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Micropolar Fluid Past a Sphere Coated with a Thin Fluid Film

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Abstract

The present study deals creeping flow of a micropolar fluid past a sphere coated with thin fluid film. On fluid-film interface we used non-zero spin boundary condition for the micro-rotation vector. Thevariation of drag force with respect to different parameters is studied and some previous results deduced from present analysis.

Keywords: Coated Sphere, Drag Force, Micropolar Fluid, Modified Bessel Function

1. Introduction

The studies involving non Newtonian fluid are interesting in the field of fundamental physical sciences and its applications in the related fields. The knowledge about non-Newtonian fluid finds its application in various industrial and even day-to-day processes such as polymer processing, coating, inkjet printing etc. Non-Newtonian fluid dynamics also helps us to understand natural phenomenon like flow of Earth's mantle, hemodynamic etc. There has been extensive literature about analytical and numerical solutions available regarding non-Newtonian flows on account of complex varieties of fluids, there is no single governing equation that can describe all the properties of every non-Newtonian fluids. In connection to this context, various models of non-Newtonian fluid are on offer such as micropolar fluid model propounded by Eringen[1]. Micropolar fluid model demonstrates the effect of local rotary and couple stresses.

The fine liquid films are almost omnipresent in nature and it is also dealt with by various modern day technologies. Therefore, an understanding of the mechanics involved with the non-Newtonian fluid dynamics is significant and finds its use in various applications. A typical thin film is often composed with the expense of liquid partially encapsulated by some solid substrate with an open surface and there the liquid stays in contact to another fluid.

Gupalo and Ryazantsev[2] studied about coated sphere and using matching asymptomatic expansion, they found the flow field and resisting force experienced by particle coated by a liquid film. Flow past a coated sphere is studied by E. Johnson[3]. In this paper he found the flow field analytically using perturbation scheme except for fluid film profile which required numerical preparation. Different Researchers Kawno and Hashimoto[4], Niefer and Kaloni[5], Choudhury and Padmavati[6], Sadhal and Johnson[7] studied the flow past a coated sphere and given different results.

2. Mathematical Formulation

Let us consider a rigid sphere of radius a coated with a thin Newtonion fluid film of radius $b(b > a)$ in an unbounded medium with origin at the centre O of the sphere. We assume that the coated sphere is stationary and a steady axisymmetric Stokes flow of micropolar fluid has been established around it by a uniform far-field flow with velocity of magnitude U along z-axis.

For outside portion (1) we have taken that the body force and body couple terms are absent. Therefore, the governing equations for outside flow are given by

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$$div\,\mathbf{v}^{(1)}=0, \quad (1)$$

$$-\nabla p^{(1)}+\kappa\nabla\times\boldsymbol{\omega}^{(1)}-(\mu_1+\kappa)\nabla\times\nabla\times\mathbf{v}^{(1)}=0, \quad (2)$$

$$-2\kappa\boldsymbol{\omega}^{(1)}+\kappa\nabla\times\mathbf{v}^{(1)}-\gamma\nabla\times\nabla\times\boldsymbol{\omega}^{(1)}$$
$$+(\alpha+\beta+\gamma)\nabla(\nabla.\boldsymbol{\omega}^{(1)})=0 \quad (3)$$

For the portion (2), inside the fluid sphere governing equation (Happel and Brenner[8]) is given as

$$\mu_2\nabla^2\mathbf{v}^{(2)}=\nabla p^{(2)},\quad div\,\mathbf{v}^{(2)}=0. \quad (4)$$

3. Stream Function Formulation

The velocity and microrotation can be taken in the spherical polar coordinates (r,θ,ϕ) as

$$\mathbf{v}^{(i)}=v_r^{(i)}(r,\theta)\hat{e}_r+v_\theta^{(i)}(r,\theta)\hat{e}_\theta \quad (5)$$

and $$\boldsymbol{\omega}^{(i)}=\upsilon_\phi^{(i)}(r,\theta)\hat{e}_\phi. \quad (6)$$

To make non-dimensionalize the equations and variables, we replace

$$r=a\tilde{r},\quad \psi^{(i)}=Ua^2\tilde{\psi}^{(i)},\ p^{(i)}=\frac{\mu U}{a}\tilde{p}^{(i)},\quad \upsilon_\phi^{(i)}=\frac{U}{a}\tilde{\upsilon}_\phi^{(i)}$$

and dropping tildes subsequently in further analysis.

Using the velocity component in terms of stream function

$$v_r^{(i)}=-\frac{1}{r^2\sin\theta}\frac{\partial\psi^{(i)}}{\partial\theta};\quad v_\theta^{(i)}=\frac{1}{r\,\sin\theta}\frac{\partial\psi^{(i)}}{\partial r}.i=1,2. \quad (7)$$

and pressure from equation (2) and using equation (7), we get

$$E^4\psi^{(1)}-NE^2(\mathrm{r}\sin\theta\,\upsilon_\phi^{(1)})=0. \quad (8)$$

Using above equation (8) in equation (3), we find that

$$\upsilon_\phi^{(1)}=\frac{1}{2r\sin\theta}[E^2\psi^{(1)}+\frac{2-N}{Nm^2}E^4\psi^{(1)}]. \quad (9)$$

Eliminating $\upsilon_\phi^{(1)}$ from equations (8) and (9) we get,

$$E^4(E^2-m^2)\psi^{(1)}=0. \quad (10)$$

Similarly, eliminating the pressure from equations (4) and using equations (7), we obtain the equation as,

$$\mathrm{E}^2(\mathrm{E}^2\ \psi^{(2)})=0, \quad (11)$$

where $E^2=\frac{\partial^2}{\partial r^2}+\frac{(1-\zeta^2)}{r^2}\frac{\partial^2}{\partial\zeta^2}$, $\zeta=\cos\theta$, $m^2=\frac{\kappa(2\mu+\kappa)}{\gamma(\mu+\kappa)}a^2$ and $N=\frac{\kappa}{\mu+\kappa}$ being the coupling number $(0\le N<1)$.

The particular regular solution of equation (10) which satisfies the uniform condition at infinity $\psi_\infty(r,\varsigma)=r^2G_2(\zeta)$ reduces to

$$\psi^{(1)}(r,\zeta)=[r^2+A_2r^{-1}+B_2r+C_2\sqrt{r}K_{3/2}(mr)]G_2(\zeta). \quad (12)$$

Substituting the value of $\psi^{(1)}$ from equation (12) in equation (9), we get microrotation component as

$$\upsilon_\phi^{(1)}(r,\zeta)=\frac{1}{r\sin\theta}[-B_2r^{-1}+\frac{m^2(\mu_1+\kappa)}{\kappa}C_2\sqrt{r}K_{3/2}(mr)]G_2(\zeta). \quad (13)$$

For the inside region of the fluid sphere, the particular regular solution of equation(11) is given by

$$\psi^{(2)}(r,\zeta)=[A_2^*r^2+B_2^*r^{-1}+C_2^*r+D_2^*r^4]G_2(\zeta). \quad (14)$$

4. Boundary Conditions

The mathematically consistent boundary condition for this problem can be taken as:

$$\psi^{(1)}=0\ \text{on}\ r=a, \quad (15)$$

$$\psi^{(2)}=0\ \text{on}\ r=a. \quad (16)$$

Continuity of tangential velocity across the surface i.e.

$$\frac{\partial\psi^{(1)}}{\partial r}=\frac{\partial\psi^{(2)}}{\partial r}\quad\text{on}\ r=a. \quad (17)$$

Continuity of the tangential stress $T_{r\theta}$ i.e.

$$T_{r\theta}^{(1)}=T_{r\theta}^{(2)}\quad\text{on}\ r=a. \quad (18)$$

Non-zero spin on the boundary (Lukaszewicz[10]), i.e.

$$\boldsymbol{\omega}^{(1)}=\frac{\tau}{2}curl\,\mathbf{v}^{(1)}$$

which on simplification provides

$$\upsilon_\phi^{(1)}=\frac{\tau}{2r\sin\theta}E^2\psi^{(1)},\ \text{on}\ r=a. \quad (19)$$

On the inner solid sphere $r = b$, the conditions of impenetrability and no slip provides

$$\psi^{(2)}(\mathrm{r},\zeta)=0 \quad (20)$$

$$\psi_r^{(2)}(\mathrm{r},\zeta)=0 \quad (21)$$

Using these boundary conditions (15)-(21) and equations (12) and (14) and solving the resulting equations we get the values of all constants which are appears in stream functions as

$$A_2 = \frac{\begin{array}{c}-6(1+m)k(-1+\tau)\{\mu_1(8+6l-6l^2-8l^3) \\ +k(4+3l-3l^2-4l^3)-\mu_2(4+6l+6l^2+4l^3)\} \\ +2m^2k\mu_2-3m(\tau-2)+6m\mu_1 \\ +6\mu_2\mu_1+3\}(2+3l+3l^2+2l^3) \\ +km^2\{k\tau m+2m\mu_1\tau+(-2+3\tau)(2\mu_1+k)\} \\ (-4-3l+3l^2+4l^3)\end{array}}{\Delta} \quad (22)$$

$$B_2 = \frac{\begin{array}{c}-3m^2(1+m)(-6\mu_2(2+3l+3l^2+2l^3)+\kappa(-4-3l+3l^2+4l^3) \\ +\mu_1(-8-6l+6l^2+8l^3))(2\mu_1-\kappa(-2+\tau))\end{array}}{\Delta} \quad (23)$$

$$C_2 = \frac{\begin{array}{c}3e^m m^{3/2}\sqrt{\frac{2}{\pi}}\kappa(-6\mu_2(2+3l+3l^2+2l^3) \\ +\kappa(-4-3l+3l^2+4l^3)+\mu_1(-8-6l+6l^2+8l^3))(-1+\tau)\end{array}}{\Delta} \quad (24)$$

$$A_2^* = \frac{\begin{array}{c}-\{3(2\mu_1+\kappa)l^3(1+2l)(m^3(\mu_1+\kappa)+2\kappa(-1+\tau) \\ +2m\kappa(-1+\tau)+m^2(\mu_1+\kappa(-1+2\tau)))\}\end{array}}{\Delta} \quad (25)$$

$$B_2^* = \frac{\begin{array}{c}-3(2\mu_1+\kappa)(2+4l+6l^2+3l^3)\left(m^3(\mu_1+\kappa)\right. \\ +2\kappa(-1+\tau)+2m\kappa(-1+\tau) \\ \left.+m^2(\mu_1+\kappa(-1+2\tau))\right)\end{array}}{\Delta} \quad (26)$$

$$C_2^* = \frac{\begin{array}{c}3(2\mu_1+\kappa)l(3+6l+4l^2+2l)(m^3(\mu_1+\kappa) \\ +2\kappa(-1+\tau)+2m\kappa(-1+\tau) \\ +m^2(\mu_1+\kappa(-1+2\tau)))\end{array}}{\Delta} \quad (27)$$

$$D_2^* = \frac{\begin{array}{c}3(2\mu_1+\kappa)(2+l)(m^3(\mu_1+\kappa)+2(\kappa+m\kappa)(-1+\tau) \\ +m^2(\mu_1+\kappa(-1+2\tau)))\end{array}}{\Delta} \quad (28)$$

Where,

$$\begin{aligned}\Delta = {} & 2[12\{\mu_2\kappa(2+3l+3l^2+2l^3)(-1+\tau) \\ & +12m\mu_2\kappa(2+3l+3l^2+2l^3)(-1+\tau) \\ & +m^3\{6\mu_1{}^2(-4-3l+3l^2+4l^3) \\ & +\kappa\{-\kappa(-4-3l+3l^2+4l^3)(-3+\tau) \\ & +6\mu_2(2+3l+3l^2+2l^3)(-2+\tau)\} \\ & -\mu_1\{12\mu_2(2+3l+3l^2+2l^3) \\ & +\kappa(-4-3l+3l^2+4l^3)(-9+2\tau)\}\} \\ & +m^2\{6\mu_1{}^2(-4-3l+3l^2+4l^3)-\mu_1\end{aligned} \quad (29)$$

5 Calculation of Drag Force

The drag force F experienced by a coated sphere of radius a can be evaluated by the formula

$$F = 2\pi a^2\int_0^\pi [T_{rr}^{(1)}\cos\theta - T_{r\theta}^{(1)}\sin\theta]_{r=a}\sin\theta d\theta \quad (30)$$

Putting the expressions for the stresses in spherical polar coordinate in above equation and evaluating the integral, we found that $F = 2\pi(2\mu_1+k)aUB_2$

$$= \frac{\begin{array}{c}6\pi(2\mu_1+k)aUm^2(1+m)[-6\mu_2(2+3l+3l^2+2l^3) \\ +\kappa(-4-3l+3l^2+4l^3)+\mu_1(-8-6l+6l^2+8l^3)](2\mu_1-\kappa(-2+\tau))\end{array}}{\Delta} \quad (31)$$

Where Δ is given by equation (29)

Case I: Drag for zero spin on the boundary (τ=0)

Putting τ=0 in equation (31) we get drag for without spin as

$$F = \frac{\begin{array}{c}6\pi(2\mu_1+k)aUm^2(1+m)(\mu_1+\kappa) \\ [-6\mu_2(2+3l+3l^2+2l^3)+(2\mu_1+\kappa)(-4-3l+3l^2+4l^3)]\end{array}}{\Delta_1} \quad (32)$$

Where

$$\begin{aligned}\Delta_1 = {} & -24m^2\mu_1{}^2-24m^3\mu_1{}^2-24m^2\mu_1\mu_2 \\ & -24m^3\mu_2\mu_1-28m^2\mu_1\kappa-36m^3\mu_1\kappa \\ & -24\mu_2\kappa-24m\mu_2\kappa-36m^2\mu_2\kappa-24m^3\mu_2\kappa \\ & -54m^2\mu_2\kappa l^2-36m^3\mu_2\kappa l^2 \\ & +6m^2\kappa^2l^2+9m^3\kappa^2l^2+24m^2\mu_2\mu_1{}^2l^3 \\ & +24m^3\mu_1{}^2l^3-24m^2\mu_2\mu_1l^3 \\ & -24m^3\mu_1\mu_2l^3+28m^2\kappa\mu_1l^3\beta \\ & +8m^2\kappa^2l^3+12m^3\kappa^2l^3\end{aligned}$$

Which agree with the result given by Gupta[11]

Case II: Drag on a Newtonian fluid sphere in the micropolar fluid (b→0)

When b=0 i.e. l=0, then coated fluid sphere becomes fluid sphere of radius a In this case the drag force comes out as:

$$F = \frac{6\pi(2\mu_1+k)aUm^2(1+m)(\mu_1+\kappa)(3\mu_2+2\mu_1+\kappa)}{\Delta_2} \quad (33)$$

Where,

$$\Delta 2 = 6\mu_{22\beta} + 3^{\mu 12\beta}$$

Which agree with the result given by Ramkissoon[12].

Case III: Drag on a rigid sphere in an unbounded micropolar fluid (b→a)

If b=a i.e. l=1 , then coated fluid sphere becomes a rigid sphere. Drag force In this case comes out as

$$F = \frac{12\pi(2\mu_1+k)aUm^2(1+m)(\mu_1+\kappa)}{\Delta_3} \quad (34)$$

Which agree with the result given by RamKissoon and Mazumdar[13]

Case IV: Drag on a Newtonian fluid sphere in an unbounded Newtonian fluid (κ→0)

Drag on a Newtonian fluid sphere comes out as

$$F = -6\pi\mu_1 Ua\frac{1+\frac{2}{3}\sigma}{1+\sigma} \quad (35)$$

Where $\sigma = \frac{\mu_1}{\mu_2}$

Which is same as the result given by Happel and Brenner[10].

6. Result and Conclusions

From the Figure 1 we conclude that drag decreases with increasing the values of viscosity μ_1 and decreases rapidly if we take non-zero spin boundary condition in place of zero boundary condition for values of $\mu_2 - 20$, $k - 0.5$, and $l - 0.5$.

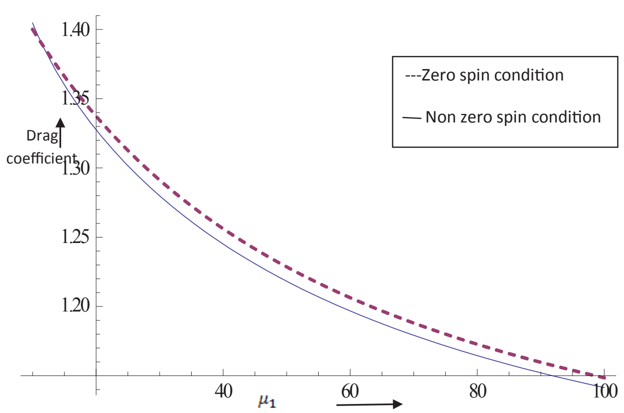

Figure 1. The comparison of Drag coefficient with non-zero and zero spin boundary condition for various values of μ_1 when $\mu_2 - 20$, $k - 0.5$, and $l - 0$.

The variation of drag coefficient on thickness of fluid coating ∈ and spin parameter τ for the values $\mu_1 - 20$, $\mu_2 - 100$, $k - 0.5$ and $m = 20$, is shown in Figure 2. From the figure it is clear that the drag coefficient decreases as thickness of fluid coating ∈ increases. It means that a sphere without fluid coating experience more drag while the present of fluid coating reduces the drag on the sphere, which agrees the result earlier reported by Johnson[2]. The variation of drag coefficient on l and vortex viscosity coefficient k for different values of μ_1 μ_2 τ and m is shown in Figure 3. From the figure it is clear that the drag decreases with increasing the values of vortex viscosity coefficient k and drag coefficient increases with increasing the values of l i.e. if $l > 1$ coated sphere behaves like as rigid sphere and obviously in this case drag increases which again shows that fluid coating reduces the drag force on the sphere. The variation of drag coefficient with respect to viscosities μ_1 and μ_2 is shown in Figure 4. From the figure it is clear that the drag decreases with increasing the values of μ_1 and drag increases with increasing the values of μ_2.

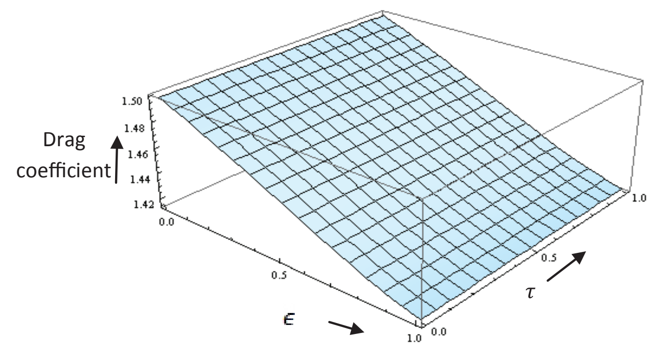

Figure 2. Drag coefficient versus thickness ∈ and τ.

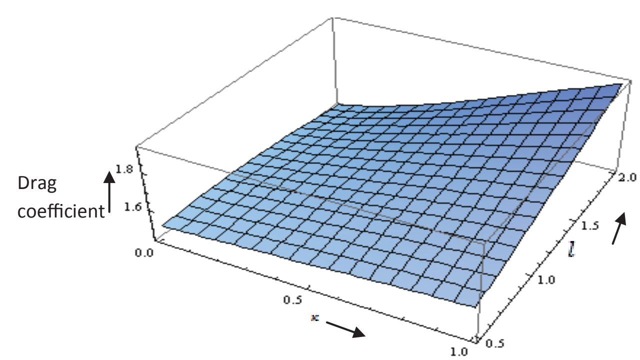

Figure 3. Drag coefficient versus k and l.

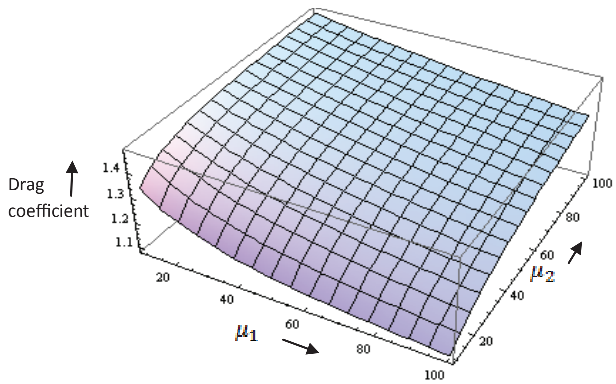

Figure 4. Drag coefficient versus viscosities μ_1 and μ_2.

7. References

1. Eringen AC. Theory of micropolar fluids. Journal of Applied Mathematics and Mechanics. 1966; 16:1–16.
2. Gupalo, Yu P, Ryazantsev YC. Flow past a sphere coated by a liquid film for small Reynolds numbers. Fluid Dynamics. 1974; 9(5):673–82.
3. Johnson RE. Stokes flow past a sphere coated with a thin fluid film. Journal of Fluid Mechanics. 1981; 110:217–38.
4. Kawano S, Hashimoto H. A numerical study on motion of a sphere coated with a thin film at intermediate Reynolds number. Journal of Fluids Engineering. ASME. 1997; 119(2):397–403.
5. Niefer R, Kaloni PN.On the motion of a micropolar fluid drop in a viscous fluid. Journal of Engineering Mathematics. 1980; 14(2):107–16.
6. Choudhuri D, Padmavati BS. A study of an arbitrary stokes flow past a fluid coated sphere in a fluid of a different viscosity. Zeitschrift fu rangew and te Mathematik und Physik. 2010; 61(2):317–28.
7. Sadhal SS, Johnson RE. Stokes flow past bubbles and drops partially coated with thin films. Part 1. Stagnant cap of surfactant film – exact solution. Journal of Fluid Mechanics. 1983; 126:237–50.
8. Happel J, Brenner H. Low Reynolds number hydrodynamics. The Hague: Martinus Nijoff Publishers; 1983.
9. Abramowitz M, Stegun IA. Handbook of mathematical functions. Dover Publications;1970.
10. Lukaszewicz G. Micropolar fluids. Theory and Applications. Boston, MA: Birkhauser. p. 1999.
11. Gupta BR, Deo S. Axisymmetric creeping flow of a micropolar fluid over a sphere coated with a thin fluid film. Journal of Applied Fluid Mechanics. 2013; 6(2):149–55.
12. Ramkissoon H. Flow of micropolar fluid past a Newtonian fluid sphere. Journal of Applied Mathematics and Mechanics/Zeitschrift fur Angew and te Mathematik und Mechanik. 1985; 65(12):635–7.
13. Ramkissoon H, Majumadar SR. Drag on axially symmetric body in the Stokes flow of micropolar fluid. Physics of Fluids. 1976; 19(1):16–21.