SMYTHIES' SAFEGUARDING FORMULA

BY K. P. SAGREIYA, I.F.S.
Chief Conservator of Forests, Madhya Pradesh

I have been asked to comment on the well-known Smythies' Safeguarding Formula *vis-à-vis* my mathematical analysis of the concept underlying Brandis' method. Hence this paper.

Smythies gives the following expression for the measure of recruitment per acre of class I trees during the first cycle—

$$x = \frac{f}{t_2}(1-z_2)\,II$$

where f is the felling cycle, t_2 is the time taken by trees to pass through class II, II is the number of class II trees per acre, and z_2 is the fraction of trees of class II that fail to reach class I or are silviculturally not available for removal. The recruitment per acre per year will be—

$$\frac{1}{t_2}(1-z_2)\,II \text{ or say } x'.$$

Smythies has proposed the use of this formula for working the forest under *volume* control, (apparently *number* control is implied).

It is obvious that as the prescribed yield is equal to the recruitment from class II there will be no reduction in the class I trees at the end of the cycle. But this formula would be workable only so long as f is less than t_2. When f is greater than t_2, all the class II trees would have passed into class I in t_2 years ; whereafter in the remaining ($f-t_2$) years of the cycle, if f is not greater than (t_2+t_3) the recruitment rate per acre per year will be—

$$\frac{1}{t_3}(1-z_3)\,III \text{ or say } y'$$

which might well be greater or less than x'. Therefore, when x' trees are removed annually in the last ($f-t_2$) years of the cycle the number of class I trees left in the forest at the end of the cycle will be less or greater than I according as x' is greater or less than y'.

As an improvement, Smythies suggests an *area* control formula.

He assumes that the forest is divided into f equi-extensive cutting sections and trees of classes I and II occur over these uniformly distributed. Although not specified, he also visualises that the rate of recruitment from class II is constant from year to year and in all cutting sections. He then argues that in the cutting section due for working in the nth year the number of exploitable trees, per acre, will be :

(*i*) I of the original class I trees,

plus (*ii*) $\frac{nx}{f}$ class II trees that would have reached class I *at the time of marking**, i.e.,

in all $I + \frac{nx}{f}$ = say N trees.

---

* The availability per acre in the nth cutting section *at the time of marking* will be only $I + (n-1)\frac{x}{f}$. When fellings proceed from one end of the section to other it will be $I + (n-\frac{1}{2})\frac{x}{f}$ trees per acre. Hence the average availability will be—

$$I + \frac{1}{f}\sum_{n=1}^{n=f}(n-\tfrac{1}{2})\frac{x}{f} = I + \frac{x}{2}.$$

From this he derives the average availability per year as $I + \frac{x}{2}$ and uses it to determine the percentage of trees of class I that could be removed from the cutting section of the year namely $\left(\frac{x}{I + \frac{x}{2}} 100 \right)$ per cent of N or say k % of N.

To this factor he adds another arbitrary factor $\pm$ A and thus gets the well-known Safeguarding formula, viz., removal of (K $\pm$ A) % of available class I trees in the year's cutting section, so that eventually not more than x class I trees, per acre, are removed in the felling cycle.

One drawback of this formula, which is admitted by Smythies, is that the yield will not be constant but will progressively increase from year to year as n will vary from 1 to f in the expression $I + \frac{nx}{f}$.

To more or less equalize the yield Smythies has suggested that cutting sections with a larger number of class II trees per acre might be worked earlier. This may not always be justifiable silviculturally. From the management view-point also, the yield should be so fixed that it is the highest realizable, will sustain and, if possible, progressively increase from cycle to cycle until the potentialities of the forest have been realized. Instead, Smythies has tried to sustain the yield at the annual recruitment to class I by proposing maintenance of trees in lower classes in a mathematically computed proportion as under :—

If I, II, III, etc., are trees (not per acre as in the earlier formula but for the whole forest) in classes, I, II, III, etc. and t_2, t_3, etc.*, are the periods for which trees remain in classes II, III, etc. and the casualties when the trees pass from classes II, III, etc., to the *next higher class* are represented by the fractions, z_2, z_3, etc., then, the yield in the felling cycle will be :

$$X = \frac{f}{t_2} (1 - z_2) \text{II}$$

$$\therefore \text{II} = \frac{X t_2}{f (1 - z_2)}$$

The loss in class II in f years is equal to recruitment from it to class I, i.e., X, *plus* the casualties, i.e., $\frac{f}{t_2} z_2 \text{II}$, or in all

$$X + \frac{f}{t_2} z_2 \text{II}$$

Substituting the value of II, this becomes

$$X + \frac{f}{t_2} z_2 \left[\frac{X t_2}{f (1 - z_2)} \right] = \frac{X}{1 - z_2}$$

If the yield is to be sustained this loss must be made good by recruitment from class III, Hence,

$$\frac{f}{t_3} (1 - z_3) \text{III} = \frac{X}{1 - z_2} \quad \therefore \text{III} = \frac{X t_3}{f (1 - z_2) (1 - z_3)}$$

Therefore, in general the ideal proportion is

$$\text{II} : \text{III} : \text{IV etc.} : : t_2 : t_3/(1 - z_3) : t_4/(1 - z_3) (1 - z_4) \text{ etc.}$$

* As Smythies symbols t, t^1, t^2, etc. and z, z^1, z^2, etc. are liable to cause confusion as t^2 can be read as square of t, I have used the symbols t_2, t_3 etc. and z_2, z_3, etc., for classes II, III, etc., respectively.

Smythies simplifies these fractions by assuming that

$$t_2 = t_3 \text{ etc.} = \text{say } t$$
$$\text{and } z_2 = z_3 \text{ etc.} = \text{say } z$$

so that

$$\text{II} : \text{III} : \text{IV, etc.} :: 1 : (1-z)^{-1} : (1-z)^{-2} : \text{etc.}$$

It will be seen that these simple formulæ have been obtained for 'normalizing' the forest to obtain an yield which is exclusively based on the rate of recruitment to class I in the first f years, without any regard to the stock-in-hand or the rate of recruitment in successive cycles. It is more or less axiomatic that so long as the diameter classes are fixed and there is a definite proportion in which trees of different sizes will tend to occur in an all-aged forest under a specific silvicultural treatment, so long it is more likely than not that t_2, t_3, etc., as also z_2, z_3, etc., will *not* be equal to any assumed t and z respectively. Forcing the crops to conform to this pattern may seriously upset the silvicultural conditions and thus jeopardise the realization of the potentialities of the forest, viz., the highest sustained yield of class I trees, which is the fundamental aim of sound forest management! At any rate there is no evidence that such manoeuvring will not adversely affect growth conditions.

The nearest approximation to check this is the assumption that in an all-aged fully-stocked forest, at any rate of a strong light-demander like teak and therefore, to a great extent of a partial shade-bearer like sal, trees of various size-classes will occur in the proportion that obtains in normal even-aged crops, and in forests where a particular utilizable species, like teak in Madhya Pradesh, comprises only 20–40% of the crop and is given preferential treatment, the lower diameter classes will be proportionately higher and the casualties amongst them less when they pass into higher classes.

Taking as an example Table 29 of teak yield tables (I.F.R., New Series, Volume IV-A, No. 1, 1940) the following figures are obtained :

Crop Diameter	Mean spacement (triangular orientation)	Hence N per acre	Survival % as class I	Survival % on reaching the next higher class ($1-z_n$%)
	ft.			
20″ and up (average 22″)	37·0	35	...	...
16″–20″ (average 18″) ...	32·5	47	74	74
12″–16″ (average 14″) ...	28·0	65	54	72
8″–12″ (average 10″) ...	22·0	105	33	62
4″–8″ (average 6″) ...	14·5	240	15	44

It will be seen that the survival percentages in the last column are far from constant.

The values of t_2, t_3, etc., in the case of the teak forests of Bori, Hoshangabad are :

D.B.H.	Age	Years in class (t_n)
inches		
4	18	...
8	36	18
12	57	21
16	80	23
20	110	30

which again shows that the assumption that the class-periods are likely to be constant is also unjustified.

To summarize, Smythies method of fixing the yield is open to the following objections :—

(*i*) It more or less completely ignores the fundamental fact that in an all-aged forest the proportion of trees of lower diameter classes and the casualties amongst them depend primarily on silvicultural treatment and if this is disturbed by manipulation the productivity of the forest is likely to be adversely affected.

(*ii*) As the rate of growth of trees and the casualties (and hence the recruitment to the harvestable size) can be determined by successive enumerations fairly accurately, the yield should be fixed after giving due consideration to the stock-in-hand as well as the rate of recruitment in the subsequent cycles and fixed at the maximum number of trees *realizable*, after which the rate of recruitment in the subsequent class periods is higher than the yield thus arrived at, and progressively increases until the potentialities of the forest are realized.

Thus taking Forest C of my article and a felling cycle of 20 years (*Ind. For.*, June 1956, statement on p. 273), the yield should not be fixed at 700 trees, the annual recruitment from class II, which is the average yield under Smythies Safeguarding Formula, but at 984 which is *realizable* for 2 cycles, whereafter, it rises to 989 in cycle III, 1,265 in cycle IV, etc., until the maximum increment of the forest is realized.

(*iii*) Limiting the yield to $\frac{f}{t_2}$ ($1-z_2$) II per cycle without any regard to the stock-in-hand and the recruitment in subsequent cycles can result in depletion of the class I trees, or *per contra* their retention for an unduly long period in which they may deteriorate in value. If the recruitment in successive cycles is less, removal of the yield derived from Smythies formula will definitely deplete the forest.

(*iv*) When the prescribed yield is fixed as a percentage of available class I trees in equi-extensive cutting sections in the first cycle it will increase from year to year at the rate the recruitment accumulates.

(*v*) Deliberately altering the proportion of trees of various diameter classes to obtain the prescribed yield can adversely affect the productivity of the forests.

All the above objections are avoided under the method proposed by me.

For comparison, I give below the calculations of yield, etc., according to the various formulæ suggested by Smythies and those arrived at by me for Forest C (q.v. *supra*) area 2,000 acres, when $f = 10$, i.e., less than t_2 which is 14.

Data :—

Class	Trees as enumerated		Years in class
	Total	Per acre	
I (20" and up)	15,000	7·5	...
II (16"–20")	12,250	6·125	14(t_2)
III (12"–16")	27,000	13·5	18(t_3)
IV (8"–12")	60,000	30·0	24(t_4)
V (4"–8")	88,000	44·0	16(t_5)

	Class I	Class II	Class III	Class IV	Class V
Assumed survival % as class I	100	80	60	40	25
Proportionate initial availability	100	125	166·2/3	250	400
Fraction surviving when reaching the *next higher class* ($1-Z_n$)	...	4/5	3/4	2/3	5/8

Therefore—

(1) Total recruitment per acre to class I in first cycle,

$$X = \frac{f}{t_2}(1-z_2)\,\text{II} = \frac{10}{14} \times \frac{4}{5} \times 6\cdot125 = 3\cdot5$$

Hence recruitment per year for the whole forest will be $3\cdot5 \times 2000/10 = 700$. (This is R_2 in my terminology).

(2) Class I trees per acre in the nth cutting section in the middle of the year of working—

$$N = \text{I} + (n-\tfrac{1}{2})\frac{x}{f} = 7\cdot5 + (n-\tfrac{1}{2})\frac{3\cdot5}{10} = 7\cdot325 + 0\cdot35\,n$$

(3) The average number of class I trees per acre in a cutting section—

$$\text{I} + \frac{x}{2} = 7\cdot5 + \frac{3\cdot5}{2} = 9\cdot25$$

Hence availability for the whole forest in a felling cycle $= 200 \times 10 \times 9\cdot25 = 18{,}500$.

$$\left(S_1 + \frac{R_2 f}{2} \text{ in my terminology}\right)$$

(4) The *percentage* of class I trees removable from a cutting section to realize × trees per acre in the felling cycle—

$$\frac{x}{\text{I}+\frac{x}{2}} \times 100 = \frac{3\cdot5}{7\cdot5+\frac{3\cdot5}{2}} \times 100 = \frac{700}{18\cdot5} \text{ or } k\%.$$

Hence actual number removable per section

$$= k\% \text{ of availability} = k\% \text{ of } 200\,(7\cdot325+0\cdot35n)$$

$$= \frac{7}{18\cdot5}(1465 + 70n).$$

The yield will thus vary from 580 (approx.) in the first year to 820 (approx.) in the tenth year.

The total yield removed will be—

$$\frac{7}{18\cdot5}(14650 + \overline{70 \times 55}) = 7000$$

i.e., fR_2 in my terminology.

(5) The number of trees, per acre, of classes II, III, etc., to sustain this yield will be :

$$II = \frac{xt_2}{f(1-z_2)} = 6 \cdot 125$$

$$III = \frac{xt_3}{f(1-z_2)(1-z_3)} = 10 \cdot 5 \text{ etc.}$$

whereas the actual number of class III trees is 13·5 per acre. In other words Smythies visualises cashing of 3 trees per acre of class III to keep down the yield in the next cycle to just 3·5 trees per acre.

The ideal (max. sustained and, thereafter progressively increasing) yield of the forest will be 1071 as shown below :—

BASIC DATA AND SYMBOLS

Class	Trees reaching class I		Years in class	Hence annual recruitment in class period
	%	No.		
I	100	15000	...	...
II	80	9800	14(t_2)	700(R_2)
III	60	16200	18(t_3)	900(R_3)
IV	40	24000	24(t_4)	1000(R_4)
V	25	22000	16(t_5)	[1375(R_5) 1500(R_6)]

(1) Recruitment in successive cycles—

I $fR^{I} = 10R_2 = 7000$

II $fR^{II} = 4R_2 + 6R_3 = 8200$

III $fR^{III} = 10R_3 = 9000$

IV $fR^{IV} = 2R_3 + 8R_4 = 9800$

V $fR^{V} = 10R_4 = 10000$

VI $fR^{VI} = 6R_4 + 4R_5 = 11500$

VII $fR^{VII} = 6R_5 + 4R_6 = 14250$

The realizable and accumulating yields for each cycle are obtained from the general formulae—

$$R_r = \tfrac{1}{2}\left[fR^n - a(R^n - R_x)\right]$$
$$\text{and } R_a = fR^n - R_r$$

Where the co-efficient 'a' is equal to the co-efficient of the first term in the above equations, i.e., 10, 4, 10, 2, etc., for cycles I, II, III, IV, etc., n is the index of R and x is the suffix of R in this term. Thus R_r for cycle IV

$$= \tfrac{1}{2}\left[fR^{IV} - 2(R^{IV} - R_3)\right] = 4820.$$

Cycle	Accruing	1. Realizable 2. Accumulating	Hence available in cycle
I	7000	1. 3500	I - 3500
		2. 3500	
II	8200	1. 3860	II - 7360
		2. 4340	
III	9000	1. 4500	III - 8840
		2. 4500	
IV	9800	1. 4820	IV - 9320
		2. 4980	
V	10000	1. 5000	V - 9980
		2. 5000	
VI	11500	1. 5300	VI - 10300
		2. 6200	
VII	14250	1. 6975	VII - 13175
		2. 7275	Stock-in-hand for cycle VIII

If the stock-in-hand is liquidated in 1, 2, etc., cycles, the realizable yield will be :

If stock-in-hand is liquidated in	Annual realizable yield in cycle						
	I	II	III	IV	V	VI	VII
1 cycle	1850	736	884	932	998	1030	1317
2 cycles	1293	1293	,,	,,	,,	,,	,,
3 cycles	$1156\frac{2}{3}$	$1156\frac{2}{3}$	$1156\frac{2}{3}$	,,	,,	,,	,,
4 cycles	$1100\frac{1}{2}$	$1100\frac{1}{2}$	$1100\frac{1}{2}$	$1100\frac{1}{2}$	,	,,	,,
5 cycles	1080	1080	1080	1080	1080	,,	,,
6 cycles	$1071\frac{2}{3}$	$1071\frac{2}{3}$	$1071\frac{2}{3}$	$1071\frac{2}{3}$	$1071\frac{2}{3}$	$1071\frac{2}{3}$	,,

In other words an yield of $1071\frac{2}{3}$ trees per year is definitely realizable for 6 cycles whereafter, 1317 trees will be realizable. This yield could be obtained by number control, leaving the extra class I trees uniformly distributed, thus—

Total realizable class I trees in cycle I $= 15000 + 3500$

Total prescribed yield ... $= 1071\frac{2}{3} \times 10$

In other words 10716/18500 or 58% of the available class I trees are realizable and not just 7000/18500 or 38% as derived from Smythies formula !