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Stochastic Stability Analysis for Networked Markov Jump System

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In the paper, we describe the development and implementation of reliable H_∞filters for a class of Networked Markov Jump Systems (NMJS) with random sensor failures that are triggered by events. The plant's nonlinear dynamic is approximated with a NMJS. Failures of sensors are described using stochastic variables. The Event-Triggered Mechanism (ETM) is introduced to NCS, which offers some positive points over other schemes. Using the event-triggered mechanism, data of sensors from the plant will be only transmitted if it contradicts the specified condition. By considering the effects of an ETM and the sensor faults, the event-based filter is developed for NMJS. The design parameters of the filter as well as sufficient conditions for its existence are given accurately based on Linear Matrix Inequality (LMI).

Keywords: ETM, LMI, Networked systems, NMJS, Stochastic filters

Introduction

Over the past few decades, significant results has been achieved by researchers and engineers in *Markov Jump Systems (MJSs)* because of its wide applications in practice.[1–7] Observer-based controller for *MJSs* has been widely investigated such as finite-time stabilization, H_∞ control, etc.[1–4] Besides this, a scheme of asynchronous H_∞ fault detection filter under discrete homogeneous Markov jump linear systems has also gained popularity.[5] The control of a asynchronous MJS has also been the interest of engineers in modeling.[6] The Hidden Markov Model (HMM) has been introduced and the stability of systems is proved by adopting the composite stochastic Lyapunov function method. The issue of finite–time blended H_∞ and asynchronous filter can be investigated for singular *MJS* based T-S fuzzy model, in which a scheme based on dynamic *ETM* is utilized to reduce the total transmission of sampled signals in the network.[7] Hence, it is necessary to investigate the properties of a networked Markov Jump system with nonlinear effects.

Because of the highly nonlinear characteristics of the most systems in the real world, the method that can effectively control them, such as nonlinear control which transforms the nonlinear systems which have high complexity to a series of linear systems through fuzzy rules, have attracted wide attention of researchers.[8–13] Investigations on the design of T-S fuzzy model based with improved stability condition of continuous T-S fuzzy system are also presented in the literature available.[10] In published literature, researchers discussed the issue of Stabilization Control of continuous nonlinear systems with time delay. While investigators focused on the design of controllers for continuous *MIMO* nonlinear systems.[11]

With the rapid development of network technology and its wide application in real life, Networked Control Systems (NCSs) raised great attention which caused new control problems at the same time, such as data packet dropouts and delays caused by the unstable factor of network, security problems that are vulnerable to malicious attacks due to the openness of the network, and others. So many researchers have done a lot of work on these problems in the past 10 years. For example, The authors addressed the problems of delay and data loss by predicting the system dynamics in a limited range. [14,15] The authors have contributed to security control, in which a mechanism that based on an adaptive *ETM* is utilized to lighten the burden of transmission in the network and a model is established considering the *NCSs* under deception attacks.[16] Besides, robust control has been widely employed to control the *NCSs*.[17–19] Among them, the most significant differences are the control of time-delay NCSs using a nonlinear model and the robust stabilization of a class of nonlinear NCSs without accounting for delays.[18,19]

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In the H_∞ control process, controller adopts H_∞ norm as the control performance index, is one of the most essential and used methods in the area of robust control, which aims at finding a controller that minimizing the H_∞ norm, and is also an optimal control method. Therefore, H_∞ control theory has been applied to various systems, such as singular systems, *NCSs*, stochastic systems and so on.[20–24] For example, as to switched systems with unknown signal disturbances, a controller is introduced to solve the H_∞ finite-time control issue in.[25] The authors utilized a new method of finite--time adaptive H_∞ control, which ensure the finite-time boundedness while meeting H_∞ level. This factor is also taken into account in this study.[26]

This article makes the following contributions:

1. New stochastic jump systems considering sensor faults are proposed that employ event-triggered schemes, which are not discussed in the published literatures.
2. In terms of *LMI*, the H_∞ filter must exist for sufficient conditions to be satisfied. Event generators and filtering can be designed together using these conditions.

Method of system modeling

Let consider the following *NMJS*:

$$\begin{cases}\dot{p}(t) = \boldsymbol{A}_{\sigma(t)}p(t) + \boldsymbol{A}_{\eta\sigma(t)}p(t-\eta(t)) + \boldsymbol{B}_{\omega\sigma(t)}\omega(t)\\ y(t) = \boldsymbol{C}_{\sigma(t)}p(t)\\ z(t) = \boldsymbol{L}_{\sigma(t)}p(t)\end{cases} \quad \ldots (1)$$

where,

$p(t) \in \mathbb{R}^n$, presents the state vector of the nonlinear plant.

$y(t) \in \mathbb{R}^m$, denotes the output vector of the model.

$z(t) \in \mathbb{R}^p$describes the estimated signal of system.

$\boldsymbol{A}_{\sigma(t)}, \boldsymbol{A}_{\eta\sigma(t)}, \boldsymbol{B}_{\omega\sigma(t)}, \boldsymbol{C}_{\sigma(t)}and\boldsymbol{L}_{\sigma(t)}$are belongs to parameter matrices with proper dimensions; $\omega(t) \in \boldsymbol{L}_2[0,\infty)$ presents the disturbance sign; A time-varying delay $\eta(t)$ is a series of values that represent value changes over the domain $[\eta_m, \eta_M]$, where η_m and η_M are non-negative scalars. Within a set $\boldsymbol{M} = 1,2,\cdots,\boldsymbol{N}$, $\sigma(t)$ represents a homogeneous Markov-jump process with finite states, then follows the probability matrix of transitions $\partial = \{\pi_{\sigma\mathrm{m}}\}$ which yields to:

$$\boldsymbol{P}_r\{\sigma(\mathfrak{k} + \nabla\mathfrak{k}) = \mathfrak{q} | \sigma(\mathfrak{k}) = \mathfrak{r}\} = \begin{cases}\pi_{\mathfrak{rq}}\nabla\mathfrak{k} + \emptyset(\nabla\mathfrak{k}), & \mathfrak{r} \neq \mathfrak{q}\\ 1 + \pi_{\mathfrak{rr}}\nabla\mathfrak{k} + \emptyset(\nabla\mathfrak{k}), & \mathfrak{r} = \mathfrak{q}\end{cases} \quad \ldots (2)$$

where,

$$lim_{\nabla\mathfrak{k}\to 0}\left(\frac{\emptyset(\nabla\mathfrak{k})}{\nabla\mathfrak{k}}\right) = 0, \quad \nabla\mathfrak{k} > 0 \quad \ldots (3)$$

$\pi_{\mathfrak{rq}} \geq 0$is the transition probability at time$(\mathfrak{k} + \nabla\mathfrak{k})$ from the mode$\mathfrak{r}$to$\mathfrak{q}$ if$\mathfrak{r} \neq \mathfrak{q}and\pi_{\mathfrak{rr}} = -\sum_{\mathfrak{q}\in\boldsymbol{M},\mathfrak{q}\neq\mathfrak{r}}\pi_{\mathfrak{rq}}$. Utilizing $\sigma(t) = i$ for simplicity. In the next section, authors present the general form of markov filter design.

Markov Filter Design

We propose a stochastic filter that has a H_∞ index, which is presented as follows:

$$\begin{cases}\dot{p}_f(t) = \mathbb{A}_{f\sigma(t)}p_f(t) + \mathbb{B}_{f\sigma(t)}\hat{y}(t)\\ z_f(t) = \mathbb{C}_{f\sigma(t)}p_f(t)\end{cases} \quad \ldots (4)$$

where,

$z_f(t) \in \mathbb{R}^p$, describes the estimated signal of filter.

$\hat{y}(t) \in \mathbb{R}^m$, denotes the original input.

$p_f(t) \in \mathbb{R}^n$, presents the state vector of the nonlinear filter.

where, $\mathbb{A}_{f\sigma(t)} \in \mathbb{R}^{n\times n}$, $\mathbb{B}_{f\sigma(t)} \in \mathbb{R}^{n\times m}$, $\mathbb{C}_{f\sigma(t)} \in \mathbb{R}^{p\times n}$ are to be calculated.

We assume the network communication takes place with a time varying delayμ_k, where$\mu_k \in [0,\bar{\mu})$, and the real number$\bar{\mu}$is greater than 0. As a result, we have the sensor measurements from the sample $y(i_0\hbar), y(i_1\hbar), y(i_2\hbar),\cdots$ will reached at the filter node at the instants

$$i_0\hbar + \mu_0, i_1\hbar + \mu_1, i_2\hbar + \mu_2,\cdots.$$

For the convince of the reader, now we denote the $\mathbb{A}_{f\sigma(t)} = \mathbb{A}_{fi}$, $\mathbb{B}_{f\sigma(t)} = \mathbb{B}_{fi}$ and so on. In Eq. (4), $\hat{y}(t)$ can be explained as:

$$\hat{y}(t) = \boldsymbol{C}_i p(i_k\hbar),\quad t \in [i_k\hbar + \mu_{i_k}, i_{k+1}\hbar + \mu_{i_{k+1}}] \quad \ldots (5)$$

where, $\hbar$ presents the sampling period,

$i_k \in \{1,2,3,\cdots\}$. At the time of transmission $i_k\hbar$and $i_{k+1}\hbar$, and $\mu_{i_k}$and$\mu_{i_{k+1}}$ are the delays caused by the network at the transmission instant. The following (5) can be rewritten considering the possibility of sensor failure:

$$\hat{y}(t) = \exists\boldsymbol{C}_i p(i_k\hbar) = \exists_\ell \boldsymbol{E}_\ell \boldsymbol{C}_i p(i_k\hbar),\quad t \in [i_k\hbar + \mu_{i_k}, i_{k+1}\hbar + \mu_{i_{k+1}}] \quad \ldots (6)$$

For more detail of sensor fault, readers can refer to Liu & Yue.[27] In order to simplify the network traffic, an event-triggered mechanism should be introduced that determines whether data which is currently sampled should be transmitted to the filter or not. The periodic sampling mechanism is known to send unnecessarily large amounts of data, which reduces bandwidth utilization. We propose a sensor-filter system with an event generator. The smart sensor sampler, which will be explained in a sequel, samples the sensor measurements on a regular basis. The decision-making algorithm is as follows:

$$[\mathscr{E}\{\bar{\exists}y((i+j)\hbar)\}-\mathscr{E}\{\bar{\exists}y(i\hbar)\}]^T\mho[\mathscr{E}\{\bar{\exists}y((i+j)\hbar)\} - \mathscr{E}\{\bar{\exists}y(i\hbar)\}] \leq \nu[\mathscr{E}\{\bar{\exists}y((i+j)\hbar)\}]^T\mho[\mathscr{E}\{\bar{\exists}y((i+j)\hbar)\}] \quad \ldots (7)$$

where, $\mho$ denotes the a symmetric non-negative definite matrix, $j=1,2,\cdots$, and$\nu\in(0,1)$. In order to be sent to the filter from the event generator, the current sampled data must vary by the specified threshold (7). Let's look at two examples for technical convenience:

Case i: If$i_k\hbar+\hbar+\bar{\mu}\geq i_{k+1}\hbar+\mu_{k+1}$, where $\bar{\mu}=\max\mu_k$, define$\mu(t)$ as follows:

$$\mu(t)=i-i_k\hbar, \quad t\in[i_k\hbar+\mu_k, i_{k+1}\hbar+\mu_{k+1}] \quad \ldots (8)$$

The following information is easily obtainable:

$$\mu_k\leq\mu(t)\leq(i_{k+1}-i_k)\hbar+\mu_{k+1}\leq\hbar+\bar{\mu} \quad \ldots (9)$$

Case ii: If$i_k\hbar+\hbar+\bar{\mu}\leq i_{k+1}\hbar+\mu_{k+1}$, t the following intervals into consideration:

$$[i_k\hbar+\mu_k, i_k\hbar+\hbar+\bar{\mu}], \quad [i_k\hbar+i\hbar+\bar{\mu}, i_k\hbar+i\hbar+\hbar+\bar{\mu}) \quad \ldots (10)$$

In view of$\mu_k\leq\bar{\mu}$, it is easy to prove the existence of a positive integer$\Theta_M$which satisfies

$$i_k\hbar+\Theta_M\hbar+\bar{\mu}<i_{k+1}\hbar+\mu_{k+1}\leq i_k\hbar+\delta_M\hbar+\hbar+\bar{\mu} \quad \ldots (3)$$

Therefore, $p(i_k\hbar)$ and $i_k\hbar+i\hbar$ with $i=1,2,\cdots,\Theta_M$ fulfill (7). Let

$$\begin{cases}\mathcal{G}_0=[i_k\hbar+\mu_k, i_k\hbar+\hbar+\bar{\mu})\\ \mathcal{G}_i=[i_k\hbar+i\hbar+\bar{\mu}, \quad i_k\hbar+i\hbar+\hbar+\bar{\mu})\\ \mathcal{G}_{\tau_m}=[i_k\hbar+\Theta_M\hbar+\bar{\mu}, i_{k+1}\hbar+\mu_{k+1})\end{cases} \quad \ldots (12)$$

where, $i=1,2,\cdots,\Theta_M-1$. The following can easily be demonstrated:

$$[i_k\hbar+\mu_k, i_{k+1}\hbar+\mu_{k+1})=\bigcup_{i=0}^{i=\Theta_M}\mathcal{G}_i \quad \ldots (13)$$

Let suppose:

$$\mu(t)=\begin{cases}i-i_k\hbar, & t\in\mathcal{G}_0\\ i-(i_k+i)\hbar, & t\in\mathcal{G}_i, i=1,2,\cdots,\Theta_M-1\\ i-i_k\hbar-\Theta_M\hbar, & i\in\mathcal{G}_{\Theta_M}\end{cases} \quad \ldots (14)$$

We can derive the following from the definition of $\mu(t)$:

$$\begin{cases}i_k\leq\mu(t)<\hbar+\bar{\mu}, & t\in\mathcal{G}_0\\ i_k\leq\bar{\mu}\leq\mu(t)<\hbar+\bar{\mu}, & t\in\mathcal{G}_i, i=1,2,\cdots,\Theta_M-1\\ i_k\leq\bar{\mu}\leq\mu(t)<\hbar+\bar{\mu}, & t\in\mathcal{G}_{\Theta_M}\end{cases} \quad \ldots (15)$$

One can observe that in the 3^{rd} row in (15) satisfies on account of $i_{k+1}\hbar+\mu_{k+1}\leq i_k\hbar+(\mu_M+1)\hbar+\bar{\mu}$.

$$0\leq\mu_k\leq\mu(t)\leq\hbar+\bar{\mu}\triangleq\mu_M, \quad i\in[i_k\hbar+\mu_k, i_{k+1}\hbar+\mu_{k+1}) \quad \ldots (16)$$

From the *Case i*, define$e_k(t)=0$. In the same consequences, from the *Case ii*, measure the mathematical error that will occur between the latest transmission moment and the current sampling moment.

$$\exists e_k(t)=\begin{cases}0, & t\in\mathcal{G}_0\\ \bar{\exists}y(i_k\hbar)-\bar{\exists}y(i_k\hbar+i\hbar), & t\in\mathcal{G}_i, i=1,2,\cdots,\Theta_M-1\\ \bar{\exists}y(i_k\hbar)-\bar{\exists}y(i_k\hbar+\Theta_M\hbar), & i\in\mathcal{G}_{\Theta_M}\end{cases} \quad \ldots (17)$$

Using the concept of (7), we can get:

$$y(i_k\hbar)=\exists\boldsymbol{C}_i p(i_k\hbar) \quad \ldots (18)$$

Taking (6) and (17) and combining (18) and (15), the filter input is as follows:

$$\hat{y}(t)=\bar{\exists}\boldsymbol{C}_i p(t-\mu(t))+\bar{\exists}e_k(t)+(\exists-\bar{\exists})\boldsymbol{C}_i p(t-\mu(t)), \quad t\in[i_k\hbar+\mu_k, i_{k+1}\hbar+\mu_{k+1}) \quad \ldots (19)$$

Let assume$\lambda(t)=\begin{bmatrix}p(t)\\ p_f(t)\end{bmatrix}$, $\tilde{z}(t)=z(t)-z_f(t)$, a filtering-error system can be combined to the following form:

$$\begin{cases}\dot{\lambda}(t)=\overline{\boldsymbol{A}}\lambda(t)+\overline{\boldsymbol{A}}_\eta\mathscr{H}\lambda(t-\eta(t))+\overline{\boldsymbol{B}}\mathscr{H}\lambda(t-\mu(t))+\overline{\boldsymbol{B}}_1 e_k(t)\\ \quad +\overline{\boldsymbol{A}}_\omega\omega(t)+\overline{\boldsymbol{B}}_t\mathscr{H}\lambda(t-\mu(t))\\ \tilde{z}(t)=\overline{\boldsymbol{L}}\lambda(t)\end{cases} \quad \ldots (20)$$

where,

$$\overline{\boldsymbol{A}}=\begin{bmatrix}\boldsymbol{A}_i & 0\\ 0 & \mathbb{A}_{fi}\end{bmatrix}, \quad \overline{\boldsymbol{A}}_\eta=\begin{bmatrix}\boldsymbol{A}_{\eta i}\\ 0\end{bmatrix},$$

$$\overline{\boldsymbol{B}}=\begin{bmatrix}0\\ \mathbb{B}_{fi}\exists\boldsymbol{C}_i\end{bmatrix}, \quad \overline{\boldsymbol{B}}_1=\begin{bmatrix}0\\ \mathbb{B}_{fi}\bar{\exists}\end{bmatrix},$$

$$\overline{\boldsymbol{A}}_\omega=\begin{bmatrix}\boldsymbol{A}_{\omega i}\\ 0\end{bmatrix}, \quad \overline{\boldsymbol{B}}_t=\begin{bmatrix}0\\ \mathbb{B}_{fi}(\exists-\bar{\exists})\boldsymbol{C}_i\end{bmatrix}$$

$$\overline{\boldsymbol{L}}=[\boldsymbol{L}_i \quad -\mathbb{C}_{fi}], \quad \mathscr{H}=[\mathscr{J} \quad 0]$$

Results

We will present the analysis for the non-linear networked systems Eq. (20) with time-varying delays using the given filter which satisfies Eq. (4). Next, we will be discussing the filter design problem in the system.

Corollary 1 For given positive scalars γ, ρ, η_m,η_M, andμ_M plant (20) is stochastically stable with the H∞ performance indexγ with the triggered conditions (7) if there exist matrices $\boldsymbol{P} > 0$,$\boldsymbol{Q}_\ell > 0$,$\boldsymbol{R}_\ell > 0 (\ell = 1,2,3)$, and $\boldsymbol{S}_{ij}$, $\boldsymbol{T}_{ij}$,$\boldsymbol{N}_{ij}$ and M_{ij}, with proper dimensions satisfying

$$\Gamma^{ij} + \Gamma^{ji} < 0, \quad i \le j \qquad \dots (21)$$

where,

$$\Gamma^{ij} = \begin{bmatrix} \Phi_{11}^{ij} & \Phi_{21}^{ij} & \Phi_{31}^{ij} & \Phi_{41}^{ij}(s) \\ ¶ & \Phi_{22}^{ij} & 0 & 0 \\ ¶ & ¶ & \Phi_{33}^{ij} & 0 \\ ¶ & ¶ & ¶ & \Phi_{44}^{ij} \end{bmatrix}$$

$(n = 1,2,3,4)$

$$\Phi_{11}^{ij} = \begin{bmatrix} \Phi_{ij1} & ¶ & ¶ & ¶ & ¶ & ¶ & ¶ & ¶ \\ \mathbf{R}_2 & \Phi_{ij2} & ¶ & ¶ & ¶ & ¶ & ¶ & ¶ \\ \mathbf{H}^T\overline{\mathbf{A}}_\mu^T\mathbf{P} & \mathbf{M}_{ij3} - \mathbf{M}_{ij2}^T & \Phi_{ij3} & ¶ & ¶ & ¶ & ¶ & ¶ \\ 0 & 0 & \mathbf{N}_{ij4} - \mathbf{N}_{ij3}^T & \Phi_{ij4} & ¶ & ¶ & ¶ & ¶ \\ \mathbf{H}^T\overline{\mathbf{B}}^T\mathbf{P} + \mathbf{T}_{ij5} - \mathbf{T}_{ij1}^T & 0 & 0 & 0 & \Phi_{ij5} & ¶ & ¶ & ¶ \\ 0 & 0 & 0 & 0 & \boldsymbol{S}_{ij6} - \boldsymbol{S}_{ij5}^T & \Phi_{ij6} & ¶ & ¶ \\ \overline{\mathbf{B}}_1^T\mathbf{P} & 0 & 0 & 0 & 0 & 0 & -\exists^T\mho\exists & ¶ \\ \overline{\mathbf{A}}_\omega^T\mathbf{P} & 0 & 0 & 0 & 0 & 0 & 0 & -\gamma^2 I \end{bmatrix} \qquad \dots (22)$$

$$\Phi_{ij1} = \boldsymbol{P}\overline{\boldsymbol{A}} + \overline{\boldsymbol{A}}^T\boldsymbol{P} + \boldsymbol{Q}_1 + \boldsymbol{Q}_2 + \boldsymbol{Q}_3 - \boldsymbol{R}_2 + \boldsymbol{T}_{ij1} + \boldsymbol{T}_{ij1}^T$$

$$\Phi_{ij2} = -\boldsymbol{Q}_1 - \boldsymbol{R}_2 + \boldsymbol{M}_{ij2} + \boldsymbol{M}_{ij2}^T, \quad \Phi_{ij3} = -\boldsymbol{M}_{ij3} - \boldsymbol{M}_{ij3}^T + \boldsymbol{N}_{ij3} + \boldsymbol{N}_{ij3}^T$$

$$\Phi_{ij4} = -\boldsymbol{Q}_2 - \boldsymbol{N}_{ij4} - \boldsymbol{N}_{ij4}^T, \quad \Phi_{ij5} = \rho\mathscr{H}^T\boldsymbol{C}_i^T\exists^T\mho\exists\boldsymbol{C}_i\mathscr{H} - \boldsymbol{T}_{ij5} - \boldsymbol{N}_{ij5}^T + \boldsymbol{S}_{ij5} + \boldsymbol{S}_{ij5}^T$$

$$\Phi_{ij6} = -\boldsymbol{Q}_3 - \boldsymbol{S}_{ij6} - \boldsymbol{S}_{ij6}^T$$

$$\Phi_{21}^{ij} = \begin{bmatrix} \bar{L} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \sqrt{\eta_{ab}}P\bar{A} & 0 & \sqrt{\eta_{ab}}P\bar{A}_\mu\mathscr{H} & 0 & \sqrt{\eta_{ab}}P\bar{B}\mathscr{H} & 0 & \sqrt{\eta_{ab}}P\bar{B}_1 & \sqrt{\eta_{ab}}P\bar{A}_\omega \\ \eta_m P\bar{A} & 0 & \eta_m P\bar{A}_\mu\mathscr{H} & 0 & \eta_m P\bar{B}\mathscr{H} & 0 & \eta_m P\bar{B}_1 & \eta_m P\bar{A}_\omega \\ \sqrt{\mu_M}P\bar{A} & 0 & \sqrt{\mu_M}P\bar{A}_\mu\mathscr{H} & 0 & \sqrt{\mu_M}P\bar{B}\mathscr{H} & 0 & \sqrt{\mu_M}P\bar{B}_1 & \sqrt{\mu_M}P\bar{A}_\omega \end{bmatrix}$$

$$\Phi_{22}^{ij} = diag\{-I, -PR_1^{-1}P, -PR_2^{-1}P, -PR_3^{-1}P\},$$

$$\Phi_{33}^{ij} = diag\{\Re_1, \Re_2, \Re_3\}$$

$$\Re_k = diag\{\underbrace{-PR_k^{-1}P, \cdots, -PR_k^{-1}P}_{m}\}, \quad k = 1,2,3$$

$$\Phi_{44}^{ij} = diag\{-\boldsymbol{R}_1, -\boldsymbol{R}_3\},$$

$$\sqrt{\eta_{ab}} = \sqrt{\eta_M - \eta_m}$$

$$\Phi_{31}^{ij}(1) = \begin{bmatrix} 0 & 0 & 0 & 0 & \check{D}_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \check{D}_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \check{D}_3 & 0 & 0 & 0 \end{bmatrix},$$

$$\check{D}_1 = \begin{bmatrix} \sqrt{\eta_{21}}\delta_1 P\widehat{D}_1\mathscr{H} \\ \vdots \\ \sqrt{\eta_{21}}\delta_m P\widehat{D}_m\mathscr{H} \end{bmatrix}$$

$$\check{D}_2 = \begin{bmatrix} \eta_m\delta_1 P\widehat{D}_1\mathscr{H} \\ \vdots \\ \eta_m\delta_m P\widehat{D}_m\mathscr{H} \end{bmatrix},$$

$$\check{D}_3 = \begin{bmatrix} \sqrt{\mu_M}\delta_1 P\widehat{D}_1\mathscr{H} \\ \vdots \\ \sqrt{\mu_M}\delta_m P\widehat{D}_m\mathscr{H} \end{bmatrix}$$

$$\widehat{D}_l = \begin{bmatrix} 0 \\ \mathbb{B}_{fi}\boldsymbol{E}_\ell\boldsymbol{C}_i \end{bmatrix}, l = 1,2,\cdots,m$$

$$\Phi_{41}^{ij}(1) = \begin{bmatrix} \sqrt{\eta_{ab}}\boldsymbol{M}_{ij}^T \\ \sqrt{\mu_M}\boldsymbol{T}_{ij}^T \end{bmatrix}, \Phi_{41}^{ij}(2) = \begin{bmatrix} \sqrt{\eta_{ab}}\boldsymbol{M}_{ij}^T \\ \sqrt{\mu_M}\boldsymbol{S}_{ij}^T \end{bmatrix},$$

$$\Phi_{41}^{ij}(3) = \begin{bmatrix} \sqrt{\eta_{ab}}\boldsymbol{N}_{ij}^T \\ \sqrt{\mu_M}\boldsymbol{S}_{ij}^T \end{bmatrix}, \Phi_{41}^{ij}(4) = \begin{bmatrix} \sqrt{\eta_{ab}}\boldsymbol{N}_{ij}^T \\ \sqrt{\mu_M}\boldsymbol{T}_{ij}^T \end{bmatrix}$$

$$\boldsymbol{M}_{ij}^T = \begin{bmatrix} 0 & \boldsymbol{M}_{ij2}^T & \boldsymbol{M}_{ij3}^T & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\boldsymbol{N}_{ij}^T = \begin{bmatrix} 0 & 0 & \boldsymbol{N}_{ij3}^T & \boldsymbol{N}_{ij4}^T & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\boldsymbol{T}_{ij}^T = \begin{bmatrix} \boldsymbol{T}_{ij1}^T & 0 & 0 & 0 & \boldsymbol{T}_{ij5}^T & 0 & 0 & 0 \end{bmatrix},$$

$$\boldsymbol{S}_{ij}^T = \begin{bmatrix} 0 & 0 & 0 & 0 & \boldsymbol{S}_{ij5}^T & \boldsymbol{S}_{ij6}^T & 0 & 0 \end{bmatrix}$$

Proof: The following candidate for the Lyapunov functional is the right choice:

$$\mathcal{O}_1(t) = \lambda^T(t)\boldsymbol{P}\lambda(t)$$

$$\mathcal{O}_2(t) = \int_{t-\eta_m}^{t} \lambda^T(\phi)\boldsymbol{Q}_1\lambda(\phi)d\phi + \int_{t-\eta_M}^{t} \lambda^T(\phi)\boldsymbol{Q}_2\lambda(\phi)d\phi + \int_{t-\mu_M}^{t} \lambda^T(\phi)\boldsymbol{Q}_3\lambda(\phi)d\phi$$

$$\mathcal{O}_3(t) = \int_{t-\eta_M}^{t-\eta_m}\int_{\phi}^{t} \dot{\lambda}^T(\phi)\boldsymbol{R}_1\dot{\lambda}(v)dvd\phi + \eta_m\int_{t-\eta_m}^{t}\int_{\phi}^{t} \dot{\lambda}^T(v)\boldsymbol{R}_2\dot{\lambda}(v)dvd\phi + \int_{t-\mu_M}^{t}\int_{\phi}^{t} \dot{\lambda}^T(v)\boldsymbol{R}_3\dot{\lambda}(v)dvd\phi$$

Notice

$$-\eta_m\int_{t-\eta_m}^{t} \dot{\lambda}^T(s)\boldsymbol{R}_2\dot{\lambda}(\phi)d\phi \le \begin{bmatrix}\lambda(t)\\ \lambda(t-\eta_m)\end{bmatrix}^T\begin{bmatrix}-\boldsymbol{R}_2 & \boldsymbol{R}_2\\ \boldsymbol{R}_2 & -\boldsymbol{R}_2\end{bmatrix}\begin{bmatrix}\lambda(t)\\ \lambda(t-\eta_m)\end{bmatrix} \quad \ldots (23)$$

With the Free--weighting matrices can be used to obtain the following results:

$$2\zeta^T(t)\boldsymbol{M}_{ij}[\lambda(t-\eta_m) - \lambda(t-\eta(t)) - \int_{t-\eta(t)}^{t-\eta_m} \dot{\lambda}(\phi)d\phi] = 0 \quad \ldots (24)$$

$$2\zeta^T(t)\boldsymbol{N}_{ij}[\lambda(t-\eta(t)) - \lambda(t-\eta_M) - \int_{t-\eta_M}^{t-\eta(t)} \dot{\lambda}(\phi)d\phi] = 0 \quad \ldots (25)$$

$$2\zeta^T(t)\boldsymbol{T}_{ij}[\lambda(t) - \lambda(t-\mu(t)) - \int_{t-\mu(t)}^{t} \dot{\lambda}(\phi)d\phi] = 0 \quad \ldots (26)$$

$$2\zeta^T(t)\boldsymbol{S}_{ij}[\lambda(t-\mu(t)) - \lambda(t-\mu_M)) - \int_{t-\mu_M}^{t-\mu(t)} \dot{\lambda}(\phi)d\phi] = 0 \quad \ldots (27)$$

These are matrices with proper dimensions called $\boldsymbol{M}_{ij}$, $\boldsymbol{N}_{ij}$, $\boldsymbol{T}_{ij}$ and $\boldsymbol{S}_{ij}$

$$-2\zeta^T(t)\mathbf{M}_{ij}\int_{t-\eta,\eta)}^{t-\eta_m} \dot{\lambda}(\phi)d\phi \le (\eta(t)-\eta_m)\zeta^T(t)\mathbf{M}_{ij}\mathbf{R}_1^{-1}\mathbf{M}_{ij}^T\zeta(t) + \int_{t-\eta(t)}^{t-\eta_m} \dot{\lambda}^T(\phi)\mathbf{R}_1\dot{\lambda}(\phi)d\phi \quad \ldots (28)$$

$$-2\zeta^T(t)\mathbf{N}_{ij}\int_{t-\eta_M}^{t-\eta(t)} \dot{\lambda}(\phi)d\phi \le -2\zeta^T(t)\mathbf{N}_{ij}\int_{t-\eta_M}^{t-\eta(t)} \dot{\lambda}(\phi)d\phi + \int_{t-\eta_M}^{t-\eta(t)} \dot{\lambda}^T(\phi)\mathbf{R}_1\dot{\lambda}(\phi)d\phi \quad \ldots (29)$$

$$-2\zeta^T(t)\boldsymbol{T}_{ij}\int_{t-\mu(t)}^{t} \dot{\lambda}(\phi)d\phi \le \mu(t)\zeta^T(t)\boldsymbol{T}_{ij}\boldsymbol{R}_3^{-1}\boldsymbol{T}_{ij}^T\zeta(t) + \int_{t-\mu(t)}^{t} \dot{\lambda}^T(\phi)\boldsymbol{R}_3\dot{\lambda}(\phi)d\phi \quad \ldots (30)$$

$$-2\zeta^T(t)\mathbf{S}_{jj}\int_{t-\mu_2}^{t-\mu(t)} \dot{\lambda}(\phi)d\phi \le \big(\mu_M - \mu(t)\big)\zeta^T(t)\mathbf{S}_{ij}\mathbf{R}_3^{-1}\mathbf{S}_{ij}^T\zeta(t) + \int_{t-\mu,\mu)}^{t-\Lambda(t)} \dot{\lambda}^T(\phi)\mathbf{R}_3\dot{\lambda}(\phi)d\phi \quad \ldots (31)$$

Now, we can define the augmented vector that yields

$$\mathscr{E}\{\mathcal{O}(t)\} \le \mathscr{E}\{\gamma^2\omega^T(t)\omega(t) - \tilde{z}^T(t)\tilde{z}(t)\} \quad \ldots (32)$$

It has been proven.

Corollary 1 provides the foundation for designing a filter, such as (20). The following Remark provides an explicit expression for the filter parameters.

Remark 1: Due to the limitations of the pages, the authors omitted some steps to get the general form Stochastic filter.

$$\widetilde{\mathbb{A}}_{fi} = \mathbb{A}_{fi}\boldsymbol{P}_{3i} \quad \ldots (33)$$

$$\widetilde{\mathbb{B}}_{fi} = \boldsymbol{P}_{2i}\mathbb{B}_{fi}\boldsymbol{P}_{2i}^{-T} \quad \ldots (34)$$

$$\widetilde{\mathbb{C}}_{fi} = \mathbb{C}_{fi}\boldsymbol{P}_{2i}^{-T} \quad \ldots (35)$$

Conclusions

A reliable H_∞ filter design based on the *Markov Jump System* is also being examined for an event-based network control system. The *event-triggered mechanism* implemented over the system, in particular, reduces the network's communication load and increases its efficiency by streamlining communication. Additionally, the fundamental stability criterion is derived, and the design of filter technique is discussed, by using the networked Markov Jump model and the probabilistic sensor faults. Lastly, the optimal filter elements are specified.

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