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Magnetohydrodynamic unsteady convective flow past an infinite vertical porous flat surface in presence of time dependent permeability and heat source

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Abstract

This paper analyzes the Magnetohydrodynamic unsteady convective flow of a viscous incompressible electrically conducting fluid past a vertical porous plate through a porous medium in presence of time dependent permeability, oscillatory suction and heat source. Employing perturbation technique, the solutions for velocity and temperature field are obtained. The effects of the pertinent parameters on velocity and temperature distribution of the flow field are studied analytically and discussed with the aid of figures for Grashof number, $G_r > 0$ corresponding to cooling of the plate. It is of interest to note that a growing magnetic parameter decelerates the velocity of the flow field at all points due to the action of Lorentz force on the flow field and an increase in heat source parameter leads to enhance the velocity of the flow field at all points. The effect of growing Grashof number for heat transfer/permeability parameter is to accelerate the velocity of the flow field in presence of heat sink while the effect reverses in presence of heat source, while the effect of increasing Prandtl number is to diminish the temperature of the flow field at all points.

Key words: Magnetohydrodynamic flow; Free convection; Time dependent permeability; Suction; Heat source.

Introduction

In recent years Magnetohydrodynamic flows with heat transfer are of great theoretical and practical interest and much attention has been given to these flows because of their varied applications in natural sciences, engineering sciences, geophysical and astrophysical studies and also in industry. Unsteady oscillatory free convective flows play an important role in chemical engineering, turbo machinery and in aerospace technology. In view of their varied applications, a number of researchers attempted to study these problems in detail. Sparrow and Cess (1961) estimated the effect of magnetic field on free convection heat transfer. Soundalgekar and Haldavnekar (1973) analyzed the MHD free convective flow in a vertical channel. Gersten and Gross (1974) have discussed the flow and heat transfer along a plane wall with periodic suction. Yamamoto and Iwamura (1976) investigated the flow with convective acceleration through a porous medium. Singh (1983) elaborated the effect of mass transfer on unsteady MHD free convective flow past an infinite vertical porous plate with variable suction. Raptis (1986) discussed the flow through a porous medium in the presence of a magnetic field. Jha and Ravindra (1991) analyzed the MHD free convection and mass transfer flow through a porous medium with heat source. Hayat *et al.* (1998) have reported the periodic unsteady flows of a non-Newtonian fluid.

Kim (2000) studied the unsteady MHD convective heat transfer past a semi infinite vertical porous moving plate with variable suction. Singh and his co-workers (2003) analyzed the heat and mass transfer in MHD flow of a viscous fluid past a vertical plate under oscillatory suction velocity. Singh and Gupta (2005) have investigated the MHD free convective flow of a viscous fluid through a porous medium bounded by an oscillatory porous plate in the slip-flow regime with mass transfer. Das *et al.* (2007) analyzed the unsteady mixed convective MHD flow and heat transfer past an accelerated vertical porous flat plate with suction using finite difference scheme. Das and Mitra (2009) estimated the effect of mass transfer on unsteady mixed



convective MHD flow past an accelerated infinite vertical plate with suction. Das and Tripathy (2010) discussed the effect of periodic suction on three dimensional flow and heat transfer past a vertical porous plate embedded in a porous medium. Das and his associates (2011) discussed the simultaneous heat and mass transfer effects on natural convection flow of a viscous incompressible fluid bounded by an oscillating porous plate in the slip flow regime. Recently, Das and his co-workers (2012) investigated the natural convection unsteady Magnetohydrodynamic mass transfer flow past an infinite vertical porous plate in presence of suction and heat sink.

The present study considers the unsteady free convective flow and heat transfer in a viscous incompressible electrically conducting fluid past a vertical porous plate through a porous medium with time dependent permeability and in presence of a transverse magnetic field. After getting the solutions for velocity and temperature of the flow field employing perturbation technique, the results obtained are discussed for Grashof number, $G_r > 0$ corresponding to cooling of the plate. The effects of the pertinent flow parameters such as magnetic parameter M, Grashof number for heat transfer $G_{\rm r}$, permeability parameter $K_{\rm p}$ and heat source parameter S on the velocity field and the effect of Prandtl number $P_{\rm r}$ on the temperature distribution of the flow field have been studied analytically and discussed with the aid of figures.

Mathematical Formulation of the Problem

Consider the unsteady flow of a viscous incompressible fluid past an infinite vertical porous plate in a porous medium of time dependent permeability and in presence of a transverse magnetic field and heat source. Let x'-axis to be along the plate in the direction of flow and y'-axis normal to it. We assume that the magnetic Reynolds number is much less than unity so that the induced magnetic field is neglected in comparison to the applied magnetic field.

Further, all the fluid properties are assumed to be constant except that of the influence of density variation with temperature. The basic flow in the medium is therefore entirely due to the buoyant force caused by temperature difference between the wall and the medium. Initially, at $t' \le 0$, the plate as well as the fluid are assumed to be at the same temperature. When t'>0, the temperature of the plate is instantaneously raised (or 292

lowered) to T'_w . Under the above assumptions and taking the usual Boussinesq's approximation into account, the governing equations for momentum and energy in non-dimensional form are:

$$\frac{1}{4}\frac{\partial u}{\partial t} - \left(1 + \varepsilon e^{i\omega t}\right)\frac{\partial u}{\partial y} = G_r T +$$

$$\frac{\partial^2 u}{\partial y^2} - \frac{u}{K_p \left(1 + \varepsilon e^{i\omega t}\right)} - M^2 u$$

$$\frac{1}{4}\frac{\partial T}{\partial t} - \left(1 + \varepsilon e^{i\omega t}\right)\frac{\partial T}{\partial y} = \frac{1}{P_r}\frac{\partial^2 T}{\partial y^2} + ST . \quad (2)$$

The necessary boundary conditions for this flow are:

$$u = 0, T = 1 + \varepsilon e^{i\omega t}, \quad at \ y = 0,$$

$$u \to 0, T \to 0, \quad as \ y \to \infty.$$
(3)

We introduced the following non-dimensional quantities in the above equation:

$$y = v_0 \frac{y'}{v}, t = \frac{1}{4} \frac{v_0^2}{v}, \omega = 4 \frac{v\omega'}{v_0^2},$$

$$u = \frac{u'}{v_0}, \quad T = \frac{(T' - T_{\infty})}{(T_w - T_{\infty})},$$

$$G_r = vg\beta \frac{(T_w - T_{\infty})}{v_0^3}, P_r = \frac{\mu C_p}{k}, M^2$$

$$= \frac{\sigma B_0^2 v}{\rho v_o^2}, S = \frac{vS'}{v_0^2}, K_p = \frac{v^2 K'}{v_0^2},$$
(4)

where ρ is the density of the fluid, ν is the kinematic coefficient of viscosity, g is the acceleration due to gravity, β is the volumetric coefficient of expansion for heat transfer, k is the thermal conductivity, T is the temperature of the fluid, T_{∞} is the temperature of the fluid at infinity, G_r is the Grashof number for heat transfer, P_r is the Prandtl number, M is the magnetic parameter, B_0 is the uniform transverse magnetic field, K_p is the permeability parameter, μ is the coefficient of viscosity, C_p is the specific heat at constant pressure, ω is the frequency of oscillation, t is the time and S is the heat source parameter.



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Method of Solution

In order to solve the system of equations (1)-(2) under boundary conditions (3), we assume the following for velocity and temperature of the flow field.

$$u(y,t) = u_0(y) + \varepsilon u_1(y)e^{i\omega t}, \quad (5)$$
$$T(y,t) = T_0(y) + \varepsilon T_1(y)e^{i\omega t}, \quad (6)$$

Substituting equations (5) - (6) into equations (1) -(2) and equating the harmonic and non-harmonic terms, we obtain

$$u_{0}'' + u_{0}' - \left(M^{2} + \frac{1}{K_{p}}\right)u_{0} = -G_{r}T_{0}, \quad (7)$$

$$u_{1}'' + u_{1}' - \left(M^{2} + \frac{1}{K_{p}} + \frac{1}{4}\omega\right)u_{1}$$

$$= -G_{r}T_{1} - u_{0}' - \frac{u_{0}}{K_{p}}, \quad (8)$$

$$T_0'' + P_r T_0' - ST_0 = 0, (9)$$

$$T_{I}'' + P_{r}T_{I}' - \frac{1}{4}\frac{i\omega}{P_{r}T_{I}} - ST_{I} = -P_{r}T_{0}', \quad (10)$$

where the primes denote the differentiation with respect to y.

The boundary conditions now reduce to

$$u_0 = u_1 = 0, \quad T_0 = T_1 = 1, \quad \text{at } y = 0,$$

 $u_0 = u_1 \to 0, \quad T_0 = T_1 \to 0, \quad \text{as } y \to \infty.$ (11)

Solving equations (7)-(10) for u and T under boundary conditions (11) and using equations (5)-(6), we get

$$u(y,t) = A_{1} \left(e^{-\lambda_{1}y} - e^{-m_{1}y} \right) +$$

$$\varepsilon \left(A_{2} e^{-\lambda_{3}y} - (A_{3} + A_{4}) e^{-\lambda_{1}y} + \right) e^{i\omega t}, \quad (12)$$

$$T(y,t) = e^{-\lambda_{1}y} +$$

$$\varepsilon \left[e^{-\lambda_{3}y} + A_{7} \left(e^{-\lambda_{3}y} - e^{-\lambda_{1}y} \right) \right] e^{i\omega t}, \quad (13)$$

$$\begin{split} \lambda_{I} &= \frac{1}{2} \left[P_{r} + \left(P_{r}^{2} + 4S \right)^{\frac{1}{2}} \right] ,\\ \lambda_{2} &= \frac{1}{2} \left[-P_{r} + \left(P_{r}^{2} + 4S \right)^{\frac{1}{2}} \right] ,\\ \lambda_{3} &= \frac{1}{2} \left[P_{r} + \left(P_{r}^{2} + 4S + i\omega P_{r} \right)^{\frac{1}{2}} \right] ,\\ \lambda_{4} &= \frac{1}{2} \left[-P_{r} + \left(P_{r}^{2} + 4S + i\omega P_{r} \right)^{\frac{1}{2}} \right] ,\\ m_{1} &= \frac{1}{2} \left[I + \left(I + \frac{4}{K_{p}} \right)^{\frac{1}{2}} \right] ,\\ m_{2} &= \frac{1}{2} \left[-I + \left(I + \frac{4}{K_{p}} \right)^{\frac{1}{2}} \right] ,\\ m_{3} &= \frac{1}{2} \left[I + \left(I + 4 \left(\frac{1}{K_{p}} + \frac{\omega}{4} \right) \right)^{\frac{1}{2}} \right] ,\\ m_{4} &= \frac{1}{2} \left[-I + \left(I + 4 \left(\frac{1}{K_{p}} + \frac{\omega}{4} \right) \right)^{\frac{1}{2}} \right] ,\\ (\lambda_{3} - \lambda_{1})(\lambda_{1} + \lambda_{4}) &= B_{5} + iC_{5} ,\\ (m_{3} - \lambda_{3})(\lambda_{3} + m_{4}) &= B_{6} + iC_{6} ,\\ (m_{3} - \lambda_{1})(\lambda_{1} + m_{4}) &= B_{7} + iC_{7} ,\\ (m_{1} - \lambda_{1})(\lambda_{1} + m_{4}) &= B_{8} + iC_{8} ,\\ (m_{3} - m_{1})(m_{1} + m_{4}) &= B_{9} + iC_{9} ,\\ B_{I} &= \frac{P_{r}}{2} + 2^{\frac{-3}{2}} \left[\left\{ \left(P_{r}^{2} + 4S \right)^{2} + \omega^{2} P_{r}^{2} \right\}^{\frac{1}{2}} + \left(P_{r}^{2} + 4S \right)^{\frac{1}{2}} \right]^{\frac{1}{2}} \end{split}$$

where

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ISSN 2277 - 5390

ISSN 2277 - 5390

Indian J. Innovations Dev., Vol. 1, No. 4 (Apr 2012)

$$\begin{split} B_{3} &= \frac{l}{2} + 2^{\frac{-3}{2}} \Biggl[\Biggl\{ \Biggl(l + \frac{4}{K_{p}} \Biggr)^{2} + \omega^{2} \Biggr\}^{\frac{l}{2}} + \Biggl(l + \frac{4}{K_{p}} \Biggr) \Biggr]^{\frac{l}{2}}, \\ B_{4} &= -\frac{l}{2} + 2^{\frac{-3}{2}} \Biggl[\Biggl\{ \Biggl(l + \frac{4}{K_{p}} \Biggr)^{2} + \omega^{2} \Biggr\}^{\frac{l}{2}} + \Biggl(l + \frac{4}{K_{p}} \Biggr) \Biggr]^{\frac{l}{2}}, \\ B_{5} &= (B_{l} - \lambda_{l})(\lambda_{l} + B_{2}) - C_{l}^{2}, \\ B_{6} &= (B_{l} + B_{4})(B_{3} - B_{l}) + (C_{l} + C_{4})(C_{l} - C_{3}), \\ B_{7} &= (B_{3} - \lambda_{l})(\lambda_{l} + B_{4}) - C_{3}^{2}, \\ B_{8} &= (m_{l} - \lambda_{l})(\lambda_{l} + B_{4}), \\ B_{9} &= (B_{3} - m_{l})(m_{l} + B_{4}) - C_{3}^{2}, \\ C_{1} &= 2^{\frac{-3}{2}} \Biggl[\Biggl\{ \Biggl(P_{r}^{2} + 4S \Biggr)^{2} + \omega^{2} P_{r}^{2} \Biggr\}^{\frac{l}{2}} \Biggr]^{\frac{1}{2}} = C_{2}, \\ C_{1} &= 2^{\frac{-3}{2}} \Biggl[\Biggl\{ \Biggl(1 + \frac{4}{K_{p}} \Biggr)^{2} + \omega^{2} \Biggr\}^{\frac{1}{2}} \Biggr]^{\frac{1}{2}} = C_{4}, \\ - \Biggl(1 + \frac{4}{K_{p}} \Biggr)^{2} + \omega^{2} \Biggr\}^{\frac{1}{2}} \Biggr]^{\frac{1}{2}} = C_{4}, \\ C_{5} &= (B_{l} + B_{2})C_{l}, \\ C_{6} &= (C_{l} + C_{4})(B_{3} - B_{l}) + (B_{l} + B_{4})(C_{3} - C_{l}), \\ C_{7} &= (B_{3} + B_{4})C_{3}, \\ C_{8} &= C_{4}(m_{l} - \lambda_{l}), C_{9} &= (B_{3} + B_{4})C_{3}, \\ A_{1} &= \frac{G_{r}}{(m_{l} - \lambda_{l})(\lambda_{l} + m_{2})}, \\ A_{2} &= \Biggl[\frac{G_{r}}{(m_{3} - \lambda_{l})(\lambda_{l} + m_{2})} \Biggr] \Biggl[1 + \frac{P_{r}\lambda_{l}}{(\lambda_{3} - \lambda_{l})(\lambda_{l} + \lambda_{4})}, \end{aligned}$$

$$A_{4} = \frac{G_{r} \left(\lambda_{1} - \frac{1}{K_{p}}\right)}{(m_{1} - \lambda_{1})(\lambda_{1} + m_{2})(m_{3} - \lambda_{1})(\lambda_{1} + m_{4})},$$

$$A_{5} = \frac{G_{r} \left(m_{1} - \frac{1}{K_{p}}\right)}{(m_{1} - \lambda_{1})(\lambda_{1} + m_{2})(m_{3} - m_{1})(m_{1} + m_{4})},$$

$$A_{6} = A_{2} - A_{3} - A_{4} + A_{5},$$

$$A_{7} = \frac{P_{r}\lambda_{l}}{(\lambda_{3} - \lambda_{1})(\lambda_{l} + \lambda_{4})}.$$

Separating the real and imaginary parts from equations (12) and (13) and taking only the real parts as they have physical significance, the velocity and temperature distribution of the flow field can be expressed in fluctuating parts as given below.

$$u(y,t) = u_0(y) + \varepsilon (N_r \cos \omega t - N_i \sin \omega t), \quad (14)$$
$$T(y,t) = T_0(y) + \varepsilon (M_r \cos \omega t - M_i \sin \omega t), \quad (15)$$

where

$$\begin{split} N_{r} &= \left(L_{1} + L_{3} + L_{5} + L_{7} + L_{9}\right), \\ N_{i} &= -\left(L_{2} + L_{4} + L_{6} + L_{8} + L_{10}\right), \\ M_{r} &= e^{-B_{1}y} \left\{ \begin{pmatrix} 1 + D_{1}B_{5} \end{pmatrix} \cos C_{1}y - \\ D_{1}C_{5} \sin C_{1}y \end{pmatrix} - D_{1}B_{5}e^{-\lambda_{i}y}, \\ M_{i} &= D_{1}C_{5}e^{-\lambda_{i}y} - e^{-B_{1}y} \\ \left\{ \begin{pmatrix} 1 + D_{1}B_{5} \end{pmatrix} \sin C_{1}y + D_{1}C_{5} \cos C_{1}y \right\}, \\ L_{1} &= D_{2}e^{-B_{1}y} \left(B_{6} \cos C_{1}y - C_{6} \sin C_{1}y\right) + \\ D_{2}e^{-B_{3}y} \left(C_{6} \sin C_{3}y - B_{6} \cos C_{3}y\right), \\ L_{2} &= D_{2}e^{-B_{1}y} \left(C_{6} \cos C_{1}y - B_{6} \sin C_{1}y\right) - \\ D_{2}e^{-B_{3}y} \left(B_{6} \sin C_{3}y + C_{6} \cos C_{3}y\right), \\ L_{3} &= D_{3}e^{-B_{1}y} \left\{ \begin{pmatrix} B_{5}B_{6} - C_{5}C_{6} \end{pmatrix} \cos C_{1}y - \\ \begin{pmatrix} B_{5}C_{6} - C_{5}B_{6} \end{pmatrix} \sin C_{3}y - \\ \begin{pmatrix} B_{5}B_{6} - C_{5}C_{6} \end{pmatrix} \cos C_{3}y \right\} \\ &+ D_{3}e^{-B_{3}y} \left\{ \begin{pmatrix} B_{5}C_{6} - C_{5}B_{6} \end{pmatrix} \sin C_{3}y - \\ \begin{pmatrix} B_{5}B_{6} - C_{5}C_{6} \end{pmatrix} \cos C_{3}y \right\} \end{split}$$

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ISSN 2277 - 5390

 $L_{4} = D_{3}e^{-B_{1}y} \begin{cases} (B_{5}B_{6} - C_{5}C_{6})\sin C_{1}y + \\ (B_{5}C_{6} - C_{5}B_{6})\cos C_{1}y \end{cases} \\ - D_{3}e^{-B_{3}y} \begin{cases} (B_{5}B_{6} - C_{5}C_{6})\sin C_{3}y + \\ (B_{5}C_{6} - C_{5}B_{6})\cos C_{3}y \end{cases} \end{cases},$ $L_{5} = D_{4}e^{-B_{3}y} \begin{cases} (B_{7}B_{5} - C_{7}C_{5}) \\ \cos C_{3}y - (B_{7}C_{5} - C_{7}B_{5})\sin C_{3}y \end{cases} \\ - D_{4}(B_{7}B_{5} - C_{7}C_{5})e^{-\lambda_{1}y} \end{cases}$

$$\begin{split} L_{6} &= D_{4}e^{-B_{4}y} \left\{ \begin{pmatrix} B_{8}C_{6} - C_{8}B_{6} \end{pmatrix} \cos C_{4}y + \\ (B_{8}B_{6} - C_{8}C_{6}) \sin C_{4}y \end{pmatrix} \right\}, \\ &- D_{4} \begin{pmatrix} B_{8}C_{6} - C_{8}B_{6} \end{pmatrix} e^{-\lambda_{1}y} \\ L_{7} &= D_{5}e^{-B_{3}y} \begin{pmatrix} B_{7} \cos C_{3}y - C_{7} \sin C_{3}y \end{pmatrix} \\ &- D_{5}B_{7}e^{-\lambda_{1}y} \end{pmatrix}, \\ L_{8} &= D_{5}e^{-B_{3}y} \begin{pmatrix} C_{7} \cos C_{3}y + B_{7} \sin C_{3}y \end{pmatrix} \\ &- D_{5}C_{7}e^{-\lambda_{1}y} \end{pmatrix}, \\ L_{9} &= D_{6}B_{9}e^{-m_{1}y} - \\ D_{6}e^{-B_{3}y} \begin{pmatrix} B_{9} \cos C_{3}y - C_{9} \sin C_{3}y \end{pmatrix} \end{pmatrix}, \\ L_{10} &= D_{6}C_{9}e^{-m_{1}y} - \\ D_{6}e^{-B_{3}y} \begin{pmatrix} C_{9} \cos C_{3}y + B_{9} \sin C_{3}y \end{pmatrix}, \\ D_{1} &= \frac{P_{r}\lambda_{1}}{\left(B_{5}^{2} + C_{5}^{2}\right)}, D_{2} = \frac{G_{r}}{\left(B_{6}^{2} + C_{6}^{2}\right)}, \\ D_{3} &= \frac{G_{r}P_{r}\lambda_{1}}{\left(B_{5}^{2} + C_{5}^{2}\right)\left(B_{6}^{2} + C_{6}^{2}\right)}, \\ D_{4} &= \frac{G_{r}P_{r}\lambda_{1}}{\left(B_{5}^{2} + C_{5}^{2}\right)\left(B_{7}^{2} + C_{7}^{2}\right)}, \\ D_{5} &= \frac{G_{r}\left(\lambda_{1} - \frac{1}{K_{p}}\right)}{\left(m_{1} - \lambda_{1}\right)\left(\lambda_{1} + m_{2}\right)\left(B_{7}^{2} + C_{7}^{2}\right)}, \\ D_{6} &= \frac{G_{r}\left(m_{I} - \frac{1}{K_{p}}\right)}{\left(m_{I} - \lambda_{1}\right)\left(\lambda_{I} + m_{2}\right)\left(B_{9}^{2} + C_{9}^{2}\right)}. \end{split}$$

Now the expressions for transient velocity and temperature distribution of the flow field for $\omega t = \frac{\pi}{2}$ are given by

$$u\left(y,\frac{\pi}{2\omega}\right) = u_0(y) - \varepsilon N_i, \qquad (16)$$
$$T\left(y,\frac{\pi}{2\omega}\right) = T_0(y) - \varepsilon M_i. \qquad (17)$$

Skin Friction τ

The skin friction at the plate (τ) in terms of amplitude and phase is given by

$$\tau = \left(\frac{\partial u}{\partial y}\right)_{y=0} = \tau_0 + \varepsilon |L| \cos(\omega t + \alpha), \quad (18)$$

where $\tau_0 = \frac{G_r}{(\lambda_1 + m_2)},$

$$L = L_r + iL_i = \left(\frac{du_I}{dy}\right)_{y=0},$$

$$\begin{aligned} \tan \alpha &= \frac{L_i}{L_r}, \\ L_r &= D_2 B_6 (B_3 - B_1) + D_2 C_6 (C_3 - C_1) + \\ D_3 (B_5 C_6 - C_5 B_6) (C_3 + C_1) - \\ D_3 (B_5 B_6 - C_5 C_6) (B_3 + B_1) - \\ D_4 C_3 (B_7 C_5 - C_7 B_5) + D_5 C_3 C_7 + D_6 C_3 C_9 + \\ (\lambda_1 - B_3) \{D_4 (B_7 B_5 - C_7 C_5) + b_5 B_7\} + \\ D_6 B_9 (B_3 - m_1), \\ L_i &= D_2 B_6 (C_3 + C_1) + \\ D_3 (B_5 B_6 - C_5 C_6) (C_3 - C_1) - \\ D_4 C_3 (B_7 B_5 - C_7 C_5) - D_5 B_7 C_3 + \\ D_6 C_3 B_9 - (B_3 - B_1) \{D_2 C_6 + \\ D_3 (B_5 C_6 - C_5 B_6)\} - (\lambda_1 - B_3) \\ \{D_5 C_7 + D_4 (B_7 C_5 - C_7 B_5)\} - \\ D_6 C_9 (B_3 - m_1). \end{aligned}$$

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Heat Flux N_u

The heat flux at the plate i.e rate of heat transfer N_u in terms of amplitude and phase is

given by,

$$N_{u} = -\left(\frac{\partial T}{\partial y}\right)_{y=0} = \lambda_{I} + \varepsilon |Q| \cos(\omega t + \delta), \quad (19)$$

where
$$Q = Q_r + iQ_i = -\left(\frac{dT_1}{dy}\right)_{y=0}$$

$$tan \,\delta = \frac{Q_i}{Q_r},$$

$$Q_r = B_1(1 + D_1 B_5) + D_1(C_1 C_5 - \lambda_1 B_5),$$

$$Q_i = C_1(1 + D_1 B_5) - D_1(B_1 C_5 + \lambda_1 C_5).$$

Results and Discussions

The problem of magnetohydrodynamic free convective flow of a viscous incompressible fluid past a vertical porous plate through a porous medium with time dependent permeability in presence of heat source has been considered. Employing perturbation technique the solutions for velocity and temperature of the flow field are obtained and the effects of the pertinent flow parameters such as magnetic parameter M, Grashof number for heat transfer G_r , permeability parameter K_p and heat source parameter S on the velocity field have been discussed and presented with the help of Fig.1-3. Further, the effect of the Prandtl number P_r on the temperature field has been presented in Fig.4.

Velocity field u

The flow parameters play an important role in the variation of the velocity of the flow field. The major flow parameters affecting the velocity of the flow field are magnetic parameter M, Grashof number for heat transfer G_r , permeability parameter K_p and heat source parameter S. The effects of these parameters on the velocity of the flow field have been discussed in Fig.1-3.

Effect of magnetic parameter M

Fig.1 elucidates the effect of magnetic parameter M on the velocity profiles of the flow field keeping other parameters of the flow field constant. The curve with magnetic parameter, M=0 corresponds to non-MHD

flow and in other two curves the magnetic parameter is taken in increasing order. The effect of growing magnetic parameter is to decelerate the velocity of the flow field to an appreciable extent due to the action of the Lorentz force on the flow field.

Effect of heat source parameter S

The heat source parameter plays a dominant role in determining the behaviour of the flow field. Fig.2 shows the effect of heat source parameter S on the velocity profiles of the flow field for S = 0, 1 and -1 keeping other parameters of the flow field constant. The curve with S = 0 corresponds to the no heat source case. This curve is in between curves with S = -1 and S = 1. Comparing the curves of the figure, it is observed that the heat source parameter enhances the velocity at all points of the flow field. The effect of heat source parameter (S > 0) on velocity profiles is very much significant compared to the heat sink parameter (S < 0).

Effect of different parameters G_r and K_p

Fig.3 depicts the effect of Grashof number for heat transfer G_r and permeability parameter K_p on the velocity profiles of the flow field. Curves 3, 4 and 5 of the figure explain the effect of G_r on the velocity of the flow field. The effect of growing G_r is to enhance the velocity of the flow field at all points in absence of heat source (S=0) and also in presence of heat sink (S= -1) but in presence of heat source the effect reverses (curves 1 and 3). Curves 2, 4 and 5 show the effect of permeability parameter $K_{\rm p}$ on the velocity field. The permeability parameter is found to enhance the velocity of the flow field at all points in absence of heat source and also in presence of heat sink (S = -1) while the effect reverses in presence of heat source (curves 1 and 2). The heat source parameter thus plays a dominant role in determining the behaviour of the flow field.

Temperature field T

The temperature of the flow field varies with the variation of the Prandtl number P_r . Fig.4 shows a plot of non-dimensional temperature against distance for three different values of the Prandtl number. Curve with $P_r = 0.025$ corresponds to mercury at 20°C, $P_r = 0.71$ for air and $P_r = 2$ for an arbitrary species at the same temperature. The effect of increasing P_r is to diminish the temperature of the flow field at all points. Higher this parameter, the sharper is the reduction in the temperature of the flow field.



ISSN 2277 - 5390

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Conclusions

The above analysis brings out the following results of physical interest on the velocity and temperature distribution of the flow field.

The effect of growing magnetic parameter is to decelerate the velocity of the flow field to an appreciable extent due to the action of Lorentz force on the flow field.

An increase in heat source parameter S leads to accelerate the velocity of the flow field at all points. The





effect of heat source parameter (S > 0) on velocity profiles is very much significant compared to the heat sink parameter (S < 0).

The effect of growing Grashof number for heat transfer $G_{\rm r}$ is to enhance the velocity of the flow field at all points in absence of heat source and in presence of heat sink while the effect reverses in presence of heat source.

The permeability parameter K_p increases the velocity of the flow field at all points in absence of heat source and in presence of heat sink while the effect reverses in presence of heat source.



The effect of increasing Prandtl number P_r leads to diminish the temperature of the flow field at all points of the flow field.

Acknowledgement

One of the authors is very much thankful to the authorities of North Orissa University for providing facilities and to pursue research work under this University.

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ISSN 2277 - 5390