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Magneto hydrodynamic convective flow and heat transfer past a vertical porous flat plate in a porous medium with constant suction and periodic variation of plate temperature

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Abstract

The objective of this paper is to analyse the unsteady magneto hydrodynamic flow of a viscous incompressible fluid past an infinite hot vertical porous plate in presence of constant suction and periodic variation of plate temperature. The governing equations of the flow field are solved employing multi-parameter perturbation technique assuming Eckert number as perturbation parameter. The effects of magnetic parameter M, Grashof number for heat transfer Gr, Eckert number Ec, Prandtl number Pr, frequency parameter ω, etc. on velocity, temperature, skin friction and heat flux are discussed with the help of figures and tables. It is observed that a growing magnetic parameter enhances the velocity of the flow field near the plate and there after the effect reverses due to the magnetic pull of the Lorentz force on the flow field. The Grashof number/permeability parameter / Eckert number has an accelerating effect on the velocity of the flow field. A growing Prandtl number / Reynolds number has a retarding effect on the temperature of the flow field at all points. The permeability parameter enhances the skin friction and decreases the magnitude of heat flux at the wall. On the other hand, the magnetic parameter enhances the skin friction as well as the magnitude of heat flux at the wall.

***Keywords**: Convective flow; MHD; Porous medium; Unsteady; Variable temperature; Vertical plate.*

Introduction

The phenomenon of magneto hydrodynamic free convection flow with heat transfer from different geometries bounded by a porous medium has several engineering and geophysical applications such as in geothermal reservoirs, underground energy transport, enhanced oil recovery, drying of porous solids, thermal insulation, packed-bed catalytic reactors, cooling of nuclear reactors, etc. Convective flows also play an important role in chemical engineering; turbo machinery and aerospace technology which may arise either due to unsteady motion of the boundary or boundary temperature. In addition to the above studies, magneto hydrodynamics has attracted the attention of a good number of researchers in view of its diverse applications in astrophysics and geophysics, such as in the study of the stellar and solar structures, interstellar matter, radio propagation through the ionosphere, etc. In engineering it finds application in MHD pumps, MHD bearings, plasma studies, nuclear reactors, etc.

In view of its wide range of applications, Soundalgekar (1974) showed the effect of free convection on steady MHD flow of an electrically conducting fluid past a vertical plate. Mansutti *et al* (1993) have discussed the steady flow of a non-Newtonian fluid past a porous plate with suction and injection. Soundalgekar (1981) analysed the transient free convection flow with mass transfer on an isothermal vertical flat plate employing finite difference scheme. Raptis and Kafousias (1982)

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investigated the flow and heat transfer in an electrically conducting fluid through a porous medium past an infinite vertical plate under the action of a transverse magnetic field. Raptis and Singh (1983) studied the MHD free convection flow past an accelerated vertical plate. Grubka and Bobba (1985) discussed the heat transfer characteristics of a continuously stretching surface with variable temperature. Elghabaty (1988) analyzed the unsteady MHD flow of a visco-elastic fluid past an infinite porous plate with oscillating temperature.

The magneto hydrodynamic transient-free convection flow past a semi-infinite vertical plate with constant heat flux was studied by Gokhale (1991). Takhar and his associates (1997) discussed the transient free convection flow past a semi-infinite vertical plate with variable surface temperature. Muthukumaraswamy and Ganesan (1998) described the unsteady flow past an impulsively started vertical plate with heat and mass transfer. Das and his co-workers (1999) analyzed the transient free convection flow past an infinite vertical plate under periodic variation of temperature. Singh *et al*. (2003) investigated the heat and mass transfer in a viscous incompressible electrically conducting fluid past a vertical plate under oscillatory suction velocity and in presence of transverse magnetic field. Pathal *et al*. (2005) estimated the unsteady mass, momentum and heat transfer in MHD free convection flow along a vertical plate suddenly set in motion. Ogulu and Prakash (2006) discussed the transfer of heat to unsteady magneto-hydrodynamic flow past an infinite vertical moving plate with variable suction. Das *et al*. (2007a) investigated the unsteady mixed convective MHD flow and heat transfer past an accelerated vertical porous flat plate with suction employing finite difference scheme. The unsteady free convective MHD flow and heat transfer of a second order fluid between two heated vertical plates through a porous medium was studied by Das and his co-workers (2007b). Das and his associates (2009) estimated the effect of mass transfer on MHD flow and heat transfer past a vertical porous plate through a porous medium under oscillatory suction and heat source. Recently, Das *et al*. (2010) analyzed the hydromagnetic free convective mass transfer flow along a vertical porous plate embedded in a porous medium with suction and periodic variation of plate temperature. More recently, Das and his team (2012) investigated the magneto hydrodynamic unsteady convective flow past an infinite vertical porous flat surface in presence of time dependent permeability and heat source.

The study reported herein analyses the unsteady flow of a viscous incompressible electrically conducting fluid past an infinite hot vertical porous plate in presence of constant suction, periodic variation of plate temperature and a transverse magnetic field. The governing equations of the flow field are solved employing multi-parameter perturbation technique assuming Eckert number as perturbation parameter. The expressions for velocity, temperature, and skin friction and heat flux at the plate in terms of Nusselt number are obtained and the effects of the pertinent parameters on the flow field are discussed quantitatively and analyzed with the aid of figures and tables.

Mathematical Formulation

Consider an unsteady free convective flow of a viscous incompressible electrically conducting fluid past a vertical porous flat plate through a porous medium in presence of a transverse magnetic field B0. The infinite hot porous plate is lying vertically on the x^*-z^* plane such that x^*-axis is oriented in the direction of buoyancy force and y^*-axis is normal to the plane of the plate. The physical sketch and geometry of the problem is shown in Fig.1. All the physical quantities are assumed to be independent of x^* as the plate is taken to be of infinite extent in the x^*-direction. The temperature of the plate is considered to vary span wise and fluctuating cosinusoidally with time and assumed to be of the form:

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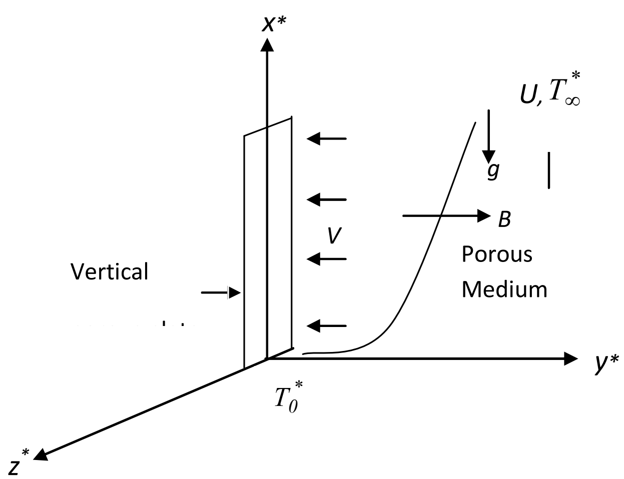

Fig.1. Physical sketch and geometry of the problem

$$T_w^*(z^*, t^*) = T_0^* + \varepsilon(T_0^* - T_\infty^*)\cos\left(\frac{\pi z^*}{l} - \omega^* t^*\right) \quad (1)$$

where T_0^*, T_∞^* and T_w^* are the mean, ambient temperature and wall temperature of the plate respectively, ω* is the frequency, t* is the time, l is the wavelength and ε is a small parameter (i.e. ε<< 1).

Let u*, v*, w* be the components of the velocity of the fluid at any point in x*, y*, z*-direction respectively. Since the wall is uniformly porous and a constant suction velocity V is applied along y*- axis and w* is independent of z*, we assume w*=0 throughout. The Joulean effect of heating is negligible as the velocity encountered in the free convection flow is small. The induced magnetic field is also negligible as the magnetic Reynolds number is assumed to be small. Further, the effect of polarization of ionized fluid is negligible as no external electric field is applied to the flow field. Under the above assumptions and using Boussinesq approximation, the governing equations of mass, momentum and energy including the dissipation function term are given by

$$\frac{\partial v^*}{\partial y^*} = 0 \qquad \Rightarrow v^* = V, V > 0 \qquad (2)$$

$$\frac{\partial u^*}{\partial t^*} + v^*\frac{\partial u^*}{\partial y^*} = \nu\left(\frac{\partial^2 u^*}{\partial y^{*2}} + \frac{\partial^2 u^*}{\partial z^{*2}}\right) + g\beta(T^* - T_\infty^*) - \frac{\nu}{K_0}u^* - \frac{\sigma B_0^2}{\rho}u^* \qquad (3)$$

$$\rho C_p\left(\frac{\partial T^*}{\partial t^*} + v^*\frac{\partial T^*}{\partial y^*}\right) = k\left(\frac{\partial^2 T^*}{\partial y^{*2}} + \frac{\partial^2 T^*}{\partial z^{*2}}\right) + \mu\left[\left(\frac{\partial u^*}{\partial y^*}\right)^2 + \left(\frac{\partial u^*}{\partial z^*}\right)^2\right] \qquad (4)$$

where g is acceleration due to gravity, T* is the temperature of the fluid, B0 is the magnetic field, σ is the electrical conductivity, Cp is the specific heat at constant pressure, k is the thermal conductivity, μ is the coefficient of viscosity, ν is the kinematic viscosity, ρ is the density, t* is the time and β is the volumetric coefficient of expansion for heat transfer.

The corresponding boundary conditions are

$$y^* = 0 \,:\, u^* = 0,\, T^* = T_0^* + \varepsilon(T_0^* - T_\infty^*)\cos\left(\frac{\pi z^*}{l} - \omega^* t^*\right)$$

$$y^* \to \infty : u^* = 0,\, T^* = T_\infty^*. \qquad (5)$$

Introducing the following non-dimensional parameters,

$$y = \frac{y^*}{l},\ z = \frac{z^*}{l},\ u = \frac{u^*}{V},\ T = \frac{T^* - T_\infty^*}{T_0^* - T_\infty^*},\quad t = \omega^* t^*,\quad E_c = \frac{C_p(T_0^* - T_\infty^*)}{V^2},$$

$$G_r = \frac{g\beta\nu(T_0^* - T_\infty^*)}{V^3},\ P_r = \frac{\mu C_p}{k},\ R_e = \frac{Vl}{\nu},\ \omega = \frac{\omega^* l^2}{\nu},\ M^2 = \frac{\sigma B_0^2 l^2}{\mu}$$

into Eqs. (2.2) - (2.4), we get

$$\frac{\omega}{R_e}\frac{\partial u}{\partial t} - \frac{\partial u}{\partial y} = \frac{1}{R_e}\left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) + G_r R_e T - \frac{1}{R_e}\left(M^2 - \frac{1}{K_p}\right)u \,, \qquad (6)$$

$$\omega\frac{\partial T}{\partial t} + R_e\frac{\partial T}{\partial y} = \frac{1}{P_r}\left(\frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right) + E_c\left[\left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial z}\right)^2\right] \qquad (7)$$

Where Pr is the Prandtl number, Gr is the Grashof number for heat transfer, Re is the Reynolds number, Ec is the Eckert number, T is the temperature and M is the magnetic parameter.

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The boundary conditions (5) reduce to

$$y=0 \quad : u=0,\ T=1+\varepsilon\cos(\pi z-t),$$

$$y\to\infty : u=0,\ T=0 \qquad (8)$$

Method of Solution

In order to reduce the system of partial differential Eqs. (6) and (7) to a system of ordinary differential equations in the non-dimensional form, we assume the following for velocity and temperature distribution of the flow field as the amplitude ε (<<1) of the temperature variation is very small.

$$f(y,z,t)=f_0(y)+\varepsilon f_1(y)e^{i(\pi z-t)} \qquad (9)$$

where f stands for u and T. Then substituting eq. (9) in eqs. (6) and (7) and equating the like powers of ε, we get

Zeroth order:

$$\frac{d^2u_0}{dy^2}+R_e\frac{du_0}{dy}-\left(M^2+\frac{1}{K_p}\right)u_0=-R_e^2G_rT_0 \qquad (10)$$

$$\frac{d^2T_0}{dy^2}+R_eP_r\frac{dT_0}{dy}+E_cP_r\left(\frac{du_0}{dy}\right)^2=0 \qquad (11)$$

The corresponding boundary conditions become

$$y=0 \quad : u_0=0,\ T_0=1,$$

$$y\to\infty : u_0=0,\ T_0=0 \qquad (12)$$

First order:

$$\frac{d^2u_1}{dy^2}+R_e\frac{du_1}{dy}-\left(M^2+\frac{1}{K_p}+\pi^2-i\omega\right)u_1=-R_e^2G_rT_1, \qquad (13)$$

$$\frac{d^2T_1}{dy^2}+R_eP_r\frac{dT_1}{dy}-\left(\pi^2-i\omega P_r\right)T_1=2E_c\frac{du_0}{dy}\frac{du_1}{dy}. \qquad (14)$$

The corresponding boundary conditions reduce to

$$y=0 \quad : u_1=0,\ T_1=1,$$

$$y\to\infty : u_1=0,\ T_1=0. \qquad (15)$$

Since eqs. (10), (11), (13) and (14) are coupled differential equations; therefore approximate solution is obtained by perturbation technique for small values of Ec as the Eckert number is small for incompressible fluid flows. Hence, we assume

$$F_0=F_{00}+E_cF_{01}+0(E_c^2),\ F_1=F_{10}+E_cF_{11}+0(E_c^2) \qquad (16)$$

Where F stands for u and T.

Using eq. (16) in eqs. (10), (11), (13) and (14) along with boundary conditions (12) and (15) and equating like powers of Ec, we obtain:

Zeroth order:

$$u''_{00}+R_eu'_{00}-\left(M^2+\frac{1}{K_p}\right)u_{00}=-R_e^2G_rT_{00} \qquad (17)$$

$$u''_{10}+R_eu'_{10}-\left(M^2+\frac{1}{K_p}-i\omega\right)u_{10}=-R_e^2G_rT_{10} \qquad (18)$$

$$T''_{00}+R_eP_rT'_{00}=0, \qquad (19)$$

$$T''_{10}+R_eP_rT'_{10}-\left(\pi^2-i\omega P_r\right)T_{10}=0. \qquad (20)$$

The corresponding boundary conditions are

$$y=0 \quad : u_{00}=0,\ T_{00}=1,\ u_{10}=0,\ T_{10}=1,$$

$$y\to\infty : u_{00}=0,\ T_{00}=0,\ u_{10}=0,\ T_{10}=0 \qquad (21)$$

First order:

$$u''_{10}+R_eu'_{01}-\left(M^2+\frac{1}{K_p}+\pi^2\right)u_{01}=-R_e^2G_rT_{01} \qquad (22)$$

$$u''_{11}+R_eu'_{11}-\left(M^2+\frac{1}{K_p}+\pi^2-i\omega\right)u_{11}=-R_e^2G_rT_{11} \qquad (23)$$

$$T''_{01}+R_eP_rT'_{01}=-P_ru'^{2}_{00}, \qquad (24)$$

$$T''_{11}+R_eP_rT'_{11}-\left(\pi^2-i\omega P_r\right)T_{11}=2u'_{00}u'_{01} \qquad (25)$$

The corresponding boundary conditions are

$$y = 0 \quad : u_{01} = 0,\ T_{01} = 0,\ u_{11} = 0,\ T_{11} = 0,$$
$$y \to \infty : u_{01} = 0,\ T_{01} = 0,\ u_{11} = 0,\ T_{11} = 0. \quad (26)$$

Solving the ordinary second order differential eqs. (17) - (20) and (22) - (25) under the boundary conditions (21) and (26) respectively, we obtain

$$T_{00} = e^{-R_e P_r y}, \quad (27)$$

$$u_{00} = A_1\left(e^{-R_e P_r y} - e^{-\lambda_1 y}\right) \quad (28)$$

$$T_{10} = e^{-\lambda_3 y}, \quad (29)$$

$$u_{10} = A_2\left(e^{-\lambda_3 y} - e^{-\lambda_5 y}\right), \quad (30)$$

$$T_{01} = A_3 e^{-2\lambda_1 y} - A_4 e^{-2R_e P_r y} + A_5 e^{-(\lambda_1 + R_e P_r)y} - A_6 e^{-R_e P_r y}, \quad (31)$$

$$u_{01} = A_7 e^{-2\lambda_1 y} + A_8 e^{-R_e P_r y} + A_9 e^{-2R_e P_r y} + A_{10} e^{-(\lambda_1 + R_e P_r)y} - A_{11} e^{-\lambda_1 y}, \quad (32)$$

$$T_{11} = A_{12} e^{-(\lambda_3 + R_e P_r)y} + A_{13} e^{-(\lambda_5 + R_e P_r)y} + A_{14} e^{-(\lambda_1 + \lambda_3)y} + A_{15} e^{-(\lambda_1 + \lambda_5)y} - A_{16} e^{-\lambda_3 y}, \quad (33)$$

$$u_{11} = A_{17} e^{-(\lambda_3 + R_e P_r)y} - A_{18} e^{-(\lambda_5 + R_e P_r)y} - A_{19} e^{-(\lambda_1 + \lambda_3)y} - A_{20} e^{-(\lambda_1 + \lambda_5)y} - A_{21} e^{-\lambda_3 y} - A_{22} e^{-\lambda_5 y} \quad (34)$$

Substituting eqs. (28), (30), (32), (34) and (16) in eq. (9) the solution for the velocity of the flow field is given by

$$u(y,z,t) = u_{00} + E_c u_{01} + \varepsilon\left(u_{10} + E_c u_{11}\right)e^{i(\pi z - t)}$$

$$= A_1\left(e^{-R_e P_r y} - e^{-\lambda_1 y}\right) + E_c\left(A_7 e^{-2\lambda_1 y} + A_8 e^{-R_e P_r y} + A_9 e^{-2R_e P_r y} + A_{10} e^{-(\lambda_1 + R_e P_r)y} - A_{11} e^{-\lambda_1 y}\right)$$

$$+ \varepsilon A_2\left(e^{-\lambda_3 y} - e^{-\lambda_5 y}\right)e^{i(\pi z - t)} + \varepsilon\Big[E_c\Big(A_{17} e^{-(\lambda_3 + R_e P_r)y} - A_{18} e^{-(\lambda_5 + R_e P_r)y} - A_{19} e^{-(\lambda_1 + \lambda_3)y} - A_{20} e^{-(\lambda_1 + \lambda_5)y} - A_{21} e^{-\lambda_3 y} - A_{22} e^{-\lambda_5 y}\Big)\Big]e^{i(\pi z - t)} \quad (35)$$

Substituting eqs. (27), (29), (31), (33) and (16) in eq. (9) the solution for the temperature of the flow field is given by

$$T(y,z,t) = T_{00} + E_c T_{01} + \varepsilon\left(T_{10} + E_c T_{11}\right)e^{i(\pi z - t)}$$
$$= e^{-R_e P_r y} + E_c\left(A_3 e^{-2\lambda_1 y} - A_4 e^{-2R_e P_r y} + A_5 e^{-(\lambda_1 + R_e P_r)y} - A_6 e^{-R_e P_r y}\right) + \varepsilon e^{-\lambda_3 y} e^{i(\pi z - t)}$$
$$+ \varepsilon E_c\left(A_{12} e^{-(\lambda_3 + R_e P_r)y} + A_{13} e^{-(\lambda_5 + R_e P_r)y} + A_{14} e^{-(\lambda_1 + \lambda_3)y} + A_{15} e^{-(\lambda_1 + \lambda_5)y} - A_{16} e^{-\lambda_3 y}\right) e^{i(\pi z - t)} \quad (36)$$

Where,

$$\lambda_1 = \frac{1}{2}\left[R_e + \sqrt{R_e^2 + 4\left(M^2 + \frac{1}{K_p}\right)}\right]$$

$$\lambda_2 = \frac{1}{2}\left[-R_e + \sqrt{R_e^2 + 4\left(M^2 + \frac{1}{K_p}\right)}\right],$$

$$\lambda_3 = \frac{1}{2}\left[R_e P_r + \sqrt{R_e^2 P_r^2 + 4\left(\pi^2 - i\omega P_r\right)}\right],$$

$$\lambda_4 = \frac{1}{2}\left[-R_e P_r + \sqrt{R_e^2 P_r^2 + 4\left(\pi^2 - i\omega P_r\right)}\right],$$

$$\lambda_5 = \frac{1}{2}\left[R_e + \sqrt{R_e^2 + 4\left(M^2 + \frac{1}{K_p} + \pi^2 - i\omega\right)}\right],$$

$$\lambda_6 = \frac{1}{2}\left[-R_e + \sqrt{R_e^2 + 4\left(M^2 + \frac{1}{K_p} + \pi^2 - i\omega\right)}\right],$$

$$A_1 = \frac{R_e^2 G_r}{(\lambda_1 - R_e P_r)(\lambda_2 + R_e P_r)},$$

$$A_2 = \frac{R_e^2 G_r}{(\lambda_5 - \lambda_3)(\lambda_3 + \lambda_6)}, A_3 = \frac{\lambda_1 P_r A_1}{2(R_e P_r - 2\lambda_1)},$$

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$$A_4 = \frac{A_1^2}{2} P_r \ , A_5 = \frac{2 R_e A_1^2 P_r^2}{\lambda_1(\lambda_1 + R_e P_r)} ,$$

$$A_6 = A_3 - A_4 + A_5 , A_7 = \frac{-R_e^2 G_r}{\lambda_1(2\lambda_1 + \lambda_2)} ,$$

$$A_8 = \frac{R_e^2 G_r (A_3 - A_4 + A_5)}{(R_e P_r - \lambda_1)(R_e P_r + \lambda_2)} , A_9 = \frac{R_e^2 G_r A_4}{(2R_e P_r - \lambda_1)(2R_e P_r + \lambda_2)} ,$$

$$A_{10} = \frac{-R_e^2 G_r A_5}{R_e P_r (R_e P_r + \lambda_1 + \lambda_2)} ,$$

$$A_{11} = A_7 + A_8 + A_9 + A_{10} , A_{12} = \frac{-2R_e P_r^2 A_1 A_2 \lambda_3}{R_e P_r (R_e P_r + \lambda_3 + \lambda_4)} ,$$

$$A_{13} = \frac{2R_e P_r^2 A_1 A_2 \lambda_5}{(R_e P_r + \lambda_5 - \lambda_3)(R_e P_r + \lambda_5 + \lambda_4)} ,$$

$$A_{14} = \frac{2 P_r A_1 A_2 \lambda_1 \lambda_3}{\lambda_1(\lambda_1 + \lambda_3 + \lambda_4)} ,$$

$$A_{15} = \frac{-2 P_r A_1 A_2 \lambda_1 \lambda_5}{(\lambda_1 - \lambda_3 + \lambda_5)(\lambda_1 + \lambda_4 + \lambda_5)} ,$$

$$A_{16} = A_{12} + A_{13} + A_{14} + A_{15} ,$$

$$A_{17} = \frac{2R_e^2 G_r P_r A_2 A_{12}}{(R_e P_r + \lambda_3 - \lambda_5)(\lambda_1 + \lambda_3 + \lambda_6)} ,$$

$$A_{18} = \frac{2R_e^2 G_r P_r A_2 A_{13}}{(R_e P_r)(R_e P_r + \lambda_5 + \lambda_6)} ,$$

$$A_{19} = \frac{2R_e^2 G_r P_r A_2 A_{14}}{(\lambda_1 + \lambda_3 - \lambda_5)(\lambda_1 + \lambda_3 + \lambda_6)} ,$$

$$A_{20} = \frac{2 R_e^2 G_r P_r A_2 A_{15}}{(\lambda_1 + \lambda_5 + \lambda_6)\lambda_1} ,$$

$$A_{21} = \frac{2 R_e^2 G_r P_r A_2 A_{16}}{(\lambda_3 - \lambda_5)(\lambda_3 + \lambda_6)} , .$$

$$A_{22} = A_{17} - A_{18} - A_{19} + A_{20} - A_{21}$$

Skin Friction

The skin friction at the wall in the x-direction is given by

$$\tau_x = \left.\frac{\partial u}{\partial y}\right|_{y=0} . \quad (37)$$

Using eq. (35) in eq. (37), the skin friction at the wall in x-direction becomes

$$\begin{aligned} \tau_x &= \lambda_1[A_1 - E_c(2A_7 + A_{10} - A_{11})] \\ &- R_e P_r [A_1 + E_c(A_8 - 2A_9 - A_{10})] \\ &+ \in \{\lambda_1 E_c (A_{19} + A_{20}) \\ &+ \lambda_3 [E_c (A_{19} - A_{17} + A_{21}) - A_2] \\ &+ \lambda_5 [A_2 + E_c (A_{18} + A_{20} + A_{22})] \\ &- E_c R_e P_r (A_{17} - A_{18})\} e^{i(\pi z - t)} \end{aligned} . \quad (38)$$

Heat Flux

The rate of heat transfer or the heat flux at the wall in terms of Nusselt number is given by

$$N_u = \left.\frac{\partial T}{\partial y}\right|_{y=0} \quad (39)$$

Using Eq. (36) in Eq. (39), the rate of heat transfer at the wall in terms of Nusselt number becomes

$$\begin{aligned} N_u &= R_e P_r \{[E_c(2A_4 - A_5 + A_6) - 1]\} - \lambda_1 E_c (2A_3 + A_5) \\ &+ \in \{\lambda_3 [E_c (A_{16} - A_{14} - A_{12}) - 1] - \lambda_1 E_c (A_{14} + A_{15}) \\ &- \lambda_5 E_c (A_{13} + A_{15}) - E_c R_e P_r (A_{12} + A_{13})\} e^{i(\pi z - t)} \end{aligned} \quad (40)$$

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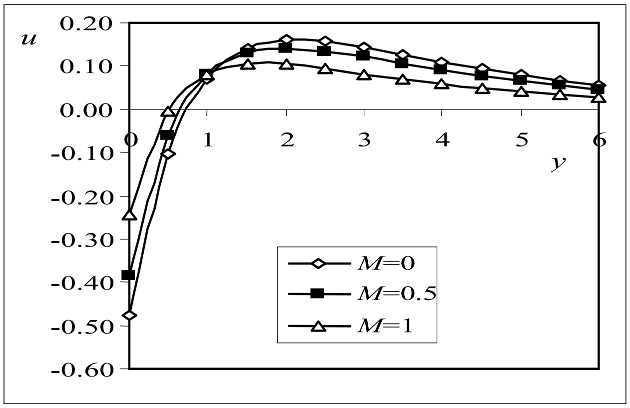

Fig.2. Velocity profiles against y *for different values of M with G_r =2,* P_r *= 0.71,* K_p *= 1, • = 5, • = 0.2,* E_c *= 0.01,* R_e *= 0.5,* t = *•/2 and* z *=0*

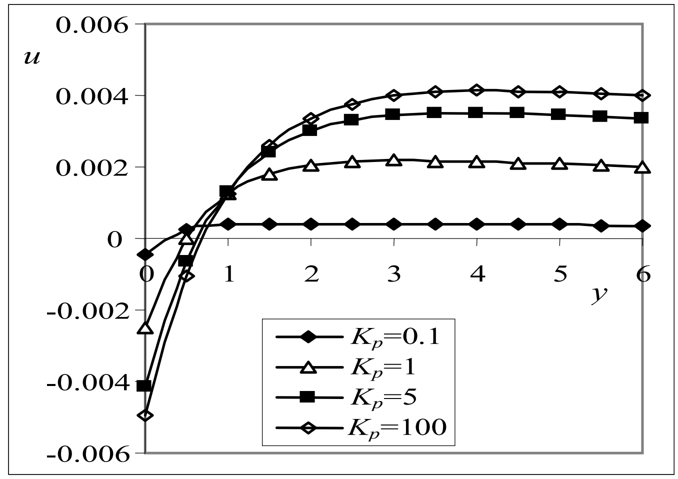

Fig.4. Velocity profiles against y for different values of K_p with M =1, P_r = 0.71, G_r =2, • = 5, • = 0.2, E_c = 0.01, R_e = 0.5, t = •/2 and z =0

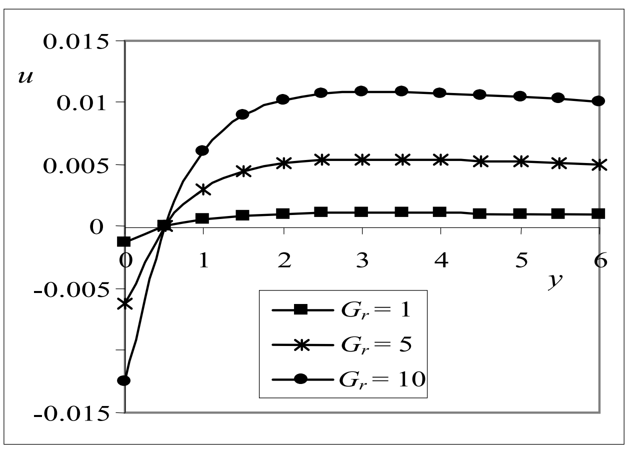

Fig.3. Velocity profiles against y for different values of G_r with M =1, P_r = 0.71, K_p = 1, • = 5, • = 0.2, E_c = 0.01, R_e = 0.5, t = •/2 and z =0

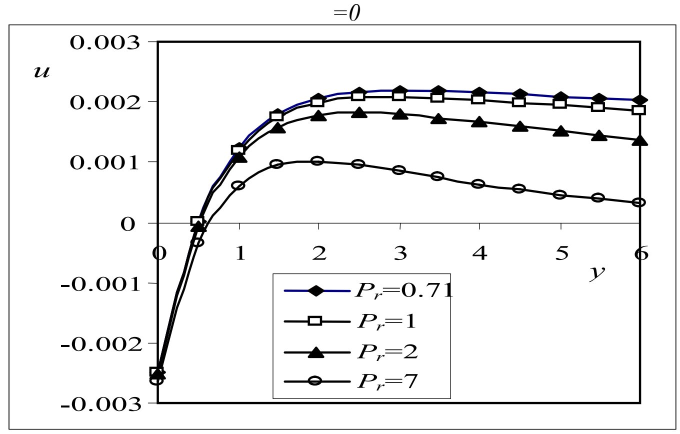

Fig.5. Velocity profiles against y for different values of P_r with M =1, K_p = 1, G_r =2, • = 5, • = 0.2, E_c = 0.01, R_e = 0.5, t = •/2 and z =0

Results and Discussions

The unsteady flow of a viscous incompressible electrically conducting fluid past an infinite hot vertical porous plate in presence of constant suction, periodic variation of plate temperature and a transverse magnetic field has been formulated and solved employing multi parameter perturbation technique and assuming Eckert number as the perturbation parameter. The expressions for velocity, temperature, skin friction and heat flux at the plate in terms of Nusselt number are obtained. The effects of magnetic parameter M, Grashof number for heat transfer Gr, Eckert number Ec, Prandtl number Pr, frequency parameter ω, etc. on velocity, temperature, skin friction and heat flux are discussed with the help of Figs. 2-8 and Tables 1-5.

The velocity of the flow field is found to change substantially with the variation of magnetic parameter M, Grashof number for heat transfer Gr, permeability parameter Kp, Prandtl number Pr and Eckert number Ec. The variations in velocity with the above mentioned parameters are shown in Figs. 2-6.

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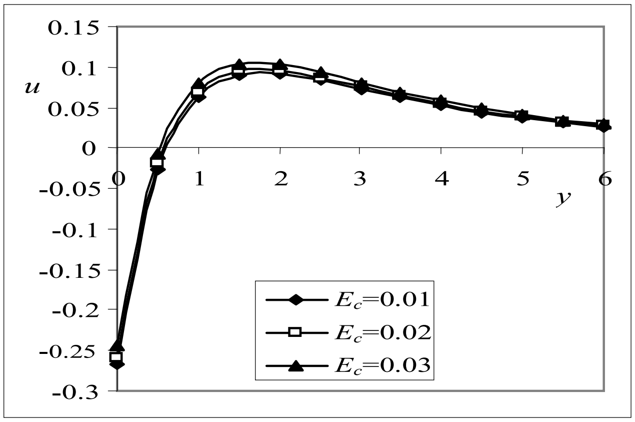

Fig.6. Velocity profiles against y for different values of E_c with M =1, K_p = 1, G_r =2, • = 5, • = 0.2, P_r = 0.71, R_e = 0.5, t = •/2 and z =0

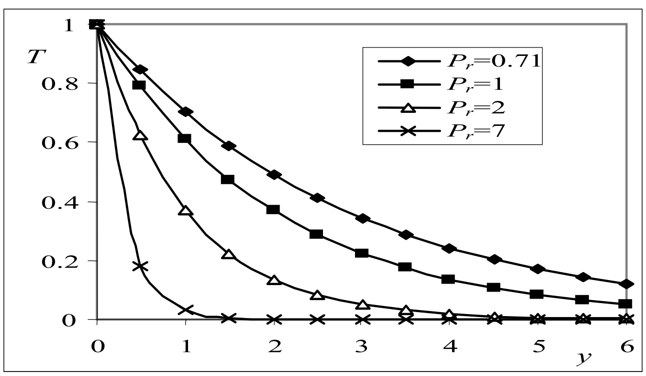

Fig.7. Temperature profiles against y for different values of P_r with M =1, K_p = 1, G_r =2, • = 5, • = 0.2, R_e = 0.5, E_c = 0.01, t = •/2 and z =0

In Fig.2, we present the effect of magnetic parameter on the velocity profiles keeping other parameters of the flow field constant. The velocity of the flow field is found to increase near the plate upto a certain distance (y=1) and there after the effect reverses. This is due to the magnetic pull of the Lorentz force acting on the flow field. Fig.3 depicts the effect of Grashof number for heat transfer Gr on the velocity profiles of the flow field. The Grashof number enhances the velocity of the flow field at all points due to the action of the free convection current in the flow field. The effect of permeability parameter Kp on the velocity of the flow field is shown in Fig.4. The permeability parameter accelerates the velocity of the flow field at all points. Fig. 5 presents the effect of Prandtl number Pr on the velocity profiles. Comparing the curves of the said figure, it is observed that the Prandtl number has a retarding effect on the velocity of the flow field at all points. The effect of Eckert number Ec on the velocity field is shown in Fig. 6. It is observed that the Eckert number has an accelerating effect on the velocity of the flow field at all points. The effect is more significant near the plate and the increase in velocity is negligible afterwards.

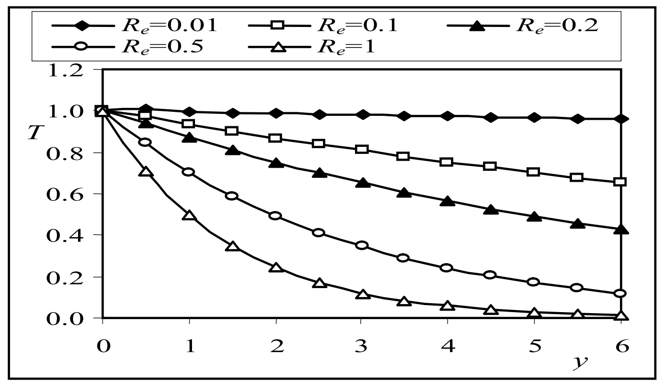

Fig.8. Temperature profiles against y for different values of R_e with M =1, K_p = 1, G_r =2, • = 5, • = 0.2, P_r = 0.71, E_c = 0.01, t = •/2 and z =0

The temperature of the flow field varies vastly with the variation of Prandtl number Pr and Reynolds number Re. Other parameters have no significant effect on the temperature field. The variations in the temperature of the flow field due to the above two parameters are shown in Figs.7 and 8 respectively. Fig.7 depicts exclusively the effect of Prandtl number on the temperature field. Comparing the curves of the figure, it is observed that a growing Prandtl number retards the temperature of the flow field at all points. Higher the Prandtl number, the more is the cooling effect in the flow field. In Fig.8, we present the variation in the temperature of the flow field due to change of Reynolds number Re. The Reynolds number has a very significant effect on the temperature field. A growing Reynolds number has a retarding effect on the temperature of the flow field at all points. It is interesting to note that for lower value of Re

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(=0.01), the decrease in temperature is very small and the temperature profile is linear and for higher value of Re, the profiles show an exponentially decrease in temperature.

The values of x-component of skin friction at the wall for different values of magnetic parameter M, permeability parameter Kp and Reynolds number Re are entered in Tables 1 and 2. Comparing the results in Table 1, it is observed that a growing permeability parameter enhances the skin friction at the wall for a particular value of the

Table 1. Variation in the value of x-component of the skin friction at the wall against K_p for different values of M with P_r = 0.71, G_r = 2, • = 5, • = 0.2, E_c = 0.01, R_e = 0.5, t = •/2, z =0

K_p	τ_x			
	M=0.5	*M* =1	*M* =5	*M* =10
0.1	0.150090	0.145057	0.082441	0.046826
0.5	0.306588	0.247038	0.093648	0.0479753
5	0.597272	0.405445	0.096870	0.049044
10	0.643663	0.420898	0.097059	0.049068
100	0.672511	0.436462	0.097230	0.049090

Table 2. Variation in the value of x-component of the skin friction at the wall against M for different values of R_e with P_r = 0.71, G_r = 2, • = 5, • = 0.2, E_c = 0.01, R_e = 0.5, t = •/2, z =0

	τ_x				
M	R_e =0.01	R_e =0.1	R_e =0.5	R_e =1	R_e =5
0	0.000199	0.019504	0.438308	1.358362	25.722178
0.5	0.000159	0.017486	0.398340	1.329353	23.298734
5	0.000039	0.003882	0.095398	0.372901	6.847573
10	0.000020	0.001970	0.048851	0.193340	4.361773
20	0.000010	0.000989	0.024631	0.098058	2.344778

magnetic parameter. It is also observed that the variation in skin friction becomes negligibly small as we increase the magnetic parameter. On the

other hand, a growing magnetic parameter decreases the skin friction at the wall for a given value of the permeability parameter. From Table 2, it is seen that an increase in magnetic parameter decreases the skin friction at the wall for a fixed value of Reynolds number. Further it is interesting

Table 3. Variation in the value of rate of heat transfer at the wall against P_r for different values of M with G_r = 2, • = 5, • = 0.2, E_c = 0.01, t = •/2, z =0

P_r	N_u				
	M=0	*M* =0.5	*M* =1	*M* =5	*M* =10
0.71	-0.242105	-0.242185	-0.242460	-0.24348	-0.24366
1	-0.343408	-0.343478	-0.343844	-0.34530	-0.34556
2	-0.708067	-0.708159	-0.708222	-0.71224	-0.71277

to observe that the skin friction increases rapidly as the Reynolds number grows in the flow field.

The variation in the value of rate of heat transfer i.e. the heat flux at the wall in terms of Nusselt number Nu for different values of magnetic parameter M, Prandtl number Pr, permeability parameter Kp and Reynolds number Re are entered in Table 3- 5. Table 3 explains the variation of heat flux against Prandtl number for different values of magnetic parameter. It is observed that a growing Prandtl number / magnetic parameter increase the magnitude of the rate of heat transfer at the wall for a given value of magnetic parameter / Prandtl number. Table 4 depicts the variation in

Table 4. Variation in the value of rate of heat transfer at the wall against K_p for different values of R_e with M=1, P_r = 0.71, G_r = 2, • = 5, • = 0.2, E_c = 0.01, t = •/2, z =0

K_p	N_u			
	R_e =0.2	R_e =0.5	R_e =1	R_e =5
0.1	-0.030617	-0.243271	-0.596893	-3.307203
0.5	-0.030532	-0.242707	-0.594594	-3.071584
5	-0.030427	-0.242167	-0.594182	-2.889890
10	-0.030415	-0.242131	-0.594469	-2.875386
100	-0.030402	-0.242105	-0.594857	-2.861767

the value of rate of heat transfer against permeability parameter for different values of Reynolds number. On careful observation, it is seen that the permeability parameter decreases the magnitude of the rate of heat transfer at the wall for a given value of Reynolds number, while a growing Reynolds number reverses the effect. Table 5 shows the variation in the value of rate of heat transfer at the wall against magnetic parameter for

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different values of permeability parameter. Comparing the results of Table 5, it is seen that the Heat flux increases in magnitude as we increase the magnetic parameter. On the other hand for a fixed value of the magnetic parameter, the permeability parameter decreases the magnitude of heat flux at the wall.

Table 5. Variation in the value of rate of heat transfer at the wall against M for different values of K_p with P_r = 0.71, G_r=2, • = 5, • = 0.2, R_e = 0.5, E_c = 0.01, t = •/2, z =0

M	N_u			
	K_p =0.1	K_p =0.5	K_p =1	K_p =2
0	-0.243241	-0.242461	-0.242104	-0.242099
0.5	-0.243249	-0.242534	-0.242186	-0.242117
5	-0.243533	-0.243487	-0.243480	-0.243476
10	-0.243672	-0.243665	-0.243664	-0.243664
20	-0.243756	-0.243756	-0.243755	-0.243755

Conclusions

On the basis of the above study, we summarize below the following results of physical interest on the flow field.

A growing magnetic parameter accelerates the velocity of the flow field near the plate upto a certain distance (y =1) and there after the effect reverses. This is due to the magnetic pull of the Lorentz force acting on the flow field.

The Grashof number for heat transfer, permeability parameter and Eckert number have an accelerating effect on the velocity of the flow field at all points.

A growing Prandtl number has a retarding effect on the velocity as well as the temperature of the flow field at all points. Higher the Prandtl number, the more is the cooling effect on the flow field.

The Reynolds number plays a very significant role on the temperature field. A growing Reynolds number has a retarding effect on the temperature of the flow field at all points. For lower value of Re (=0.01), the decrease in temperature is very small and the temperature profile is linear and for higher value of Re, the profiles show an exponentially decrease in temperature.

A growing permeability parameter enhances the skin friction at the wall for a fixed value of the magnetic parameter, while for a given value of the permeability parameter an increase in magnetic parameter decreases the skin friction at the wall. Again, the variation in skin friction becomes negligibly small as we increase the magnetic parameter.

For a fixed value of Reynolds number, the magnetic parameter decreases the skin friction at the wall. Further, it is interesting to observe that the skin friction increases rapidly as the Reynolds number grows in the flow field. The permeability parameter decreases the magnitude of the rate of heat transfer at the wall for a given value of Reynolds number, while a growing Reynolds number reverses the effect.

A growing magnetic parameter increases the magnitude of the rate of heat transfer at the wall for a fixed value of Prandtl number / permeability parameter. On the other hand for a fixed value of the magnetic parameter, the permeability parameter decreases the magnitude of heat flux, while the Prandtl number increases it at the wall.

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