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A Novel Approach to Ensure Robust Stability using Unsymmetric Lyapunov Matrix for 2-D Discrete Model

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This paper addresses the issue of ensuring the asymptotic stability of the two-dimensional discrete Roesser model. Most of the emphasis is given now a days on the stability analysis of two-dimensional discrete models because of their wide variety of real time applications. When it comes of ensuring the stability of any system, the most generalized method is to use symmetric Lyapunov function. There have been a lot of published articles in which the stability of the system has been ensured using the symmetrical Lyapunov function, but use of unsymmetrical Lyapunov function has not been adopted due to the computational complexity. One of the very popular two dimensional discrete model is the Roesser model, which is structurally different from other two dimensional discrete models and has its wide applications in the field of image processing. In this article the stability of a 2-D discrete Roesser model has been ensured using the unsymmetrical Lyapunov function, which is a more generalized way of ensuring the stability of any system. Accordingly, new stability conditions have been developed, which is an extension of the previously reported methods in which the stability is made certain using the symmetrical Lyapunov matrix. In some cases, it has been shown numerically that it is difficult to ensure stability using the symmetric Lyapunov matrix but still, the stability for such cases may be ensured using the unsymmetric Lyapunov matrix. In addition, symmetrical Lyapunov matrix stability conditions have also been derived using the unsymmetric Lyapunov matrix. The stability criteria have been checked and ensured based on newly developed stability conditions by considering two different examples. An effort has been put in reducing the conservatism with the new stability conditions.

Keywords: Asymptotic stability analysis, Roessermodel, Two-dimensional discrete system, Unsymmetric lyapunov function

Introduction

In the recent years more research is directed towards the analysis of multidimensional systems because of its wide range of applications in many areas such as image processing, signal processing,[1–3] thermal process,[4] water steam heating,[2] gas absorption[5] and digital filtering,[2,6] river pollution modelling,[7] and other general multi variable based class. In all these systems, the information is represented in more than one variable. Stability is a major concern during analyzing the performance of any such dynamical system.

In the presented approach to remain more concise, the focus is on the 2-D linear discrete Roesser model, which is considered the most versatile and widely used model for image processing applications.[8] In the case of a Roesser model the state vectors are decomposed into horizontal and vertical state vector sets which is the basic requirement for understanding any two-dimensional image.

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There are so many underline factors, which may result in adversely affecting the performance as well as the stability of any system under consideration. It may be because of the component ageing, modelling errors, the difference in the actual point of presumed and actual processes or some other neglected factors. In most systems, stability has been ensured using the concept of the Lyapunov function,[9] which states that if the system's energy decreases with time or if the system returns to the equilibrium point or the origin in due course of time, then the system is bound to become stable.

According to the Lyapunov stability theorem, to achieve sufficient asymptotic stability conditions, scalar function of the system should require to be greater than zero and to be a positive definite symmetric function.[10]

Literature Review

Many researchers have given their significant contributions in ensuring the stability of two-dimensional discrete systems, specifically those described by the Roesser model. It is not only the linear systems[7,11] but also the non-linear systems,[12–14] in which the image properties are prone to get vary which might affects the stability parameters and therefore the study of stability analysis in such cases must be given due importance. Many nonlinear controllers have been designed using the concept of Lyapunov stability theory as proposed in the paper.[15] Considerable attention has been given to the stability concerns of non-linear two-dimensional discrete systems.[16–18] The non-linear 2-D FMLSS model is investigated, where the author[19] establishes sufficient conditions for exponential stability. The paper proves the direct and converse theorem on exponential stability using Lyapunov functions, and introduces new findings on stability and controller design using passivity theory to stabilize the system. A certain real time application-based system as addresses in also need to be stabilized, using the concept of Lyapunov stability theory.[16]

A popular way to ensure stability is using the symmetrical Lyapunov matrix. But the use of Unsymmetric Lyapunov function offers more flexibility for ensuring the stability rather than symmetric Lyapunov functions. Further, author feels that unsymmetrical Lyapunov function offers a more generalized approach of dealing with the different type of instabilities such as noise and uncertainty,[20] parameter variations[21], and oscillatory instabilities.[12] But so far none of the researcher has addressed the issues of ensuring the stability using the unsymmetrical Lyapunov function doe to the computational complexity, and this motivates us to carry out our research in this direction.

Few researchers have investigated stability analysis using unsymmetric Lyapunov function for the other 2-D discrete models,[22] and they found out that when the symmetric Lyapunov function fails to establish the stability of any system, the stability of such systems may be ensured by unsymmetric Lyapunov function.

As unsymmetrical Lyapunov matrix gives a more generalized way of ensuring the stability of any system, and many relevant areas of two-dimension discrete systems advocate the usage of such a generalized Lyapunov equation and ensuring the stability using unsymmetrical Lyapunov function has not been addressed for the 2-D discrete Roesser model this motivates us to carry out this work. In the following section we will discuss on the main results.

Notations

$\mathbb{R}^n$ and $\mathbb{R}^m$ represents n and m dimensional real vector space

$\mathbb{R}^{n\times m}$ and $\mathbb{R}^{n\times n}$ are representing real matrices of $n\times m$ and $n\times n$ dimensions 0 is an appropriate dimensioned null matrix

I_n and stands for the identity matrix of $n\times n$ and $m\times m$ dimensions respectively

G^T is representing the transpose of matrix G

$G>0$ matrix indicates that G is a positive definite symmetric (PDS) matrix

$G<0$ matrix indicates that G is a negative definite symmetric (NDS) matrix

Materials and Methods

Problem Formulation and Preliminaries

The considered system is a 2-D discrete Roesser model with zero input and zero initial conditions.

$$\begin{bmatrix} x_p^h(k,l+1) \\ x_p^v(k+1,l)\end{bmatrix} = \left(\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22}\end{bmatrix} + \Delta A\right)\begin{bmatrix} x_p^h(k,l) \\ x_p^v(k,l)\end{bmatrix} \quad \dots (1)$$

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22}\end{bmatrix} \quad \dots (2)$$

where, the state vectors $x_p^h(k,l)\epsilon\mathbb{R}^n$, $x_p^v(k,l)\epsilon\mathbb{R}^m$ represent the horizontal and vertical states of the system, respectively with appropriate dimensions. The dimensions of A_{11}, A_{12}, A_{21} and A_{22} given are respectively by $A_{11}\epsilon\mathbb{R}^{m\times m}$, $A_{12}\epsilon\mathbb{R}^{n\times m}$, $A_{21}\epsilon\mathbb{R}^{m\times n}$, $A_{22}\epsilon\mathbb{R}^{n\times n}$. The matrices ΔA represent uncertainties in the parameter.

Under the zero-boundary criterion, the transfer function matrix of a 2-D discrete-time system can be represented as.[23]

$$\mathcal{G}(\mathcal{Z}_1,\mathcal{Z}_2) \neq 0\ \forall(\mathcal{Z}_1,\mathcal{Z}_2) \in \overline{U}^2 \quad \dots (3)$$

The system described by Eq. (1) achieves asymptotic stability if and only if.

$$\det\begin{bmatrix} I_n - A_{11}\mathcal{Z}_1 & -\mathcal{Z}_1 A_{12} \\ -\mathcal{Z}_2 A_{21} & I_m - A_{22}\mathcal{Z}_2\end{bmatrix} \neq 0 \text{for} (\mathcal{Z}_1,\mathcal{Z}_2)\epsilon\overline{U}^2 \quad \dots (4)$$

$$\det(\mathcal{Z}_1 I_n \oplus \mathcal{Z}_2 I_m) - A) \neq 0 \text{ for } (\mathcal{Z}_1,\mathcal{Z}_2)\epsilon\overline{U}^2 \quad \dots (5)$$

where,

$$\overline{U}^2 = \{\mathcal{Z}_1,\mathcal{Z}_2 : |\mathcal{Z}_1| \leq 1, |\mathcal{Z}_2| \leq 1\} \quad \dots (6)$$

the identity matrix is $I_n = \mathrm{diag}(I_{nh}, I_{nv})$ and $I_m = \mathrm{diag}(I_{mh}, I_{mv})$
'det' is an abbreviation for determinant.

Theorem 1[22]. If there exists a PDSM (positive definite symmetric matrix) ($P_p = P_p^T = P_p^h \oplus P_p^v$), then the system Eq. (1) is asymptotically stable.

$$\text{If,} P_p - A^T P_p A > 0 \quad ...(7)$$

where, $A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$ and A^T represents the transpose of matrix A, 0 for null, positive definite is abbreviated as PD.

Theorem 2. Given PDSM $W_{0_1} \in R^{n\times n}$ and $W_{1_0} \in R^{n\times n}$ with $I_n - W_{0_1} - W_{1_0} \geq 0$ or $I_n - W_{0_1} - W_{1_0} = 0$ system Eq. (1) is asymptotically stable, if there exists a PDSM $.P_p = P_p^h \oplus P_p^v = P_p^{\frac{T}{2}} P_p^{\frac{1}{2}} \in R^{n\times n}$ such that

$$\Theta = \begin{bmatrix} P_p^{\frac{T}{2}} W_{0_1} P_p^{\frac{1}{2}} & 0 \\ 0 & P_p^{\frac{T}{2}} W_{1_0} P_p^{\frac{1}{2}} \end{bmatrix} - A^T P_p A > 0 \quad ...(8)$$

The $\oplus$ symbol represents the direct sum of matrices. In this context, it signifies that P_p is a block diagonal matrix formed by combining two submatrices, P_p^h and P_p^v. The submatrix P_p^h signifies the horizontal direction, while P_p^v signifies the vertical direction.

Remarks 1. Theorem 2 is a generalization of Theorem 1.

Main Result

The main result of this paper is given in the following Theorem.

Theorem 3. The 2-D system as represented by Eq. (1) is asymptotically stable if there exists a PDSM P_p, The existence of a PDSM matrix P_p ensures the asymptotic stability of the 2-D system defined by Eq. (1).which is not necessarily symmetric, for the given matrices W_{0_1} and W_{1_0} such that $I_n - W_{0_1} - W_{1_0} \geq 0$, and the givenconditions must be hold true.

$$M = \begin{bmatrix} \begin{bmatrix} \left(\frac{P_p+P_p^T}{2}\right)^{\frac{T}{2}} W_{0_1} \left(\frac{P_p+P_p^T}{2}\right)^{\frac{1}{2}} & 0 \\ 0 & \left(\frac{P_p+P_p^T}{2}\right)^{\frac{T}{2}} W_{1_0} \left(\frac{P_p+P_p^T}{2}\right)^{\frac{1}{2}} \end{bmatrix} & -A^T P_p \\ -P_p^T A & \left(\frac{P_p+P_p^T}{2}\right) \end{bmatrix} > 0 \quad ...(9)$$

$$\left(\frac{P_p+P_p^T}{2}\right) > 0 \quad ...(10)$$

Given Eq. (9) represents a matrix M, constructed using the PDSM matrix P_p, its transpose, and square root of these matrices. The condition $M > 0$ indicates that the resulting matrix M must be positive definite for the given conditions to hold true.

Proof:

The asymptotic stability condition for the system as represented by the Eq. (1) taking the initial conditions as zero and $u(k,l) = 0$is given by

$$\begin{bmatrix} x_p^h(k,l+1) \\ x_p^v(k+1,l) \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} x_p^h(k,l) \\ x_p^v(k,l) \end{bmatrix} \quad ... (11)$$

Where

$$\begin{bmatrix} x_p^h(k,l+1) \\ x_p^v(k+1,l) \end{bmatrix} = x_2(k,l) = A x_1(k,l),$$

$$x_1(k,l) = \begin{bmatrix} x_p^h(k,l) \\ x_p^v(k,l) \end{bmatrix} \text{and} A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \quad ... (12)$$

In order to proof Theorem 3,a positive definite symmetric Lyapunov function has been considered which is not necessarily symmetric such that

$$\mho\left(x_1(k,l)\right) = x_1^T(k,l) \left(\frac{P_p+P_p^T}{2}\right)^{\frac{T}{2}} W_\zeta \left(\frac{P_p+P_p^T}{2}\right)^{\frac{1}{2}} x_1(k,l) \quad ... (13)$$

In order to ascertain the stability conditions for the given system we calculate the value $\Delta\mho(x_1(k,l))$ which is defined as

$$\Delta\mho(x_1(k,l)) = \mho^h\left(x_p^h(k,l+1)\right) + \mho^v\left(x_p^v(k+1,l\right.$$ –ʊhxphk,l–ʊvxpvk,l ... (14)

$$= \left(x_p^{h^T}(k,l+1)\right) P_p^h \left(x_p^h(k,l+1)\right) + \left(x_p^{v^T}(k+1,l)\right) P_p^v \left(x_p^v(k+1,l)\right) - \left(x_p^{h^T}(k,l)\right) P_p^h \left(x_p^h(k,l)\right) - \left(x_p^{v^T}(k,l)\right) P_p^v \left(x_p^v(k,l)\right) \quad ... (15)$$

On putting the values of $x_p^h(k,l+1)$and $\left(x_p^v(k+1,l\right.$ from Eq.(12) in Eq.(15) we will get

$$= -A^T P_p A - x_1^T(k,l) P_{p}^{\frac{T}{2}} W_{0_1} P_{p}^{\frac{1}{2}} x_1(k,l) - x_2^T(k,l) P_{p}^{\frac{T}{2}} W_{1_0} P_{p}^{\frac{1}{2}} x_2(k,l) + 2A^T P_p A \quad \text{... (16)}$$

which on simplification results in

$$= -A^T \left(\frac{P_p + P_p^T}{2}\right) A$$

$$- x_1^T(k,l) \left(\frac{P_p + P_p^T}{2}\right)^{\frac{T}{2}} W_{0_1} \left(\frac{P_p + P_p^T}{2}\right)^{\frac{1}{2}} x_1(k,l)$$

$$- x_2^T(k,l) \left(\frac{P_p + P_p^T}{2}\right)^{\frac{T}{2}} W_{1_0} \left(\frac{P_p + P_p^T}{2}\right)^{\frac{1}{2}} x_2(k,l) \times$$

$$x_2^T(k,l)(k,l) + A^T(P_p + P_p^T)A \quad \text{... (17)}$$

then on applying the Schur complements in Eq. (17) results in Eq. (18), where M is as specified in Eq. (9). This completes the proof.

$$\Delta v(x_1(k,l)) = -\left[x_1^T(k,l) x_2^T(k,l) A^T\right]^T M \begin{bmatrix} x_1(k,l) \\ x_2(k,l) \\ A \end{bmatrix} \quad \text{... (18)}$$

It may be observed that if we put $\left(\frac{P_p + P_p^T}{2}\right) = P_p$ the results of Eq. (9) may be recovered from Eq. (8)

As a result, $\Delta v(x_p(k,l))$ function in equation (14) satisfies $\Delta v(x_1(k,l)) \leq 0$ if Eq. (9) holds true and $\Delta v(x_1(k,l)) = 0$ only if $x_p^h(k,l) = 0, x_p^v(k,l) = 0$.

Now, for any non-negative integer K

$$\sum_{k+l=K+1} v\left(x_1(k,l)\right) = \sum_{k+l=K+1} v^h\left(x_p^h(k,l)\right) + \sum_{k+l=K+1} v^v\left(x_p^v(k,l)\right) \quad \text{... (19)}$$

$$= v^h\left(x_p^h(k+1,0)\right) + v^v\left(x_p^v(0,l+1)\right) \leq v^h\left(x_p^h(0,l)\right) + v^v\left(x_p^v(k,0)\right) = \sum_{k+l=i} v\left(x_1(k,l)\right) \quad \text{...(20)}$$

$$\sum_{k+l=i+1} v\left(x_1(k,l)\right) \leq \sum_{k+l=i} v\left(x_1(k,l)\right) \quad \text{... (21)}$$

where, use is made of $\left(x_p^h(k+1,0)\right) = 0, \left(x_p^v(0,l+1\right)=0$.

consequently

$$\lim_{k\to\infty \text{ and } l\to\infty} x_1(k,l) = \lim_{k+l\to\infty} x_1(k,l) = 0 \quad \text{... (22)}$$

This completes the proof of Theorem 3.

Special Case

By entitle $W_{11} = W_{11}^T = I_n$, as a specific case of Theorem 3, the following result is obtained.

Corollary 1.

System Eq. (1) is asymptotically stable if there exists a matrix $P_p = P_p^T = P_p^h \oplus P_p^v$ with appropriate dimensions, which need not be necessarily symmetric, such that

$$Q_p = \begin{bmatrix} \left(\frac{P_p + P_p^T}{2}\right) & -A^T P_p \\ -P_p^T A & \left(\frac{P_p + P_p^T}{2}\right) \end{bmatrix} > 0 \quad \text{... (23)}$$

$$P_p = \left(\frac{P_p + P_p^T}{2}\right) > 0 \quad \text{... (24)}$$

Proposition 1.

Theorem 3 is equivalent to Theorem 2.

Corollary 2.

The system described by (1) is asymptotically stable if there exists PDSM $P_p^h \epsilon \mathbb{R}^{n\times n}$ and $P_p^v \epsilon \mathbb{R}^{m\times m}$, not necessarily symmetric such that the conditions specified below holds good

$$P_p - A^T P_p A > 0 \quad \text{... (25)}$$

$$P_p = \left(\frac{P_p + P_p^T}{2}\right) > 0 \quad \text{... (26)}$$

$$P_p - A^T \left(\frac{P_p + P_p^T}{2}\right) A > 0 \quad \text{... (27)}$$

where,

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, P_p = P_p^h \oplus P_p^v = \begin{bmatrix} P_p^h & 0 \\ 0 & P_p^v \end{bmatrix} \quad \text{... (28)}$$

The stability criteria (Theorems 1) is based on the new stability condition has been proposed in which the matrix P_p is not required to be a symmetric matrix.

Result and Discussion

In this section some examples have been discussed which demonstrates the applicability of Corollary 1.

Example 1.

As an example, consider Eq. (1)

$$A = \begin{bmatrix} 0 & 6 & 0.03 \\ -0.4 & 0.9 & 0 \\ 0.03 & 0 & 0.03 \end{bmatrix} \quad \text{... (29)}$$

In this example, while putting $P_{\wp}^{h} = P_{\wp}^{h^T}$ the required conditions of Corollary 1 has been recovered. Further, according to this condition the system will be stable if

$$P_{\wp}^{h} - A^{11^T} P_{\wp}^{h} A^{11} > 0 \quad \text{... (30)}$$

There by implying

$$\det\left(P_{\wp}^{h} - A^{11^T} P_{\wp}^{h} A^{11}\right) > 0 \quad \text{... (31)}$$

$$P_{\wp} = P_{\wp}^{h} \oplus P_{\wp}^{v} = \begin{bmatrix} P_{\wp_{11}}^{h} & P_{\wp_{12}}^{h} & 0 \\ P_{\wp_{21}}^{h} & P_{\wp_{22}}^{h} & 0 \\ 0 & 0 & P_{\wp_{11}}^{v} \end{bmatrix} \quad \text{... (32)}$$

$$\mathrm{P}_{\wp} = \begin{bmatrix} 1 & -0.03 & 0 \\ -0.05 & 12 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{... (33)}$$

Now that the matrix is not symmetric with $P_{\wp} = P_{\wp}^{h} \oplus P_{\wp}^{v}$ given by Eq. (33), the matrix $\left(P_{\wp} + P_{\wp}^{T}\right)/2$ becomes

$$P_{\wp} = \frac{P_{\wp} + P_{\wp}^{T}}{2} = \begin{bmatrix} 1 & -0.04 & 0 \\ -0.04 & 12 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{... (34)}$$

Where d et (A^{11}) denotes the determinant of the matrix A^{11} is 2×2. Presently, $a_{11}^{1} = 0, a_{12}^{1} = 6, a_{21}^{1} = -0.4, a_{22}^{1} = 0.9$

Ones obtain

$$= -0.16\left[P_{\wp_{22}}^{\mathrm{h}} + \left(10.8P_{\wp_{22}}^{\mathrm{h}} - 18P_{\wp_{11}}^{\mathrm{h}}\right) + 11.56\sqrt{-(P_{\wp_{12}}^{h} + 0.19P_{\wp_{11}}^{h})(P_{\wp_{12}}^{h} + 1.22P_{\wp_{11}}^{h})}\right] \times \left[P_{\wp_{22}}^{\mathrm{h}} + \left(10.8P_{\wp_{22}}^{\mathrm{h}} - 18P_{\wp_{11}}^{\mathrm{h}}\right) - 11.56 - (\wp 12\mathrm{h} + 0.19\wp 11\mathrm{h})(\wp 12\mathrm{h} + 1.22\wp 11\mathrm{h})\right. \quad \text{... (35)}$$

In view of Eq. (35) and Eq. (33)

$$-1.220P_{\wp_{11}}^{h} < P_{\wp_{12}}^{h} < -0.19P_{\wp_{11}}^{h} \quad \text{... (36)}$$

which satisfies Eq. (36) and provides a feasible solution. Therefore, based on the analysis of this specific example, it can be concluded that Corollary 1 does not provide an existence for assessing asymptotic stability.

$$-1.22P_{\wp_{11}}^{h} \le P_{\wp_{12}}^{h} \quad \text{... (37)}$$

$$Q_{p} = \begin{bmatrix} 1 & -0.04 & 0 & -0.02 & 4.8 & -0.03 \\ -0.04 & 12 & 0 & -5.9 & -10.62 & 0 \\ 0 & 0 & 1 & -0.03 & 0.009 & -0.03 \\ -0.02 & -5.9 & -0.03 & 1 & -0.04 & 0 \\ 4.8 & -10.62 & 0.009 & -0.04 & 12 & 0 \\ -0.03 & 0 & -0.03 & 0 & 0 & 1 \end{bmatrix} \quad \text{... (38)}$$

which is a positive definite.

To summarize, if Eq. (37) holds, criterion Eq. (7) does not have a feasible solution in the context under evaluation. On the contrary, a feasible solution for the criterion Eq. (23) is obtained when Eq. (37) is satisfied.

Example 2.

To illustrate the importance of the criterion Eq. (7) in reaching a stability condition. Consider a specific example of a system Eq. (1) with

$$A = \begin{bmatrix} 1 & -1.5 & 0 \\ -0.1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{... (39)}$$

It is necessary to fulfil the criteria of Theorem 1 to satisfy

$$P_{\wp}^{h} - A^{11^T} P_{\wp}^{h} A^{11} > 0 \quad \text{... (40)}$$

$$\det\left(P_{\wp}^{h} - A^{11^T} P_{\wp}^{h} A^{11}\right) > 0 \quad \text{... (41)}$$

where, det (A^{11}) stands for the determinant of the matrix A^{11} is 2×2. Presently, $a_{11}^{1} = 1, a_{12}^{1} = -1.5, a_{21}^{1} = -0.1, a_{22}^{1} = 0$.

In this example, we will now apply Corollary 1, the existence of $P_{\wp}^{h} = P_{\wp}^{h^T}$ satisfying Eq. (40) requires[24]

$$P_{\wp} = P_{\wp}^{h} \oplus P_{\wp}^{v} = \begin{bmatrix} P_{\wp_{11}}^{h} & P_{\wp_{12}}^{h} & 0 \\ P_{\wp_{21}}^{h} & P_{\wp_{22}}^{h} & 0 \\ 0 & 0 & P_{\wp_{11}}^{v} \end{bmatrix} \quad \text{... (42)}$$

$$P_{\wp} = P_{\wp}^{h} \oplus P_{\wp}^{v} = \begin{bmatrix} 1 & -1 & 0 \\ -3 & 12 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{... (43)}$$

Now that the matrix is not symmetric with $P_{\wp} = P_{\wp}^{h} \oplus P_{\wp}^{v}$ given by Eq. (43), the matrix $\left(P_{\wp} + P_{\wp}^{T}\right)/2$ becomes

$$\frac{P_{\wp} + P_{\wp}^{T}}{2} = \begin{bmatrix} 1 & -2 & 0 \\ -2 & 12 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{... (44)}$$

which is PD (positive definite) satisfying the condition

$$P^{\mathrm{h}}_{\wp_{11}} \geq \left|P^{\mathrm{h}}_{\wp_{12}}\right|, P^{v}_{\wp_{11}} \geq 0 \quad \text{... (45)}$$

With $P^{h}_{\wp} \oplus P^{v}_{\wp}$ given by Eqs. (43), (45) is satisfied. The matrix Q_p in Eq. (23) takes the form

$$Q_p = \begin{bmatrix} 1 & -2 & 0 & -1.3 & 2.2 & 0 \\ -2 & 12 & 0 & 1.5 & -1.5 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ -1.3 & 1.5 & 0 & 1 & -2 & 0 \\ 2.2 & -1.5 & 0 & -2 & 12 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} \quad \text{... (46)}$$

which is PD (positive definite). All conditions stated in Corollary 1 are fulfilled. Corollary 1 thus verifies asymptotic stability in this example.

Conclusions

New stability conditions have been proposed in this research article for the 2-D discrete Roesser model using the unsymmetrical Lyapunov function and accordingly, new stability conditions have been developed. The proposed criteria are unique in that it does not require the Lyapunov function to be symmetric. This makes it more general and applicable to a wider range of systems. The use of unsymmetrical Lyapunov function gives a more generalized way of ensuring the stability conditions and helps in reducing conservatism. Further it has also been shown that the previously reported stability conditions can also be recovered from the newly developed stability conditions. The advantage of the presented criteria for ensuring the stability using unsymmetric asymptotic stability conditions is illustrated by considering two different examples. Through these examples it has been shown that even if the conditions of ensuring the stability using symmetric Lyapunov function has been violated, the stabilities in such cases can be ensured using the unsymmetrical Lyapunov function. As a part of Future Scope, the issues of addressing the stability of an uncertain 2-D discrete models can be dealt with by designing a controller based on this unsymmetrical Lyapunov function.

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Conflict of Interest

The authors declare that they have no conflict of interest in the publication of this research.

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