Indian J. Innovations Dev., Vol.1, No.6 (June 2012) ISSN 2277 - 5390

# Radiation effect of an unsteady MHD free convective flow past an infinite vertical plate with heat, mass transfer and constant suction

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## Abstract

Free convection MHD flow of a viscous incompressible fluid past an infinite vertical plate with heat and mass in the presence of radiation has been studied. The dimensionless governing equations are solved using Laplace transfer technique. The plate temperature and mass concentrations are raised harmonically. The result is obtained for velocity, temperature, concentration, Nusselt number, Sherwood number and skin friction. The effect of various material parameters are discussed on flow variable and presented by graphs.

***Keywords**:* Constant Suction; Free convection; Vertical plate; Heat transfer; Radiation Mass transfer; MHD.

## Introduction

Free convection flows past different types of vertical bodies are studied because of their wide applications and hence it has attracted the attention of numerous investigators and scientists. Literature on unsteady MHD convection heat transfer with or without Hall currents are very extensive due to its technical importance in the scientific community. The radiation effects on MHD flow and heat transfer problems have become more important industrially. At high operating temperature, radiation effect can be quite significant. Many processes in engineering areas occur at high temperatures and knowledge of radiation heat transfer becomes very important for the design of the pertinent equipment. Nuclear power plants, gas turbines and the various propulsion devices for aircraft, missiles, satellites, and space vehicles are examples of such engineering areas.

Heat and mass transfer of a moving vertical plate with suction was studied by Erickson et al., 1996 and Gupta, 1977 with different conditions. Convective heat transfer in an electrically conducting fluid at stretching surface studied by Vajravelu and Hadjinicolaou (1997). Unsteady free convection flow past a vertical porous plate was investigated by Helmy (1998). Acharya *et al*. (2000) have studied free convection and mass transfer flow through a porous medium bounded by vertical infinite surface with constant suction and heat flux. But in those studies they considered the flow to be steady. Coming back to unsteady case, (Young, 2000) investigated unsteady MHD convective heat transfer past a semi-infinite vertical porous moving plate with variable suction. Chamka considered the case taking mass transfer with heat absorption (Muthu Cumaraswamy and kumar, 2004) investigated heat and mass transfer effect on moving vertical plate in presence of Thermal Radiation. In this paper we have discussed free convection MHD flow of a viscous incompressible fluid past an infinite vertical plate with heat and mass in the presence of radiation.

### Formulation of the Problem

Consider an unsteady two dimensional free convective flow of an electrically conducting, radiating, viscous and incompressible fluid past an infinite vertical porous plate with constant suction. $X_1$-axis is taken along the vertical plate in upward direction and the $Y_1$-axis is taken normal to the plate in the direction of applied uniform magnetic field of strength Ho. The magnetic permeability is constant throughout the field. There exist free convection current in the vicinity of the plate. It is assumed that a fluid has constant properties and the variation of density with temperature and mass concentration is considered only in the body force term. All the variable in this flow are the function of $y_1$ and time $t_1$ only as the plate is infinite length. Initially, the


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Indian J. Innovations Dev., Vol.1, No.6 (June 2012) ISSN 2277 - 5390

temperature and mass concentration at the plate are respectively $T_p$ and $C_p$ and the temperature and mass concentration of fluid are respectively $T_\infty$ and $C_\infty$. At time $t_1 > 0$, it is assumed that the temperature and mass concentration at the plate rise to $T_1 = T_p + \epsilon(T_p - T_\infty)e^{i\omega_1 t_1}$ and $C_1 = C_p + \epsilon(C_p - C_\infty)e^{i\omega_1 t_1}$. The fully developed flow of a radiating gas by usual Boussinesq's approximation the unsteady flow governed by following equation.

$$\frac{\partial v_1}{\partial y_1} = 0 \qquad (1)$$

$$\frac{\partial u_1}{\partial t_1} + v_1\frac{\partial u_1}{\partial y_1} = g\beta(T_1 - T_\infty) + g\beta_c(C_1 - C_\infty) + \nu\frac{\partial^2 u_1}{\partial y_1^2} - \frac{\sigma B_0^2 u_1}{\rho} \qquad (2)$$

$$\frac{\partial T_1}{\partial t_1} + v_1\frac{\partial T_1}{\partial y_1} = k\frac{\partial^2 T_1}{\partial y_1^2} - \frac{\partial q_r}{\partial y_1} \qquad (3)$$

$$\frac{\partial C_1}{\partial t_1} + v_1\frac{\partial C_1}{\partial y_1} = D'\frac{\partial^2 C_1}{\partial y_1^2} \qquad (4)$$

The initial and boundary conditions of the problem are

$$\left.\begin{array}{l} t_1 \le 0: \; u_1 = 0 \quad T_1 = T_\infty \quad C_1 = C_\infty \\ \text{for all } y_1 \\ t_1 > 0: \\ u_1 = V_0, \quad T_1 = T_p + \epsilon(T_p - T_\infty)e^{i\omega_1 t_1}, \\ C_1 = C_p + \epsilon(C_p - C_\infty)e^{i\omega_1 t_1} \quad \text{at} \\ y_1 = 0 \\ u_1 \to 0, T_1 \to T_\infty, C_1 \to C_\infty \quad \text{as} \quad y_1 \to \infty \end{array}\right\} \qquad (5)$$

where $\rho$ is the density, g acceleration due to gravity, $\beta$ is the co-efficient of thermal expansion, k is the thermal conductivity, $q_r$ the radiation flux, $\nu$ the kinematic viscosity, $\sigma$ is electrical conductivity, $B_0$(= $H_0\,\mu$ e) is the electromagnetic induction. $\beta_C$ is the co-efficient of expansion of mass and D is the diffusion constant.

Since the plate is assumed to be porous and through it suction with uniform velocity occurs, Equation (1) integrates to $v_1 = -V_0$ is the constant suction velocity. In a optically thick limit, the fluid does not absorb its own emitted radiation that is there is no self-absorption, but it does absorb radiation emitted by the boundaries.

$$\frac{\partial q_r}{\partial y'} = 4(T_1 - T_\infty)\int_0^\infty k_{\lambda w}\left(\frac{de_{b\lambda}}{dT'}\right)d\lambda = 4I(T_1 - T_\infty) \qquad (6)$$

where $K_{\lambda w}$ is the absorption co-efficient, $e_{b\lambda}$ is the Plank's function and subscription w refers the value of the wall (plate).

On introducing the following non-dimensional quantities

$$\left.\begin{array}{l} t = \dfrac{t_1 V_0^2}{4\nu}, y = \dfrac{y_1 V_0}{\nu}, u = \dfrac{u_1}{V_0}, Pr = \dfrac{\nu}{k}, Sc = \dfrac{\nu}{D} \\ T = \dfrac{T_1 - T_\infty}{T_p - T_\infty}, C = \dfrac{C_1 - C_\infty}{C_p - C_\infty}, \omega = \dfrac{4\nu\omega_1}{V_0^2}, N = \dfrac{16I\nu}{V_0^2} \\ M = \dfrac{\nu B_0^2\sigma}{\rho V_0^2}, Gr = \dfrac{\nu g\beta(T_p - T_\infty)}{V_0^3}, Gm = \dfrac{\nu g\beta_c(C_p - C_\infty)}{V_0^3} \end{array}\right\} (7)$$

where $Gr$ is Grashof number, $Gm$ modified Grashof number, M is magnetic number, $Pr$ is Prandtl number, Sc is Schmidt number, T is non-dimensional temperature, C is non-dimensional mass concentration, $\omega$ is oscillating frequency and N is the radiation parameter.

With the help of equation (7) the equations (2) to (5) reduce to

$$\frac{\partial u}{\partial t} - 4\frac{\partial u}{\partial y} = 4\frac{\partial^2 u}{\partial y^2} + 4GrT + 4GmC - 4Mu \qquad (8)$$

$$\frac{\partial T}{\partial t} \quad 4\frac{\partial T}{\partial y} = \frac{4}{Pr}\frac{\partial^2 T}{\partial y^2} \quad NT \qquad (9)$$

$$\frac{\partial C}{\partial t} - 4\frac{\partial C}{\partial y} = \frac{4}{Sc}\frac{\partial^2 C}{\partial y^2} \qquad (10)$$

With the following boundary conditions

$$\left.\begin{array}{l} u = 0, T = 0 \\ C = 0 \; for \; all \; y = 0 \; and \; t \le 0 \end{array}\right\}$$


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Indian J. Innovations Dev., Vol.1, No.6 (June 2012) ISSN 2277 - 5390

$$t > 0 : u = 0, T = 1 + \varepsilon e^{it\omega}, C = 1 + \epsilon\, e^{it\omega} \quad at\ y = 0$$

(11)

$$u = 0, T = 0, C = 0 \qquad as \qquad y \to \infty$$

## Solution

We solve the governing equations in an exact form with the help of using Laplace transforms of equations (8) to (10) using condition (11), we have

$$4\frac{d^2\bar{u}}{dy^2} + 4\frac{d\bar{u}}{dy} - (s + 4M)\bar{u} = 4Gr\bar{T} + 4Gm\bar{C}$$

(12)

$$4\frac{d^2\bar{T}}{dy^2} + 4Pr\frac{d\bar{T}}{dy} - (NPr + Pr.s)\bar{T} = 0$$

(13)

$$4\frac{d^2\bar{C}}{dy^2} + 4Sc\frac{d\bar{C}}{dy} - Sc.s\bar{C} = 0$$

(14)

Where s is the Laplace transform parameter

The boundary conditions reduce to

$$u = 0, \bar{T} = \frac{1}{s} + \frac{\epsilon}{s - \omega i}, \bar{C} = \frac{1}{s} + \frac{\epsilon}{s - \omega i} \quad at\ y = 0$$ (15)

$$\bar{u} = 0, \bar{T} = 0, \bar{C} = 0 \qquad as \qquad \bar{y} \to \infty$$

By solving equations (12) to (14) using (15), we get

$$\bar{u} = -\left\{4Gm\left(\frac{1}{s} + \frac{\epsilon}{s - \omega i}\right)\frac{1}{(Sc-1)s - 4M} + 4Gr\left(\frac{1}{s} + \frac{\epsilon}{s - \omega i}\right)\frac{1}{(Pr-1)s - (4M - NPr)}\right\}e^{-\frac{\left(1+\sqrt{1+s+4M}\right)y}{2}}$$

$$+4Gm\left(\frac{1}{s} + \frac{\epsilon}{s - \omega i}\right)\frac{1}{(Sc-1)s - 4M}e^{-\frac{\left(Sc+\sqrt{Sc^2+Sc.s}\right)y}{2}}$$

$$+4Gr\left(\frac{1}{s} + \frac{\epsilon}{s - \omega i}\right)\frac{1}{(Pr-1)s - (4M - NPr)}e^{-\frac{\left(Pr+\sqrt{Pr^2+NPr+Pr.s}\right)y}{2}}$$

(16)

$$\bar{T} = \left(\frac{1}{s} + \frac{\epsilon}{s - \omega i}\right)e^{-\left(Pr+\sqrt{Pr^2+NPr+Pr.s}\right)y/2}$$ (17)

$$\bar{C} = \left(\frac{1}{s} + \frac{\epsilon}{s - \omega i}\right)e^{-\left(Sc+\sqrt{Sc^2+Sc.s}\right)y/2}$$ (18)

Again by taking inverse Laplace transform of equations (16) –(18), we get

$$u = -\frac{Gm}{2M}\left(e^{-2\eta\sqrt{t}}\right)\left[e^{\frac{4Mt}{Sc-1}+2\eta\sqrt{t+\frac{4tMSc}{Sc-1}}}\, erfc\left(\eta + \sqrt{t + \frac{4tMSc}{Sc-1}}\right) + e^{\frac{4Mt}{Sc-1}-2\eta\sqrt{t+\frac{4tMSc}{Sc-1}}}\, erfc\left(\eta - \sqrt{t + \frac{4tMSc}{Sc-1}}\right) - e^{2\eta\left(\sqrt{t+4Mt}\right)}.erfc\left(\eta + \sqrt{t + 4Mt}\right) - e^{-2\eta\left(\sqrt{t+4Mt}\right)}.erfc\left(\eta - \sqrt{t + 4Mt}\right)\right]$$


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$$-\frac{Gm\epsilon}{4M-\omega Sci+\omega i}\left(e^{-(2\eta\sqrt{t})}\right)\left[e^{\frac{4Mt}{Sc-1}+2\eta\sqrt{t+\frac{4tMSc}{Sc-1}}}.erfc\left(\eta+\sqrt{t+\frac{4tMSc}{Sc-1}}\right)+\right.$$
$$e^{\frac{4Mt}{Sc-1}-2\eta\sqrt{t+\frac{4tMSc}{Sc-1}}}.erfc\left(\eta-\sqrt{t+\frac{4tMSc}{Sc-1}}\right)-$$
$$e^{t\omega i-2\eta\sqrt{t(4M+1+\omega i)}}.erfc\left(\eta-\sqrt{t(4M+1+\omega i)}\right)-e^{t\omega i+2\eta\sqrt{t(4M+1+\omega i)}}.erfc\left(\eta+\right.$$
$$\left.\left.\sqrt{t(4M+1+\omega i)}\right)\right]$$

$$-\frac{2Cr}{4M-PrN}\left(e^{-(4Mt+t+2\eta\sqrt{t})}\right)\left[e^{t\left(1+\frac{Pr(4M-N)}{Pr-1}\right)+2\eta\sqrt{t+\frac{Pr\,t(4M-N)}{Pr-1}}}erfc\left(\eta+\sqrt{t+\frac{Pr\,t(4M-N)}{Pr-1}}\right)+\right.$$
$$e^{t\left(1+\frac{Pr(4M-N)}{Pr-1}\right)-2\eta\sqrt{t+\frac{Pr\,t(4M-N)}{Pr-1}}}erfc\left(\eta-\sqrt{t+\frac{Pr\,t(4M-N)}{Pr-1}}\right)-e^{t(1+4M)+2\eta\sqrt{t(1+4M)}}erfc\left(\eta+\right.$$
$$\left.\left.\sqrt{t(1+4M)}\right)-e^{t(1+4M)-2\eta\sqrt{t(1+4M)}}erfc\left(\eta-\sqrt{t(1+4M)}\right)\right]$$

$$\frac{2Cr\epsilon}{4M-NPr-Pr\omega i+\omega i}\left(e^{-(4Mt+t+2\eta\sqrt{t})}\right)\left[e^{t\left(1+\frac{Pr(4M-N)}{Pr-1}\right)+2\eta\sqrt{t+\frac{Pr\,t(4M-N)}{Pr-1}}}erfc\left(\eta+\right.\right.$$
$$\left.\sqrt{t+\frac{Pr\,t(4M-N)}{Pr-1}}\right)+e^{t\left(1+\frac{Pr(4M-N)}{Pr-1}\right)-2\eta\sqrt{t+\frac{Pr\,t(4M-N)}{Pr-1}}}erfc\left(\eta-\sqrt{t+\frac{Pr\,t(4M-N)}{Pr-1}}\right)-$$
$$e^{t(1+4M+\omega i)+2\eta\sqrt{t(1+4M+\omega i)}}erfc\left(\eta+\sqrt{t(1+4M+\omega i)}\right)-$$
$$\left.e^{t(1+4M+\omega i)-2\eta\sqrt{t(1+4M+\omega i)}}erfc\left(\eta-\sqrt{t(1+4M+\omega i)}\right)\right]$$

$$+\frac{Gm}{2M}\left(e^{-(2\eta Sc\sqrt{t})}\right)\left[e^{\left(\frac{4Mt}{Sc-1}\right)+2\eta\sqrt{tSc^2+\frac{4MSct}{Sc-1}}}.erfc\left(\eta\sqrt{Sc}+\sqrt{tSc+\frac{4Mt}{Sc-1}}\right)+\right.$$
$$e^{\left(\frac{4Mt}{Sc-1}\right)-2\eta\sqrt{tSc^2+\frac{4MSct}{Sc-1}}}.erfc\left(\eta\sqrt{Sc}-\sqrt{tSc+\frac{4Mt}{Sc-1}}\right)-e^{2\eta Sc\sqrt{t}}.erfc\left(\eta\sqrt{Sc}+\sqrt{tSc}\right)-$$
$$\left.e^{-2\eta Sc\sqrt{t}}.erfc\left(\eta\sqrt{Sc}-\sqrt{tSc}\right)\right]$$

$$+\frac{2Gm\epsilon}{(4M-Sc\omega i+\omega i)}\left(e^{-(2\eta Sc\sqrt{t})}\right)\left[e^{\left(tSc+\frac{4Mt}{Sc-1}\right)+2\eta\sqrt{tSc^2+\frac{4MSct}{Sc-1}}}.erfc\left(\eta\sqrt{Sc}+\right.\right.$$
$$\left.\sqrt{tSc+\frac{4Mt}{Sc-1}}\right)+e^{\left(tSc+\frac{4Mt}{Sc-1}\right)-2\eta\sqrt{tSc^2+\frac{4MSct}{Sc-1}}}.erfc\left(\eta\sqrt{Sc}-\sqrt{tSc+\frac{4Mt}{Sc-1}}\right)-$$
$$e^{(\omega ti)+2\eta\sqrt{Sc^2t+Sc\omega ti}}.erfc\left(\eta\sqrt{Sc}+\sqrt{Sct+\omega ti}\right)-$$
$$\left.e^{(\omega ti)-2\eta\sqrt{Sc^2t+Sc\omega ti}}.erfc\left(\eta\sqrt{Sc}-\sqrt{Sct+\omega ti}\right)\right]$$


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$$+\frac{2Gr}{4M-PrN}\left(e^{-\left(Prt+Nt+2\eta Pr\sqrt{t}\right)}\right)\left[e^{\left(Prt+\frac{t(4M-N)}{Pr-1}\right)+\left(2\eta\sqrt{tPr^2+\frac{Prt(4M-N)}{Pr-1}}\right)}erfc\left(\eta\sqrt{Pr}+\sqrt{Prt+\frac{t(4M-N)}{Pr-1}}\right)+e^{\left(Prt+\frac{t(4M-N)}{Pr-1}\right)-\left(2\eta\sqrt{tPr^2+\frac{Prt(4M-N)}{Pr-1}}\right)}erfc\left(\eta\sqrt{Pr}-\sqrt{Prt+\frac{t(4M-N)}{Pr-1}}\right)-e^{Prt+tN+2\eta\sqrt{Pr^2t+PrNt}}erfc\left(\eta\sqrt{Pr}+\sqrt{Prt+Nt}\right)-e^{Prt+tN-2\eta\sqrt{Pr^2t+PrNt}}erfc\left(\eta\sqrt{Pr}-\sqrt{Prt+Nt}\right)\right]$$

$$+\frac{2Gr\epsilon}{4M-PrN-Pr\omega i+\omega i}\left(e^{-\left(Prt+Nt+2\eta Pr\sqrt{t}\right)}\right)\left[e^{\left(Prt+\frac{t(4M-N)}{Pr-1}\right)+\left(2\eta\sqrt{tPr^2+\frac{Prt(4M-N)}{Pr-1}}\right)}erfc\left(\eta\sqrt{Pr}+\sqrt{Prt+\frac{t(4M-N)}{Pr-1}}\right)+e^{\left(Prt+\frac{t(4M-N)}{Pr-1}\right)-\left(2\eta\sqrt{tPr^2+\frac{Prt(4M-N)}{Pr-1}}\right)}erfc\left(\eta\sqrt{Pr}-\sqrt{Prt+\frac{t(4M-N)}{Pr-1}}\right)-e^{t(Pr+N+\omega i)+2\eta\sqrt{t(Pr^2+NPr+Pr\omega i)}}.erfc\left(\eta\sqrt{Pr}+\sqrt{t(Pr+N+\omega i)}\right)-e^{t(Pr+N+\omega i)-2\eta\sqrt{t(Pr^2+NPr+Pr\omega i)}}.erfc\left(\eta\sqrt{Pr}-\sqrt{t(Pr+N+\omega i)}\right)\right] \quad ..\ (19)$$

$$T=\frac{e^{-2\eta Pr\sqrt{t}}}{2}\left[e^{-2\eta\sqrt{tPr^2+tNPr}}.erfc\left(\eta\sqrt{Pr}-\sqrt{t(Pr+N)}\right)+e^{2\eta\sqrt{tPr^2+tNPr}}.erfc\left(\eta\sqrt{Pr}+\sqrt{t(Pr+N)}\right)+\epsilon\left(e^{t\omega i-2\eta\sqrt{tPr^2+tNPr+tPr\omega i}}.erfc\left(\eta\sqrt{Pr}-\sqrt{t(Pr+N+\omega i)}\right)+e^{t\omega i+2\eta\sqrt{tPr^2+tNPr+tPr\omega i}}.erfc\left(\eta\sqrt{Pr}+\sqrt{t(Pr+N+\omega i)}\right)\right)\right] \quad \ldots\ (20)$$

$$C=\frac{e^{-2\eta Sc\sqrt{t}}}{2}\left[e^{-2\eta Sc\sqrt{t}}.erfc\left(\eta\sqrt{Sc}-\sqrt{tSc}\right)+e^{2\eta Sc\sqrt{t}}.erfc\left(\eta\sqrt{Sc}+\sqrt{tSc}\right)+\epsilon\left(e^{t\omega i-2\eta\sqrt{tSc^2+tSc\omega i}}.erfc\left(\eta\sqrt{Sc}-\sqrt{t(Sc+\omega i)}\right)+e^{t\omega i+2\eta\sqrt{tSc^2+tSc\omega i}}.erfc\left(\eta\sqrt{Sc}+\sqrt{t(Sc+\omega i)}\right)\right)\right] \quad \ldots.\ (21)$$


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where $\eta = \frac{y}{4\sqrt{t}}$

The non-dimensional shearing stress at the wall from the equations (19) is given by

$$\tau = \left(\frac{\partial u}{\partial \eta}\right)_{\eta=0} =$$

$$\frac{Gm\sqrt{t}}{M}\left[e^{\frac{4Mt}{Sc-1}}\left(erfc\left(\sqrt{t+\frac{4MtSc}{Sc-1}}\right)+erfc\left(-\sqrt{t+\frac{4MtSc}{Sc-1}}\right)\right)-\left(erfc\left(\sqrt{t+4Mt}\right)+erfc\left(-\sqrt{t+4Mt}\right)\right)\right]-\frac{Gm}{2M}\left[2\sqrt{t+\frac{4MtSc}{Sc-1}}\;e^{\frac{4Mt}{Sc-1}}\left(erfc\left(\sqrt{t+\frac{4MtSc}{Sc-1}}\right)-erfc\left(-\sqrt{t+\frac{4MtSc}{Sc-1}}\right)\right)-2\sqrt{t+4Mt}\left(erfc\left(\sqrt{t+4Mt}\right)-erfc\left(-\sqrt{t+4Mt}\right)\right)\right]+$$

$$\frac{4Gm\varepsilon\sqrt{t}}{4M-\omega Sci+\omega i}e^{-(1+4M)\varepsilon}\left[e^{\frac{4MSct}{Sc-1}}\left(erfc\left(\sqrt{t+\frac{4MtSc}{Sc-1}}\right)+erfc\left(-\sqrt{t+\frac{4MtSc}{Sc-1}}\right)\right)-e^{t(4M+1+\omega i)}\left(erfc\left(\sqrt{t+4Mt+\omega ti}\right)+erfc\left(-\sqrt{t+4Mt+\omega ti}\right)\right)\right]$$

$$-\frac{2Gm\varepsilon}{4M-\omega Sci+\omega i}e^{-(1+4M)\varepsilon}\left[2\sqrt{t+\frac{4MtSc}{Sc-1}}\,.\,e^{\frac{4MSct}{Sc-1}}\left(erfc\left(\sqrt{t+\frac{4MtSc}{Sc-1}}\right)-erfc\left(-\sqrt{t+\frac{4MtSc}{Sc-1}}\right)\right)-2\sqrt{t+4Mt+\omega ti}\;e^{t(4M+1+\omega i)}\left(erfc\left(\sqrt{t+4Mt+\omega ti}\right)-erfc\left(-\sqrt{t+4Mt+\omega ti}\right)\right)\right]+$$

$$\frac{4Gr\sqrt{t}}{4M-PrN}\left[e^{\frac{t(4M-N)}{Pr-1}}\left(erfc\left(\sqrt{t+\frac{Prt(4M-N)}{Pr-1}}\right)+erfc\left(-\sqrt{t+\frac{Prt(4M-N)}{Pr-1}}\right)\right)-\left(erfc\left(\sqrt{t+4Mt}\right)+erfc\left(-\sqrt{t+4Mt}\right)\right)\right]-\frac{2Gr}{4M-PrN}\left[2\sqrt{t+\frac{Prt(4M-N)}{Pr-1}}\;e^{\frac{t(4M-N)}{Pr-1}}\left(erfc\left(\sqrt{t+\frac{Prt(4M-N)}{Pr-1}}\right)-erfc\left(-\sqrt{t+\frac{Prt(4M-N)}{Pr-1}}\right)\right)-2\sqrt{t+4Mt}\left(\left(erfc\left(\sqrt{t+4Mt}\right)-erfc\left(-\sqrt{t+4Mt}\right)\right)\right)\right]$$


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Indian J. Innovations Dev., Vol.1, No.6 (June 2012) ISSN 2277 - 5390

$$+\frac{4Gr\varepsilon\sqrt{t}}{4M-NPr-Pr\omega i+\omega i}\left[e^{\frac{t(4M-PrN)}{Pr-1}}\left(erfc\left(\sqrt{t+\frac{Pr\,t(4M-N)}{Pr-1}}\right)+erfc\left(-\sqrt{t+\frac{Pr\,t(4M-N)}{Pr-1}}\right)\right)-\right.$$

$$\left.e^{t\omega i}\left(erfc\left(\sqrt{t(1+4M+\omega i)}\right)+erfc\left(-\sqrt{t(1+4M+\omega i)}\right)\right)\right]-$$

$$\frac{2Gr\varepsilon}{4M-NPr-Pr\omega i+\omega i}\left[2\sqrt{t+\frac{Pr\,t(4M-N)}{Pr-1}}\;e^{\frac{t(4M-PrN)}{Pr-1}}\left(erfc\left(\sqrt{t+\frac{Pr\,t(4M-N)}{Pr-1}}\right)-\right.\right.$$

$$\left.erfc\left(-\sqrt{t+\frac{Pr\,t(4M-N)}{Pr-1}}\right)\right)-2\sqrt{t(1+4M+\omega i)}\;e^{t\omega i}\left(erfc\left(\sqrt{t(1+4M+\omega i)}\right)-\right.$$

$$\left.\left.erfc\left(-\sqrt{t(1+4M+\omega i)}\right)\right)\right]-$$

$$\frac{4GmSc\sqrt{t}}{Sc^2-Sc+4M}\left[e^{\frac{4Mt}{Sc-1}}\left(erfc\left(\sqrt{tSc+\frac{4Mt}{Sc-1}}\right)+erfc\left(-\sqrt{tSc+\frac{4Mt}{Sc-1}}\right)\right)-\left(erfc\left(\sqrt{Sct}\right)+\right.\right.$$

$$\left.\left.erfc\left(-\sqrt{Sct}\right)\right)\right]+$$

$$\frac{Gm}{2M}\left[2\sqrt{tSc^2+\frac{4ScMt}{Sc-1}}\;e^{\frac{4Mt}{Sc-1}}\left(erfc\left(\sqrt{tSc+\frac{4Mt}{Sc-1}}\right)-erfc\left(-\sqrt{tSc+\frac{4Mt}{Sc-1}}\right)\right)-\right.$$

$$\left.2Sc\sqrt{t}\;\left(erfc\left(\sqrt{Sct}\right)-erfc\left(-\sqrt{Sct}\right)\right)\right]-\frac{4GmSc\varepsilon\sqrt{t}}{4M-Sc\omega i+\omega i}\left[e^{\frac{4Mt}{Sc-1}}erfc\left(\sqrt{tSc+\frac{4Mt}{Sc-1}}\right)+\right.$$

$$\left.erfc\left(-\sqrt{tSc+\frac{4Mt}{Sc-1}}\right)-e^{t\omega i}\left(erfc\left(\sqrt{Sct+\omega ti}\right)+erfc\left(-\sqrt{Sct+\omega ti}\right)\right)\right]$$

$$\frac{2Gm\varepsilon}{4M-Sc\omega i+\omega i}\left[2\sqrt{tSc^2+\frac{4ScMt}{Sc-1}}\;e^{\frac{4Mt}{Sc-1}}\left(erfc\left(\sqrt{tSc+\frac{4Mt}{Sc-1}}\right)-erfc\left(-\sqrt{tSc+\frac{4Mt}{Sc-1}}\right)\right)-\right.$$

$$\left.2\sqrt{Sc^2t+Sc\omega ti}\,.e^{t\omega i}\left(erfc\left(\sqrt{Sct+\omega ti}\right)-erfc\left(-\sqrt{Sct+\omega ti}\right)\right)\right]$$

$+$

$$-\frac{4GrPr\sqrt{t}}{4M-PrN}\left[e^{\frac{t(4M-PrN)}{Pr-1}}\left(erfc\left(\sqrt{Prt+\frac{t(4M-N)}{Pr-1}}\right)+erfc\left(-\sqrt{Prt+\frac{t(4M-N)}{Pr-1}}\right)\right)-\right.$$

$$\left.\left(erfc\left(\sqrt{Prt+Nt}\right)+erfc\left(-\sqrt{Prt+Nt}\right)\right)\right]+$$

$$\frac{2Gr}{4M-PrN}\left[2\sqrt{tPr^2+\frac{Prt(4M-N)}{Pr-1}}\;e^{\frac{t(4M-PrN)}{Pr-1}}\left(erfc\left(\sqrt{Prt+\frac{t(4M-N)}{Pr-1}}\right)-\right.\right.$$

$$\left.\left.erfc\left(-\sqrt{Prt+\frac{t(4M-N)}{Pr-1}}\right)\right)-2\sqrt{Pr^2t+NPrt}\left(erfc\left(\sqrt{Prt+Nt}\right)-erfc\left(-\sqrt{Prt+Nt}\right)\right)\right]-$$

$$\frac{4Gr\varepsilon Pr\sqrt{t}}{4M-PrN-Pr\omega i+\omega i}\left[e^{\frac{t(4M-PrN)}{Pr-1}}\left(erfc\left(\sqrt{Prt+\frac{t(4M-N)}{Pr-1}}\right)+erfc\left(-\sqrt{Prt+\frac{t(4M-N)}{Pr-1}}\right)\right)-\right.$$

$$\left.e^{t\omega i}\left(erfc\left(\sqrt{Prt+Nt+\omega ti}\right)+erfc\left(-\sqrt{Prt+Nt+\omega ti}\right)\right)\right]+$$

$$\frac{2Gr\varepsilon}{4M-PrN-Pr\omega i+\omega i}\left[2\sqrt{tPr^2+\frac{Prt(4M-N)}{Pr-1}}\;e^{\frac{t(4M-PrN)}{Pr-1}}\left(erfc\left(\sqrt{Prt+\frac{t(4M-N)}{Pr-1}}\right)-\right.\right.$$

$$\left.\left.erfc\left(-\sqrt{Prt+\frac{t(4M-N)}{Pr-1}}\right)\right)-2\sqrt{Pr^2t+NPrt+\omega ti}\,e^{t\omega i}\left(erfc\left(\sqrt{Prt+Nt+\omega ti}\right)-\right.$$

$$\left.\left.erfc\left(-\sqrt{Prt+Nt+\omega ti}\right)\right)\right] \qquad (22)$$


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the dimensional rate of heat transfer/ Nusselt Number,

$$Nu=-\left(\frac{\partial T}{\partial \eta}\right)_{\eta=0}=$$
$$Pr\sqrt{t}\left[erfc\left(-\sqrt{t(Pr+N)}\right)+erfc\left(\sqrt{t(Pr+N)}\right)+e^{ti\omega}\left(erfc\left(-\sqrt{t(Pr+N+\omega i)}\right)+\right.\right.$$
$$\left.\left.erfc\left(\sqrt{t(Pr+N+\omega i)}\right)\right)\right]-$$
$$\left[\sqrt{tPr^2+tPrN}\left(erfc\left(\sqrt{t(Pr+N)}\right)-erfc\left(-\sqrt{t(Pr+N)}\right)\right)-\frac{4\sqrt{Pr}}{\sqrt{\pi}}e^{-t(Pr+N)}+\right.$$
$$\left.\sqrt{tPr^2+tPrN+tPr\omega i}\,.e^{ti\omega}\left(erfc\left(\sqrt{t(Pr+N+\omega i)}\right)-erfc\left(-\sqrt{t(Pr+N+\omega i)}\right)\right)\right] \quad ..(23)$$

The dimensionless rate of mass transfer/ Sherwood Number,

$$Sh=-\left(\frac{\partial C}{\partial \eta}\right)_{\eta=0}=Sc\sqrt{t}\left[erfc\left(-\sqrt{tSc}\right)+erfc\left(\sqrt{tSc}\right)+e^{ti\omega}\left(erfc\left(-\sqrt{t(Sc+\omega i)}\right)+\right.\right.$$
$$\left.\left.erfc\left(\sqrt{t(Sc+\omega i)}\right)\right)\right]-$$
$$\left[Sc\sqrt{t}\left(-erfc\left(-\sqrt{tSc}\right)+erfc\left(\sqrt{tSc}\right)\right)-\frac{2\sqrt{Sc}}{\sqrt{\pi}}e^{-tSc}+\right.$$
$$\left.e^{ti\omega}\sqrt{tSc^2+tSc\omega i}\left(-erfc\left(-\sqrt{t(Sc+\omega i)}\right)+erfc\left(\sqrt{t(Sc+\omega i)}\right)\right)-\frac{2\sqrt{Sc}}{\sqrt{\pi}}e^{-t(Sc+\omega i)}\right] \quad ..(24)$$

In order to get the physical insight into the problem, the numerical value of $u, \tau$ and other Physical quantities have been calculated from the above equations. At the time of evaluation of the expressions, it is observed that the argument of the error function is complex and hence, it is separated into real and imaginary parts by using the following formula and then the complimentary error function up to twenty terms is calculated.

$$erfc\,(a+ib)=erf\,(a)+\frac{\exp\left(-a^2\right)}{2a\pi}[1-\cos\left(2ab\right)+i\sin\left(2ab\right)]$$
$$+\frac{2\exp\left(-n^2/4\right)}{\pi}\sum_{n=1}^{\infty}\left[f_n(a,b)+ig_n(a,b)\right]+\varepsilon(a,b) \quad ....(25)$$

Where $f_n=2a-2a\cosh(nb)\cos(2ab)+n\sinh(nb)\sin(2ab)$, $g_n=2a\cosh(nb)\sin(2ab)+n\sinh(nb)\cos(2ab)$ and $|\varepsilon(a,b)|\approx 10^{-16}|erf\,(a+ib)|$

**Discussion of Results**

In this paper we have studied the effect of radition

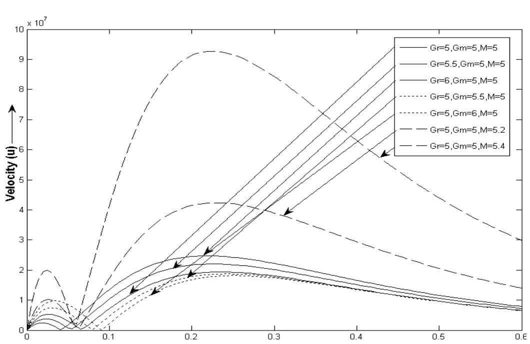


*Fig1*: Effect of Gr,Gm and M on velocity profile when N=1,Pr=2 and t=1

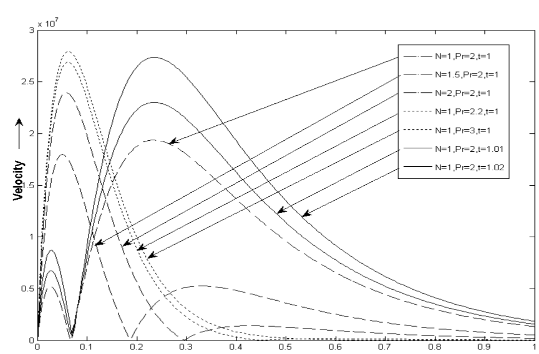


*Fig 2*. Effect of N,Pr and t on velocity profile when Gr=5,Gm=5 and M=5


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on unsteady MHD free convective flow past an infinite vertical plate with heat, mass transfer and constant suction. The effect of parameters Gr, Gm, M, N, t, Pr, ω, Sc, on flow characteristics has been presented in the figures 1-9.

**Velocity profile** The mean velocity profiles is depicted in Fig1-2. Fig. 1 shows the effect of the parameters Gr, Gm & M on velocity at any point of the fluid, when N=1,Pr=2& t=1.It is observed that the velocity decreases with both the parameters Gr & Gm very near the plate and then increases away from the plate. But the velocity increases for the parameter M throughout diminishing to zero at a small distance from the plate.

Fig. 2 Shows the effect of the parameters N, Pr &t on velocity at any point of the fluid, when Gr=5, Gm=5 and M=5. It is observed that the velocity increases with both the parameters Pr & N very near the plate and decreases away from the plate. But the velocity increases for the parameter t throughout diminishing to zero at a small distance from the plate.

**Temperature profile** The temperature profile is depicted in Fig. 3. Fig. 3 shows the effect of the parameters Pr, N, ω and t on heat at any point of the fluid, when Є=0.02. It is observed that the temperature falls with increase of the parameters Prandtl number (Pr), Oscillating frequency (ω) and time (t).

**Concentration profile:** The Concentration profile is depicted in Fig. 4. Fig. 4 shows the effect of the parameters Sc, ω and t on concentration at any point of the fluid, when Є=0.02. It is noticed that

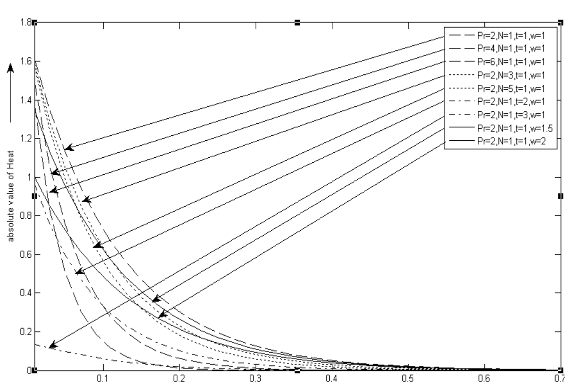


*Fig 3*. Effect of N,Pr, $\omega$ and t on heat profile when $\epsilon = 0.02$

the Concentration decreases with increase of all the parameters Schmidt number (Sc), Oscillating frequency (ω) and time (t).

**Nusselt Number:** The Nusselt number is depicted in Fig. 5. Fig.5 shows the effect of the parameters Pr,ω

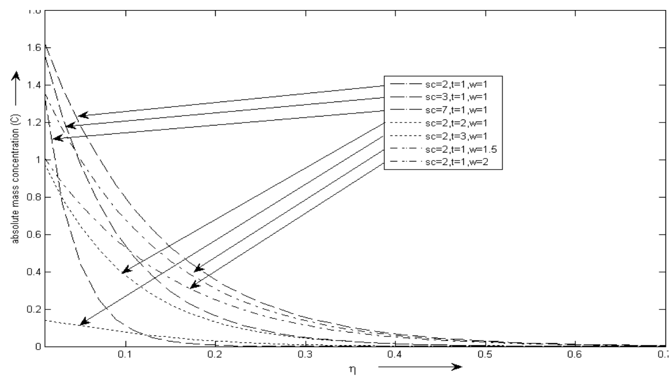


*Fig 4***.** Effect of Sc, $\omega$ and t on mass concentration profile when $\epsilon = 0.02$

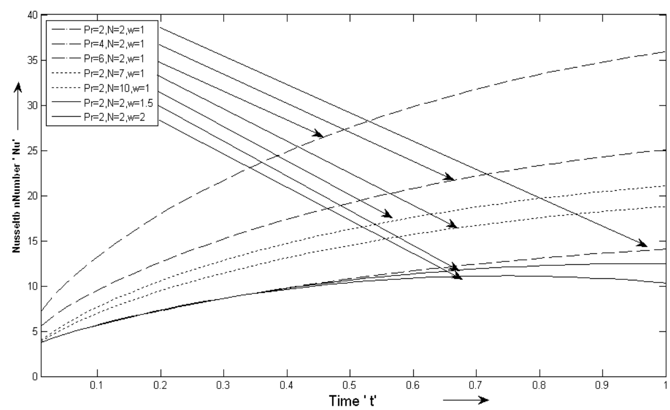


*Fig 5*. Effect of Pr, $\omega$ and N on Nusselt Number when $\epsilon = 0.02$

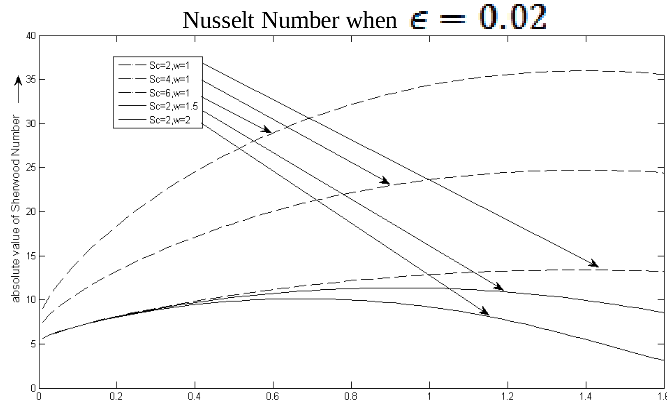


*Fig 6***.** Effect of Sc and $\omega$ on Sherwood Number when $\epsilon = 0.02$

and N on Nusselt number at any point of the fluid, when


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Indian J. Innovations Dev., Vol.1, No.6 (June 2012) ISSN 2277 - 5390

∈=0.02. It is noticed that the Nusselt number increases with the increase of the parameter Prandtl number (Pr) and Radiation parameter (N) but decreases with the Oscillating frequency(ω).

**Sherwood Number:** The Sherwood number is depicted in Fig. 6. Fig. 6 shows the effect of the parameters Sc and ω on Sherwood number at any point of the fluid, when ∈=0.02. It is noticed that the Sherwood number increases with the increase of the parameter Schmidt number (Sc), but decreases with the Oscillating frequency (ω).

**Shearing Stress:** The Shearing Stress is depicted in Fig 7 - 9. Fig. 7 shows the effect of the parameters Gr and N on Shearing Stress at any point of the fluid, when M=1, Sc=2, Pr=2, Gm=5, ∈=0.02 and ω=1. It is noticed that the Shearing Stress increases with the increase of the parameter Grashof number (Gr), but decreases with the Radiation parameter (N).

Fig. 8 shows the effect of the parameters ω and M on Shearing Stress at any point of the fluid, when Gr=5, Sc=2, Pr=2, Gm=5, ∈=0.02 and N=1 . It is noticed that the Shearing Stress decreases with decrease of the parameter Oscillating frequency(ω), but increases with the Magnetic number(M). Figure-(9) shows effect of the parameters Sc, Pr and Gm on Shearing Stress at any point of the fluid, when Gr=5, ω=1, N=1, ∈=0.02 and M=1. Shearing Stress has no effect. For the parameters Schmidt number (Sc) & modified Grashof number (Gm).

**Sherwood Number:** The Sherwood number is depicted in Fig. 6. Fig. 6 shows the effect of the parameters Sc and ω on Sherwood number at any point of the fluid, when ∈=0.02. It is noticed that the Sherwood number increases with the increase of the parameter Schmidt number (Sc) ,but decreases with the Oscillating frequency(ω).

**Shearing Stress:** The Shearing Stress is depicted in Figures 7-9. Fig. 7 shows the effect of the parameters Gr and N on Shearing Stress at any point of the fluid, when M=1, Sc=2, Pr=2, Gm=5, ∈=0.02 and ω=1. It is noticed that the Shearing Stress increases with the increase of the parameter Grash of number (Gr), but decreases with the Radiation parameter (N).

Fig. 8 shows the effect of the parameters ω and M on Shearing Stress at any point of the fluid, when Gr=5, Sc=2,Pr=2,Gm=5, ∈=0.02 and N=1 . It is noticed that the Shearing Stress decreases with decrease of the parameter Oscillating frequency(ω), but increases with the Magnetic number(M).

Fig. 9 shows effect of the parameters Sc, Pr and Gm on Shearing Stress at any point of the fluid, when Gr=5, ω=1, N=1, ∈=0.02 and M=1. Shearing Stress has no effect. For the parameters Schmidt number (Sc) & modified Grashof number (Gm).

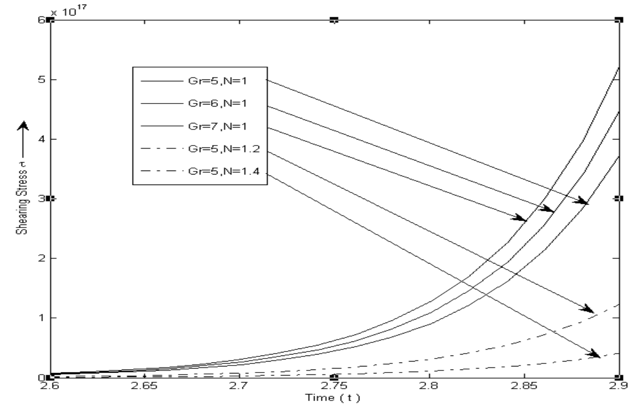


*Fig 7*. Effect of Gr and N on shearing stress when M=1,Sc=2,Pr=2,Gm=5 , $\epsilon = 0.02$ and ω=1

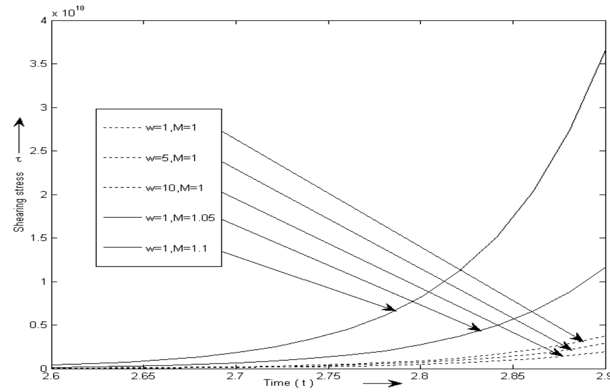


*Fig 8*. Effect of ω and M on shearing stress when Gr=5,Sc=2,Pr=2,Gm=5 , $\epsilon = 0.02$ and N=1

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