

Scientific Workplace Software (SWPS) in Computation of a Truncated Distribution

Osowole, I. Oyedeji

Department of Statistics, University of Ibadan, Nigeria
dosowole@yahoo.com

Abstract

Truncated distributions have been used in numerous industrial settings and in many practical situations. This study, using the Scientific Workplace software (SWPS), highlights an algorithm for computing the variance of any given truncated distribution with particular reference to the standard normal density function. The efficiency of the truncated standard normal density is found to improve according to the inverse values of the upper and lower truncation points. SWPS exhibits user friendliness, flexibility and speedy implementation of truncation problems.

Keywords: Truncated distribution, Screening inspection, Tolerance limits, Multistage production process, Scientific workplace software

1. Introduction

In a variety of desired applications the mean and variance of distributions are truncated in various ways [1]. End goods are subject to checking before being shipped to the buyer. In normal practice, a product can be judged compliant if it is within specified tolerance limits, and consequently shipped to the buyer. If it fails, the item is discarded, thrown away or possibly reworked; resulting in truncated original shipping to the buyer. A further illustration is in a multistage production process, where examination is conducted at every phase of production. If only compliant items are passed on to the next stage, these result in a truncated distribution [2]. Likewise, if dates of birth of kids enlisted in a school are desired, these would normally be liable to truncation with respect to all kids in that zone, as the school registers on a particular date in stipulated age range. Authentic information on birth dates of youngsters in the territory, before or after the school's cut-off dates, might not be accessible [3]. Measured traffic from three locations on a state-of-the-art Ethernet has equally been shown to be several truncated distributions [4].

Censoring is sampling conducted to record information about items, existing outside the specified limit of real values [5]. Several authors who have contributed to the study of truncated distributions include [6, 7, 8, 9, 10, 11, 12, 13].

Since evaluation of moments of truncated distributions usually involves manual painstaking efforts, the objective is therefore to highlight the use of the "Scientific Workplace Software (SWPS)" to handle truncated versions of any distribution with particular reference to variance determinations of distributions. The variance is chosen because of its usefulness in the assessment of precision levels of a given function. The algorithm presented is illustrated with the use of the Standard Normal density function.

2. The Variance of a Truncated Distribution

Following the work of Saralees Nadarajah and Samuel Kotz [2], let Y^* be a random variable representing the truncated version of a continuous variable Y with probability density function (p.d.f) and cumulative distribution function (c.d.f) specified by $f(\cdot)$ and $F(\cdot)$, respectively over the interval $[a, b]$, where $-\infty < a < b < \infty$. The p.d.f, c.d.f, mean, and variance of Y^* are given by

$$g_{Y^*}(y^*) = \begin{cases} \frac{f(y)}{F(b) - F(a)}, & \text{if } a \leq y \leq b \\ 0, & \text{otherwise} \end{cases} \quad (2.1)$$

$$G_{Y^*}(y^*) = \frac{\int_a^y f(t)dt}{F(b) - F(a)} = \frac{F(\max(\min(y, b), a)) - F(a)}{F(b) - F(a)} \tag{2.2}$$

$$\text{Mean}(Y^*) = E(Y^*) = \int_a^b y^* g_{Y^*}(y^*) dy = \frac{\int_a^b y f(y) dy}{F(b) - F(a)} \tag{2.3}$$

$$\text{Variance}(Y^*) = V(Y^*) = \int_a^b [y - E(y)]^2 g_{Y^*}(y^*) dy = \frac{\int_a^b [y - E(y)]^2 f(y) dy}{F(b) - F(a)} \tag{2.4}$$

3. Evaluating the Variance of the Truncated Standard Normal Distribution using Scientific Work- place (SWPS)

The following is the algorithm in SWPS for computing (2.4):

- Click the toggle math- text ‘Math/Text’ button to switch to “M”, the mathematics option
- Click the integration function “∫” button to initiate the integration procedure
- Click the “N_x” button to enter the lower limit for the integration
- Type “a”-the lower limit(say)
- Click the “N^x” button to enter the upper limit for the integration
- Type “b”-the upper limit(say)
- Enter completely the function to be integrated
- Place the insertion point to the right of the function to be integrated
- From the maple menu drop down box, choose evaluate

4. Numerical Illustration

Computation of the variance of the standard normal distribution truncated at different points of a and b will be considered. The choice of the standard normal distribution is based on the fact that the parameters are easily specified as μ = 0 and σ² = 1 respectively. The different values of a and b (where a < b) to be considered are shown in the table below:

Table.1 Selected Truncation Values for the Standard Normal Density

| | | | | | | | | | |
|----------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| a | -3.00 | -2.75 | -2.50 | -2.25 | -2.00 | -1.75 | -1.50 | -1.25 | -1.00 |
| b | 3.00 | 2.75 | 2.50 | 2.25 | 2.00 | 1.75 | 1.50 | 1.25 | 1.00 |

To evaluate the variance of the Standard Normal density at the different truncation points, the following are example entries for some of the variances

$$\frac{\int_{-3}^3 [y - E(y)]^2 \frac{e^{-\frac{y^2}{2}}}{\sqrt{2\pi}} dy}{F(3) - F(-3)}, \quad \frac{\int_{-2.75}^3 [y - E(y)]^2 \frac{e^{-\frac{y^2}{2}}}{\sqrt{2\pi}} dy}{F(3) - F(-2.75)}, \quad \frac{\int_{-2.50}^3 [y - E(y)]^2 \frac{e^{-\frac{y^2}{2}}}{\sqrt{2\pi}} dy}{F(3) - F(-2.50)}, \quad \frac{\int_{-2}^3 [y - E(y)]^2 \frac{e^{-\frac{y^2}{2}}}{\sqrt{2\pi}} dy}{F(3) - F(-2)}$$

$$\frac{\int_{1.75}^3 [y - E(y)] \frac{1}{\sqrt{\pi}} dy}{F(3) - F(-1.75)}, \frac{\int_{-1.50}^3 [y - E(y)]^2 \frac{e^{-\frac{y^2}{2}}}{\sqrt{2\pi}} dy}{F(3) - F(-1.50)}, \frac{\int_{-1.25}^3 [y - E(y)]^2 \frac{e^{-\frac{y^2}{2}}}{\sqrt{2\pi}} dy}{F(3) - F(-1.25)} \text{ and } \frac{\int_{-1}^3 [y - E(y)]^2 \frac{e^{-\frac{y^2}{2}}}{\sqrt{2\pi}} dy}{F(3) - F(-1)}$$

respectively.

5. Result and Discussion

The table below shows the variance of the Standard Normal density for different truncation values.

Table. 2 Variance of the Standard Normal Density at different truncation points

| b | a | | | | | | | | |
|------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| | -3.00 | -2.75 | -2.50 | -2.25 | -2.00 | -1.75 | -1.50 | -1.25 | -1.00 |
| 3.00 | 0.97334 | 0.96153 | 0.94244 | 0.91412 | 0.87572 | 0.82862 | 0.77725 | 0.72944 | 0.69612 |
| 2.75 | 0.96153 | 0.94969 | 0.93053 | 0.90208 | 0.86350 | 0.81609 | 0.76427 | 0.71581 | 0.68156 |
| 2.50 | 0.94244 | 0.93053 | 0.91126 | 0.88260 | 0.84367 | 0.79575 | 0.74316 | 0.69360 | 0.65778 |
| 2.25 | 0.91412 | 0.90208 | 0.88260 | 0.85359 | 0.81410 | 0.76533 | 0.71151 | 0.66022 | 0.62203 |
| 2.00 | 0.87572 | 0.86350 | 0.84367 | 0.81410 | 0.77375 | 0.72367 | 0.66801 | 0.61417 | 0.57250 |
| 1.75 | 0.82862 | 0.81609 | 0.79575 | 0.76533 | 0.72367 | 0.67173 | 0.61343 | 0.55601 | 0.50959 |
| 1.50 | 0.77725 | 0.76427 | 0.74316 | 0.71151 | 0.66801 | 0.61343 | 0.55152 | 0.48935 | 0.43676 |
| 1.25 | 0.72944 | 0.71581 | 0.69360 | 0.66022 | 0.61417 | 0.55601 | 0.48935 | 0.42105 | 0.36076 |
| 1.00 | 0.69612 | 0.68156 | 0.65778 | 0.62203 | 0.57250 | 0.50959 | 0.43676 | 0.36076 | 0.29113 |

From the table above, the highest variance was obtained when a = -3 and b = 3 while the least was obtained when a = -1 and b = 1. The variance was approximately 50% for the following truncation pairs a = -1.75, b =1.00; a= -1.50, b=1.25; a= -1.00, b=1.75 and a = -1.25, b=1.50 respectively.

6. Conclusion

The variance of the truncated standard normal density has been shown to reduce as b (upper truncation point) is reducing and as a (lower truncation point) is increasing. The highest precision point is therefore obtained when a = -3 and b = 3. The implementation of the algorithm provided in estimating the variance of the truncated standard normal density for different truncation pairs using SWPS reveals the general flexibility, user friendliness and simplicity of the software to handle truncation problems.

7. Acknowledgement

The author is grateful to the Economic Research Unit, Indian Statistical Institute, Kolkata under the headship of Prof. Manoranjal Pal for offering him a place under the Post Doctoral Study platform. This paper was written while the author is still at the institute.

8. References

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