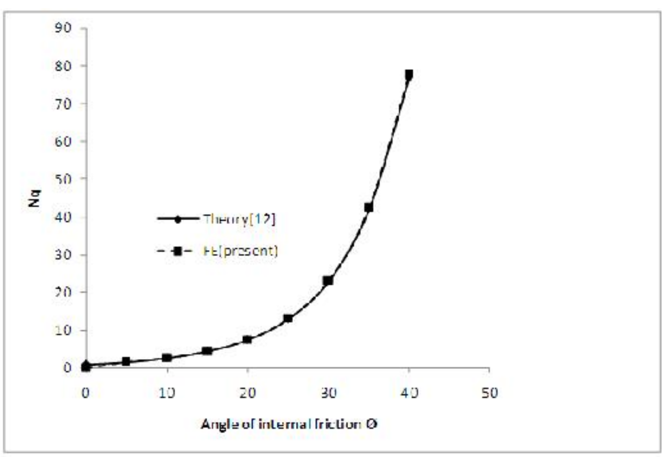

Fig.8. N_q for rough strip footing

For the same soil with surcharge q_0, Sokolovsky (Smith, 1980) gives the value of N_q as:

$$N_q = \tan^2\left(45 + \frac{\phi}{2}\right)e^{\pi \tan \phi} \tag{29}$$

Rough Strip Footings

For rough footings without cohesion and surcharge (c=q=0), Terzaghi (Scott, 1974) has shown that the failure stress q_f is:

$$q_f = \frac{1}{2}B\gamma N_\gamma \tag{30}$$

Where based on the work of Bowles [12] we have:

$$N_\gamma = \tan\frac{\phi}{2}\left(\frac{K_{p\gamma}}{\cos^2\phi} - 1\right) \tag{31}$$

and

$$K_{p\gamma} = \tan^2\left(45 + \frac{\phi}{2}\right) \tag{32}$$

Substitution of equation (32) into equation (31) gives N_γ as:

$$N_\gamma = \tan\frac{\phi}{2}\left(\frac{\tan^2(45 + \frac{\phi}{2})}{\cos^2\phi} - 1\right) \tag{33}$$

For a rough c-ϕ soil with weight γ, we use Terzaghi's equation [12] as:

$$q_f = cN_c s_c + q_0 N_q + \frac{1}{2}\gamma B N_\gamma s_\gamma \tag{34}$$

Where for a strip footing considered in the present work we have $s_c = s_\gamma = 1$.

For the type of soil considered in equation (34) we have N_q by Terzaghi as (Bowles, 1988):

$$N_q = \frac{a^2}{2\cos^2(45 + \frac{\phi}{2})} \tag{35}$$

in which

$$a = \exp\left[\left(0.75\pi - \frac{\phi}{2}\right)\tan\phi\right] \tag{36}$$

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Then the parameter N_c is obtained from equation (35) as:

$$N_c = (N_q - 1)\cot\phi \quad (37)$$

and N_γ is the same expression as in equation (33).

Remark: A look at equation (33) reveals that the parameter N_γ is independent of cohesion c and weight γ. It is however, dependent on the angle of internal friction ϕ. For brevity we recommend that equation (33) be used for the smooth footings from (Jiki, 2008) calculation of N_γ for both smooth and rough strip footings.

Results and Discussion

The soil properties such as angle of internal friction ϕ, cohesion c, Young's modulus E and Poisson's ratio ν were obtained using a tri- axial compression test on thirty soil samples from soils around the university of Agriculture. These results from (Desai, 1981) were used for both analytical and numerical evaluation of the bearing capacity parameters for the soils around the university. The results obtained were presented as Figs.3-5 for smooth footings. Those for rough footings are shown here as Figs.6-8.

Figs.3-5 show soil parameters Nc, Nq and N_γ for smooth strip footing model; while Figs.6-8 show the same parameters for rough strip footings. It can be seen that the theoretical and finite element solutions both compare very well, with a difference of about only 1% for Nc and Nq. Three mesh refinements were made using curved iso-parametric elements as shown in Figs.1&2. However, we have observed from Figs.5&6 that for the parameter N_γ, as the angle of internal friction ϕ increases, the agreement between theoretical and finite element solutions has reduced to about 3%. The reason for this we will find out in another study.

Conclusions

From the above findings on the present study, we conclude as follows:

1. The finite element method can be used as a unifying method to unify the scatter in equations that exist in the literature for bearing capacity parameters Nc,Nq and N_γ by various authors.
2. Charts for these parameters can be prepared rapidly from finite element analyses, which can be used for specific soil properties and conditions.
3. The parameters generated here can serve as a data bank for soil properties within the University for Use when and where the need arises.
4. With this information available, the dependence on laboratory work for preliminary design of shallow strip footings can be reduced.

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