Production Planning using Fuzzy Meta–Goal Programming Model

A. K. Bhargava[1], S. R. Singh[2] and Divya Bansal[3*]

[1]Department of Mathematics, M.M.H. College, Ghaziabad - 201001, Uttar Pradesh, India. Affiliated: C.C.S. University, Meerut - 250001, Uttar Pradesh, India; dr.bhargavaak@gmail.com
[2]Department of Mathematics, C.C.S. University, Meerut - 250001, Uttar Pradesh, India; shivrajpundir@gmail.com
[3]Banasthali University, Rajasthan, India; divyabansal.2008@gmail.com

Abstract

In this paper, fuzzy meta-goal programming model is proposed in a manufacturing plant where the objective is to determine the number of units to be produced and hence sold within the given restrictions. Here, three types of meta-goals are considered which are assumed to have fuzzy bounds to manipulate the degree of attainment for the prioritised goals. Hypothetical production planning problem is used to illustrate the same where, LINDO 11.0 optimizer solver is used to draw results of the problem. The decision-maker is given flexibility by means of meta-goals in expressing their preferences. The production planning problem solved using the proposed model shows that the decision-maker can know establish target values not only for goals but also for relevant achievement functions which are assumed fuzzy in nature thus, providing a new dimension to programming by allowing the decision-maker to establish requirements on different achievement function, rather than limiting his/her opinions to the requirements of a single variant. This approach can be applied to more realistic problems on being more flexible than usual GP, namely, textile, coal, sugar industries, etc.

Keywords: Fuzzy Goal Programming, Fuzzy Meta-Goal Programming Method, Manufacturing Plant, Meta Goals, Production Planning

1. Introduction

After the Second World War, the Industrial world faced a depression to solve the various industrial problems. Industrialist learnt that the techniques of OR can conveniently apply to solve industrial problems. Then onwards, various models of OR/GP have been developed to solve industrial problems. In fact GP models are helpful to the managers to solve various problems; they face in their day to day work. These models are used to minimize the cost of production, increase the productivity and use the available resources carefully and for healthy industrial growth. Goal programming technique is an efficient method for evaluating international expansion sites and making selection decisions.

In today's complex organizational environment, the decision maker is regarded as one who attempts to achieve a set of objectives to the fullest possible extent in an environment of conflicting interests, incomplete information and limited resources as studied by Simon[1]. To handle multi-objective decision making a unique approach known as Goal Programming (GP) which reflects the Simon's theory of "satisficing" is widely applied techniques for modern decision-making problems. The advantage of using Goal Programming over other techniques in dealing with real-world decision problems is that it reflects the way managers actually make decisions. Goal Programming allows decision maker to incorporate environmental, organizational and managerial consideration into model through goal levels and priorities. Goal Programming, although far from a panacea, often represents a substantial improvement in the modeling and analysis of the real life situation.

Interest in Goal Programming has increased significantly in the recent past as has its actual implementation. The initial development of the concept of Goal Programming

---

*Author for correspondence

was due to Charnes and Cooper[2], in a discussion of which appeared in 1961 although Charnes, Cooper and Ferguson claim that the idea actually originated in 1955. In essence they proposed a model and approach for dealing with certain linear programming problems in which conflic-tion "goals of management" were included as constraints. Since it might well be impossible to satisfy exactly all such goals, one attempts to minimize the sum of the absolute values of the deviation from such goals.

Goal Programming (GP) also known as Multiple-Criteria Decision Making (MCDM) technique is an extension of linear programming problem involving mul-tiple objectives. Each of these objectives is given a goal or target value to be achieved. GP model seeks to minimize the deviations between the desired goals and the actual results to be obtained according to the assigned priori-ties, that is, certain function of the unwanted deviation variables is minimized[3,4].

2. Methodology

Many literatures present different arguments in favor or against the use of the two GP models: Preemptive Goal Programming and Weighted Goal Programming. For example, Gass[5] described the following reasons of how it may be unrealistic for the decision maker to model a multiple objective problem using the preemptive struc-ture. First, it may be very difficult for the decision maker to set absolute goal hierarchy levels because this assumes infinite trade-offs between different levels. Second, the sequential solution technique may cut-off some parts of the solution space, which might be of interest to the deci-sion maker. On the other hand, a decision maker may find determining absolute goal priority levels in some situations more straight forward than determining pre-cise weights for the goals, this might make weighted GP less favourable compared to the preemptive priority GP[6,7]. Rad et al.[8] for products of Hafez tile factory during one year suggested an aggregate production planning model with the goals viz. maximizing production capacity of fac-tories, minimizing production cost and also providing the market demands. Silva da[9] proposed multi-choice mixed integer goal programming optimization for real problems in a sugar and ethanol milling company.

Providing crisp definition of goal priorities, in prac-tice, is not an easy task, where multi-objective decision problems are considered. Uncertainty may be inher-ent in relative importance relations among the goals or alternatively the perception of the relative importance relations among the goals may be vague form the decision maker's point of view. The decision space and correlation between objectives may also have effects on the definition of importance relations among the goals. Hence, there is a need to develop a FGP model which takes into account these uncertainties and provides a flexible decision mak-ing tool. Often, in certain cases the aspiration levels and/ or priority factors of the decision-makers and sometimes even the weights to be assigned to the goals are impre-cise in nature. In such situations the concepts of fuzzy set theory are useful to achieve desired results[10].

Bellman and Zadeh[11] set the basic principles of deci-sion making in fuzzy environments, which have been used as building blocks of fuzzy linear programming[12]. Since early 1980s, fuzzy sets have been used in GP mod-els to represent uncertain knowledge about a certain parameter[13,14] and to represent a satisfaction degree of the decision maker with respect to his/her preference struc-ture[15–24]. These researchers investigated different types of decision problem using Fuzzy Goal Programming (FGP). Bhargava, Singh and Bansal[25–27] gave the application of FGP for production planning in industry in order to max-imize the production capacity, profit, minimize the extra finishing labor and furnace hours and the manufacturing capacity with different operational constraints, including strategic aim of the company, profit goal, limit on finish-ing and furnace hours needed, cups manufactured with target values being imprecise in nature; FGP approach to frozen food product distribution of small and medium enterprises considering three major objectives, viz. achieving the total distribution of five products of frozen foods to three different locations, maximizing total prof-its and minimizing the total manufacturing costs, bakery production to maximize the daily sales profits, minimize overtime and maximize the utility of machines during the daily production of a small and medium enterprise in producing muffins, cupcakes, brownies, cream puff, cheese tarts and egg tarts. Yaghoobi, Jones and Tamiz[28] applied the conventional MINMAX approach to solve FGP problem in GP. While Goal Programming requires the definite aspiration values set by decision maker for each objective that he/she wishes to achieve, in the Fuzzy Goal Programming (FGP) model all these aspiration lev-els are specified in an imprecise manner. Hannan in 1981 and 1982 assigns aspiration values for the membership functions of the fuzzy goals (which restricts the mem-bership function from full achievement, i.e., unity) and

uses the additive property to aggregate the deviational variables of the membership functions to minimize them. These functions are defined on the interval [0, 1]. For example, if the i-th goal is attained, then the membership function has the value of 1 and the decision maker is fully satisfied, otherwise the membership function assumes the value between 0 and 1. Throughout this paper a fuzzy goal is considered as a goal with imprecise aspiration level.

Iskander M. G.[29] proposed an exponential membership functions in Fuzzy Goal Programming utilized within two main forms of Fuzzy Goal Program. It showcased the comparison between the lexicographical minimization model and the model with a preemptive goal hierarchy applied to a fuzzy textile production planning problem, where the membership functions of the fuzzy goals are considered exponential with either increasing or decreasing rate of change. It can be concluded that, if the decision-maker is not sure about the tolerance limits of the fuzzy data, it is preferable to use the lexicographical minimization model, in order to minimize the sensitivity of the change in results due to the change in data. Also, the lexicographical minimization model is recommended, when the desirable achievement levels for the goals having top priority level are extremely limited, so as to avoid having infeasible solution when the model with a preemptive goal hierarchy is used. This conclusion is applicable whether in the case of increasing or the case of decreasing rate of change. On the other hand, if the decision-maker is seeking to get different set of results based on different tolerance limits that reflect alternative production scenarios, then the model with a preemptive goal hierarchy can be preferable. Finally, whether in the case of increasing or the case of decreasing rate of change, if an optimal solution exists for each of the two models, the trade-off between not getting zero for any membership function and achieving large values for the membership functions with high priority levels is a main criterion for choosing between the two models. Taghizadeh et al.[30] suggested optimization of production planning using FGP fulfilling two objectives, reducing the cost of production, increase revenue leading to increased profits using TIVARY simple collective models and collective weighted method. Hajikarimi et al.[31] used the FGP method in order to improve productivity where productivity factors including factors such as data, process, output, business cycles, competitors and government policies are modeled and a systematic model to improve factors in the field of three productivity areas: materials, capital and human resource is provided resulting in better picture of

the real systems of production and sales. Ighravwe et al.[32] formulated a bi-objective programming-based facility layout design problem having the objectives to minimise workforce costs and maximise efficiency improvement in a layout. In this procedure, Fuzzy Goal Programming and big-bang big-crunch algorithm is used in order to generate a Pareto solution. The proposed model was tested using a small-scale sachet water production enterprise data.

It is clear that real world practitioners of Goal Programming never accept the first solution of a model as the definitive one. In this sense, some kind of sensitivity analysis is always carried out, taking into account some feedback from the decision-maker. Nevertheless, in many cases this sensitivity analysis is carried out using the same GP variant, while other parameters are changed. The purpose of this paper is to provide the users with a formulation, which allows to explore in a more comfortable way, different possible satisficing solutions using several variants at the same time.

In order to address the above problem it has been suggested to formulate a GP model based upon not in a single variant but in a mix of variants. Another way to address this problem is to use Meta-GP model.

Meta-goal is considered as a simultaneous cognitive evaluation on the degree of achievements for original decision goals considered in a GP model. The meta-goal expressed as the utility function of the model, evaluates undesired deviation of each of the goal function d_i in order to communicate concisely with decision-makers the overall status of decision outcomes.

As different goals contribute differently to the final decision, thus, the sensitivity corresponding to the deviations is represented by appropriate weighting factors. Further, in order to ensure that all goal functions are analyzed on the same scale, normalization is one of the popular technique used as it helps to determine the trade offs between different decision goals[33]. But the problem here is deviations may not be fixed and thus required to be fuzzy in nature.

3. Developing Model

The purpose of this paper is to develop different possible satisficing solutions using several variants using Fuzzy Meta-Goal Programming model. It has been shown how this approach can be more flexible than the usual GP models allowing to the decision-makers to establish target values not only for the goals but also for another

criterion functions assuming fuzziness. For this three types of Fuzzy Meta-Goals are considered:

Type 1: A Fuzzy Meta-Goal relating to percentage sum of unwanted deviations.

Type 2: A Fuzzy Meta-Goal relating to the maximum percentage deviation, and

Type 3: A Fuzzy Meta-Goal relating to the percentage of unachieved goals.

The Fuzzy Meta-Goal Programming approach can be very helpful for a decision-maker to clarify his/her knowledge of the problem situation and of his/her own preferences.

Let us consider the following general setting:

$$f_i(\mathbf{x}) + n_i - p_i = t_i, i = 1, 2, ..., l$$

$$g_i(\mathbf{x}) \leq b_j, j = 1, 2, ..., m$$

$$\mathbf{x} \in R^n. \tag{1}$$

Where $f_i(\mathbf{x})$ are concave and $g_i(\mathbf{x})$ are convex functions with l goals and m constraints. Further, it is assumed that the unwanted deviation variables are the negative ones. These unwanted deviation variables can be minimized following a lexicographic, weighted or minmax option. The achievement function of the weighted goal programming model is represented as:

$$\sum_{i=1}^{l} w_i \frac{n_i}{t_i} \tag{2}$$

Where w_i represents a preferential weight and the deviation variables have been normalized, by dividing them among their corresponding target values. If the minmax variant is chosen, then the achievement function becomes:

$$\underset{i=1,2,...,l}{Max} \left\{ w_i \frac{n_i}{t_i} \right\} \tag{3}$$

Representing the minimum maximum percentage weighted deviation from the target values which can be achieved.

Further it is assumed that the decision-maker may want to give aspiration levels for the final values of these achievement functions that is giving goals of the original goals. There are three types of meta-goals can be defined:

Type 1: The percentage sum of unwanted deviation variables cannot surpass a certain bound Q_1, thus, imposing the following constraint:

$$\sum_{i=1}^{l} w_i \frac{n_i}{t_i} \leq Q_1 \tag{4}$$

Type 2: The maximum percentage deviation variables cannot surpass a certain bound Q_2, thus, imposing the following set of constraints:

$$\underset{i=1,2,...,l}{Max} \left\{ w_i \frac{n_i}{t_i} \right\} \leq Q_2 \Leftrightarrow \begin{cases} w_i \frac{n_i}{t_i} - D \leq 0, i = 1, 2, ..., l, \\ D \leq Q_2, \end{cases} \tag{5}$$

Where D represents the maximum percentage weighted deviation.

Type 3: The percentage of achieved goals cannot surpass a certain bound Q_3, thus, imposing the following set of constraints:

$$n_i - M_i y_i \leq 0, i = 1, 2, ..., l$$

$$\frac{\sum_{i=1}^{l} y_i}{l} \leq Q_3 \tag{6}$$

Where y_i are binary variables and M_i represent arbitrarily large values that the corresponding attributes cannot achieve with:

$$\sum_{i=1}^{l} y_i$$

Representing the number of goals that have not been fully achieved.

Further, let us suppose that a type 1 meta-goal is imposed on the set:

$$L_u^{(1)} = C\{1, 2, ..., l\}.$$

Then, the goal takes the form:

$$\sum_{i \in L_u^{(1)}} w_i \frac{n_i}{t_i} \leq Q_u^{(1)}. \tag{7}$$

Similarly, for a type 2 meta-goal on the set $L_v^{(2)}$ and type 3 meta-goal on the set $L_w^{(3)}$, we have:

$$w_i \frac{n_i}{t_i} - D_v \leq 0, i \in L_v^{(2)}, D_v \leq Q_v^{(2)} \tag{8}$$

and

$$n_i - M_i y_i, i \in L_w^{(3)}$$

$$\frac{\sum_{i \in L_w^{(3)}} y_i}{card\left(L_w^{(3)}\right)} \leq Q_w^{(3)}$$

$$y_i \in \{0, 1\}, i \in L_w^{(3)} \tag{9}$$

respectively.

The Meta-Goal Programming model as given by Rodriguez-Uria, Caballero, Ruiz and Romero[34] with r1 type 1 meta-goals, r2 type 2 meta-goals, r3 type 3 meta-goals. In this way, the following meta-GP or $[GP]^2$ model is proposed.

$$Min\left\{\beta_1^{(1)},...,\beta_{r1}^{(1)},\beta_1^{(2)},...,\beta_{r2}^{(2)},\beta_1^{(3)},...,\beta_{r3}^{(3)}\right\}$$

Subject to:

$$f_i(x)+n_i-p_i=t_i, \qquad i=1,2,...,l$$

$$g_i(x)\le b_j, \qquad j=1,2,...,m$$

$$\sum_{i\in L_u^{(1)}}w_i\frac{n_i}{t_i}+a_u^{(1)}-\beta_u^{(1)}=Q_u^{(1)}, \quad u=1,2,...,r_1$$

$$w_i\frac{n_i}{t_i}-D_v\le 0, i\in L_v^{(2)}, \qquad v=1,2,...,r_2$$

$$D_v+a_v^{(2)}-\beta_v^{(2)}=Q_v^{(2)}, \qquad v=1,2,...,r_2$$

$$n_i-M_iy_i, i\in L_w^{(3)}, \qquad w=1,2,...,r_3$$

$$\frac{\sum_{i\in L_w^{(3)}}y_i}{card\left(L_w^{(3)}\right)}+a_w^{(3)}-\beta_w^{(3)}=Q_w^{(3)}, \qquad w=1,2,...,r_3$$

$$y_i\in\{0,1\}, i\in L_w^{(3)}, \qquad w=1,2,...,r_3$$

$$n_i,p_i\ge 0, \qquad i\in 1,2,...,l$$

$$x\in R^n$$

$$a_u^{(1)},\beta_u^{(1)},a_v^{(2)},\beta_v^{(2)},a_w^{(3)},\beta_w^{(3)}\ge 0. \qquad (10)$$

Using the concept of Fuzzy Goal Programming, let us assume that all the three meta-goals defined above have fuzzy bounds. Here, we define only right-sided (positive deviations penalized) linear function as:

$$\mu[f_i(x)]=\begin{cases}1 & f_i(x)\le t_i \\ 1-\dfrac{f_i(x)-t_i}{s_{max}} & t_i\le f_i(x)\le t_i+s_{max} \\ 0 & f_i(x)\ge t_i+s_{max}\end{cases}$$

Where $\mu[f_i(x)]$ represents the fuzzy membership function with respect to the *i-th* goal.
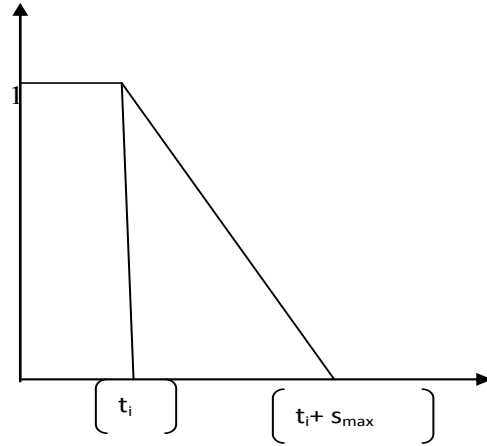

Figure 1. Right-sided linear function.

Assuming that the Q goals consisting of right-sided membership functions give the following algebraic formulation given by Yaghoobi, Jones and Tamiz[28]:

$$Min\,Z=\sum_{i=1}^{Q}\frac{k_is_i}{s_{max}}$$

Subject to:

$$f_i(x)-s_i\le t_i \qquad i=1,...,Q$$

$$\mu_i+\frac{s_i}{s_{max}}=1 \qquad i=1,...,Q$$

$$x\in F$$

$$s_i,\mu_i\ge 0 \qquad i=1,...,Q \qquad (11)$$

Where s_{max} represents the positive deviation level beyond the goal at which total dissatisfaction occurs. μ_i represents the fuzzy membership function achieved level for the i-th goal.

Thus, the three Fuzzy Meta-Goals are defined as follows:

FMG1: $\sum_{i\in L_u^{(1)}}w_i\frac{n_i}{t_i}\lesssim Q_u^{(1)};i\in L_u^{(1)}$

The membership function for FMG1 is defined as:

$$\mu[\sum_{i\in L_u^{(1)}}w_i\frac{n_i}{t_i}]=\begin{cases}1 & ;\sum_{i\in L_u^{(1)}}w_i\frac{n_i}{t_i}\le Q_u^{(1)} \\ 1-\dfrac{\sum_{i\in L_u^{(1)}}w_i\frac{n_i}{t_i}-Q_u^{(1)}}{\beta_{u\,max}^{(1)}} & ;Q_u^{(1)}\le\sum_{i\in L_u^{(1)}}w_i\frac{n_i}{t_i}\le Q_u^{(1)}+\beta_{u\,max}^{(1)} \\ 0 & ;\sum_{i\in L_u^{(1)}}w_i\frac{n_i}{t_i}\ge Q_u^{(1)}+\beta_{u\,max}^{(1)}\end{cases}$$

FMG2: $\underset{i=1,2,...,l}{Max}\left\{w_i\dfrac{n_i}{t_i}\right\}\lesssim Q_v^{(2)}$

$$w_i\frac{n_i}{t_i}-D_v\le 0, i\in L_v^{(2)},$$

$$D_v\lesssim Q_v^{(2)}$$

For FMG2, the membership function is defined as:

$$\mu[D_v]=\begin{cases}1 & ;D_v\le Q_v^{(2)}\\[4pt]1-\dfrac{D_v-Q_v^{(2)}}{\beta_{v\max}^{(2)}} & ;Q_v^{(2)}\le D_v\le Q_v^{(2)}+\beta_{v\max}^{(2)}\\[4pt]0 & ;D_v\ge Q_v^{(2)}+\beta_{v\max}^{(2)}\end{cases}$$

FMG3: $n_i-M_iy_i\le 0, i\in L_w^{(3)}$

$$\frac{\sum_{i\in L_w^{(3)}}y_i}{card\left(L_w^{(3)}\right)}\lesssim Q_w^{(3)}$$

$$y_i\in\{0,1\}, i\in L_w^{(3)}$$

Again for FMG3, the membership function is defined as:

$$\mu[\frac{\sum_{i\in L_w^{(3)}}y_i}{card(L_w^{(3)})}]=\begin{cases}1 & ;\dfrac{\sum_{i\in L_w^{(3)}}y_i}{card(L_w^{(3)})}\le Q_w^{(3)}\\[8pt]1-\dfrac{\frac{\sum_{i\in L_w^{(3)}}y_i}{card(L_w^{(3)})}-Q_w^{(3)}}{\beta_{w\max}^{(3)}} & ;Q_w^{(3)}\le\dfrac{\sum_{i\in L_w^{(3)}}y_i}{card(L_w^{(3)})}\le Q_w^{(3)}+\beta_{w\max}^{(3)}\\[8pt]0 & ;\dfrac{\sum_{i\in L_w^{(3)}}y_i}{card(L_w^{(3)})}\ge Q_w^{(3)}+\beta_{w\max}^{(3)}\end{cases}$$

Thus, FMGP with r_1 type 1 meta-goals, r_2 type 2 meta goals and r_3 type 3 meta goals is formulated as:

$$Min\left\{\sum_{u=1}^{r_1}\frac{\beta_u^{(1)}}{\beta_{u\max}^{(1)}},\sum_{v=1}^{r_2}\frac{\beta_v^{(2)}}{\beta_{v\max}^{(2)}},\sum_{w=1}^{r_3}\frac{\beta_w^{(3)}}{\beta_{w\max}^{(3)}}\right\}$$

Subject to:

$$f_i(x)+n_i-p_i=t_i, \qquad i=1,2,...,l$$

$$g_i(x)\le b_j, \qquad j=1,2,...,m$$

$$\sum_{i\in L_u^{(1)}}w_i\frac{n_i}{t_i}-\beta_u^{(1)}\le Q_u^{(1)}, \qquad u=1,2,...,r_1$$

$$\mu(\sum_{i\in L_u^{(1)}}w_i\frac{n_i}{t_i})+\frac{\beta_u^{(1)}}{\beta_{u\max}^{(1)}}=1, \qquad u=1,2,...,r_1$$

$$w_i\frac{n_i}{t_i}-D_v\le 0, i\in L_v^{(2)}, \qquad v=1,2,...,r_2$$

$$D_v-\beta_v^{(2)}\le Q_v^{(2)}, \qquad v=1,2,...,r_2$$

$$\mu(D_v)+\frac{\beta_v^{(2)}}{\beta_{v\max}^{(2)}}=1 \qquad v=1,2,...,r_2$$

$$n_i-M_iy_i\le 0, i\in L_w^{(3)}, \qquad w=1,2,...,r_3$$

$$\frac{\sum_{i\in L_w^{(3)}}y_i}{card\left(L_w^{(3)}\right)}-\beta_w^{(3)}\le Q_w^{(3)}, \qquad w=1,2,...,r_3$$

$$\mu(\frac{\sum_{i\in L_w^{(3)}}y_i}{card\left(L_w^{(3)}\right)})+\frac{\beta_w^{(3)}}{\beta_{w\max}^{(3)}}=1 \qquad w=1,2,...,r_3$$

$$y_i\in\{0,1\}, i\in L_w^{(3)}, \qquad w=1,2,...,r_3$$

$$n_i,p_i\ge 0, \qquad i\in 1,2,...,l$$

$$x\in R^n$$

$$\beta_u^{(1)},\beta_v^{(2)},\beta_w^{(3)}\ge 0. \qquad (12)$$

4. An Illustrative Example

To illustrate the functioning of the Fuzzy Meta-Goal Programming approach, let us consider the following set of goals and constraints for a production planning problem, where variables x_1 and x_2 represents the number of unit of product A and B manufactured, respectively.

4.1 Goals

$6250\,x_1+5000\,x_2+n_1-p_1=200000$; (Profit level)

$1375\,x_1+1025\,x_2+n_2-p_2=36000$; (Working capital available)

$120\,x_1+180\,x_2+n_3-p_3=4000$; (Annual labour hours available)

$400\,x_1+n_4-p_4=2000$; (Annual labour hours available to manufacture product A)

$450\,x_2+n_5-p_5=2000$; (Annual labour hours available to manufacture product B)

$35\,x_1+35\,x_2+n_6-p_6=1000$; (Machine hours available)

$x_1+x_2+n_7-p_7=15$; (Minimum number of units of A and B together to be manufactured)

4.2 Constraints

$6250\,x_1 + 5000\,x_2 \geq 75000$; (Profit-break-even point)

The normalizing factor will be the respective target value for each goal. The unwanted deviation variables for this example are as follows:

$$n_1, p_2, p_3, p_4, p_5, n_6 + p_6 \text{ and } n_7$$

Thus, weighted goal programming problem is defined as:

Min: $n_1/200000 + p_2/36000 + p_3/4000 + p_4/2000 +$
   $p_5/2000 + (n_6 + p_6)/1000 + n_7/15$

Subject to:

$6250\,x_1 + 5000\,x_2 + n_1 - p_1 = 200000$;
$1375\,x_1 + 1025\,x_2 + n_2 - p_2 = 36000$;
$120\,x_1 + 180\,x_2 + n_3 - p_3 = 4000$;
$400\,x_1 + n_4 - p_4 = 2000$;
$450\,x_2 + n_5 - p_5 = 2000$;
$35\,x_1 + 35\,x_2 + n_6 - p_6 = 1000$;
$x_1 + x_2 + n_7 - p_7 = 15$;
$6250\,x_1 + 5000\,x_2 \geq 75000$;
$x_i, n_j, p_j \geq 0$ (i = 1, 2, 3, 4, 5; j = 1, 2, . . ., 11)

Using the goal programming methodology, the solution is:

$x_1 = 8.44$, $x_2 = 4.44$;
$n_1 = 125000$, $p_1 = 0$, $n_2 = 19833.3$, $p_2 = 0$, $n_3 = 2186.67$,
   $p_3 = 0$, $n_4 = 0$,
$p_4 = 1377.78$, $n_5 = 0$, $p_5 = 0$, $n_6 = 548.89$, $p_6 = 0$, $n_7 = 2.1$,
   $p_7 = 0$.

Objective function (Total deviation): 2.003

Let us assume that the decision-maker does not consider acceptable the above solution. Thus, the above weighted goal programme is extended to a Meta-Goal Programme with the following three meta-goals:

MG1: The percentage maximum deviation from all goals should be less than or equal to 2.10.

MG2: The maximum percentage deviation from any goal should be less than or equal to 0.60.

MG3: Number of goals unsatisfied should be less than or equal to 4.

Thus, the meta-GP formulation is given as:

Min: $(\beta_1) + (\beta_2) + (\beta_3)$
Subject to:
$6250\,x_1 + 5000\,x_2 + n_1 - p_1 = 200000$;

$1375\,x_1 + 1025\,x_2 + n_2 - p_2 = 36000$;
$120\,x_1 + 180\,x_2 + n_3 - p_3 = 4000$;
$400\,x_1 + n_4 - p_4 = 2000$;
$450\,x_2 + n_5 - p_5 = 2000$;
$35\,x_1 + 35\,x_2 + n_6 - p_6 = 1000$;
$x_1 + x_2 + n_7 - p_7 = 15$;
$6250\,x_1 + 5000\,x_2 \geq 75000$;
$n_1 - 200000\,D \leq 0$
$p_2 - 36000\,D \leq 0$
$p_3 - 4000\,D \leq 0$
$p_4 - 2000\,D \leq 0$
$p_5 - 2000\,D \leq 0$
$(n_6 + p_6) - 1000\,D \leq 0$
$n_7 - 15D \leq 0$
$n_1 - 2000000\,y_1 \leq 0$
$p_2 - 360000\,y_2 \leq 0$
$p_3 - 40000\,y_3 \leq 0$
$p_4 - 20000\,y_4 \leq 0$
$p_5 - 20000\,y_5 \leq 0$
$(n_6 + p_6) - 10000\,y_6 \leq 0$
$n_7 - 150\,y_7 \leq 0$
$n_1/200000 + p_2/36000 + p_3/4000 + p_4/2000 + p_5/2000$
   $+ (n_6 + p_6)/1000 + n_7/15 + \alpha_1 - \beta_1 = 2.10$
$D + \alpha_2 - \beta_2 = 0.60$
$(y_1 + y_2 + y_3 + y_4 + y_5 + y_6 + y_7)/7 + \alpha_3 - \beta_3 = 4/7$
$x_i, n_j, p_j \geq 0$ (i = 1, 2, 3, 4, 5; j = 1, 2, . . ., 11)
$y_i \in \{0,1\}$

Where, $y_i = \begin{cases} 1, & \text{if goal i is not satisfied} \\ 0, & \text{otherwise} \end{cases}$, i = 1, 2, ..., 11.

The solution for the above meta-goal programming problem is the following:

$x_1 = 8.44$, $x_2 = 4.44$;
$n_1 = 125000$, $p_1 = 0$, $n_2 = 19833.3$, $p_2 = 0$, $n_3 = 2186.67$,
   $p_3 = 0$, $n_4 = 0$,
$p_4 = 1377.78$, $n_5 = 0$, $p_5 = 0$, $n_6 = 548.89$, $p_6 = 0$, $n_7 =$
   3.56, $p_7 = 1.447$;
$y_1 = 1$, $y_2 = 0$, $y_3 = 0$, $y_4 = 1$, $y_5 = 0$, $y_6 = 1$, $y_7 = 1$;
   $D = 0.689$;
$\alpha_1 = 0$, $\alpha_2 = 0$, $\alpha_3 = 0$;
$\beta_1 = 0$, $\beta_2 = 0.088$, $\beta_3 = 0$.

Now, let us assume that that the target values of the meta-goals is assumed to be fuzzy in nature and thus define the three meta-goals as follows:

FMG1: The percentage maximum deviation from all goals should lie within the range of 1.80 to 2.10.

FMG2: The maximum percentage deviation from any goal should lie within the range of 0.58 to 1.25

FMG3: Number of goals unsatisfied should lie within the range of 3 to 5.

Thus, the membership functions are defined as:

Min: $(\beta_1)/0.3 + (\beta_2)/0.67 + (\beta_3)/3$

Subject to:

$6250\, x_1 + 5000\, x_2 + n_1 - p_1 = 200000;$

$1375\, x_1 + 1025\, x_2 + n_2 - p_2 = 36000;$

$120\, x_1 + 180\, x_2 + n_3 - p_3 = 4000;$

$400\, x_1 + n_4 - p_4 = 2000;$

$450\, x_2 + n_5 - p_5 = 2000;$

$35\, x_1 + 35\, x_2 + n_6 - p_6 = 1000;$

$x_1 + x_2 + n_7 - p_7 = 15;$

$6250\, x_1 + 5000\, x_2 \geq 75000;$

$n_1 - 200000\, D \leq 0$

$p_2 - 36000\, D \leq 0$

$p_3 - 4000 D \leq 0$

$p_4 - 2000\, D \leq 0$

$p_5 - 2000\, D \leq 0$

$(n_6 + p_6) - 1000\, D \leq 0$

$n_7 - 15D \leq 0$

$n_1 - 2000000\, y_1 \leq 0$

$p_2 - 360000\, y_2 \leq 0$

$p_3 - 40000\, y_3 \leq 0$

$p_4 - 20000\, y_4 \leq 0$

$p_5 - 20000\, y_5 \leq 0$

$(n_6 + p_6) - 10000\, y_6 \leq 0$

$n_7 - 150\, y_7 \leq 0$

$n_1/200000 + p_2/36000 + p_3/4000 + p_4/2000 + p_5/2000 + (n_6 + p_6)/1000 + n_7/15 - \beta_1 \leq 1.80$

$\mu_1 + \beta_1/0.3 = 1$

$D - \beta_2 \leq 0.58$

$\mu_2 + \beta_2/0.67 = 1$

$(y_1 + y_2 + y_3 + y_4 + y_5 + y_6 + y_7)/7 - \beta_3 \leq 3/7$

$\mu_3 + \beta_3/2 = 1$

$\mu_1 \leq 1 - ((n_1/200000 + p_2/36000 + p_3/4000 + p_4/2000 + p_5/2000 + (n_6 + p_6)/1000 + n_7/15) - 1.8)/0.3$

$\mu_2 \leq 1 - ((D - 0.58)/0.67)$

$\mu_3 \leq 1 - (((y_1 + y_2 + y_3 + y_4 + y_5 + y_6 + y_7)/7 - 3/7)/2)$

$x_i, n_j, p_j \geq 0\ (i = 1, 2, 3, 4, 5; j = 1, 2, \ldots, 11)$

$y_i \in \{0,1\}$

Where, $y_i = \begin{cases} 1, & \text{if goal i is not satisfied} \\ 0, & \text{otherwise} \end{cases}$, $i = 1, 2, \ldots, 11.$

The solution for the above Fuzzy Meta-Goal Programming problem is the following:

$x_1 = 8.44,\ x_2 = 4.44;$

$n_1 = 125000,\ p_1 = 0,\ n_2 = 19833.3,\ p_2 = 0,\ n_3 = 2186.67,\ p_3 = 0,\ n_4 = 0,$

$p_4 = 1377.78,\ n_5 = 0,\ p_5 = 0,\ n_6 = 548.89,\ p_6 = 0,\ n_7 = 2.11,\ p_7 = 0;$

$y_1 = 1,\ y_2 = 0,\ y_3 = 0,\ y_4 = 1,\ y_5 = 0,\ y_6 = 1,\ y_7 = 1;$

$D = 0.689;$

$\beta_1 = 0.2035,\ \beta_2 = 0.1089,\ \beta_3 = 0.143;$

$\mu_1 = 0.32,\ \mu_2 = 0.84,\ \mu_3 = 0.93.$

5. Conclusion

The above solution is close to weighted goal programming and Meta-Goal Programming problem. From the above solution one can conclude that only three goals namely goals 2, 3, 5 are fully satisfied. The FMGs 2 and 3 are much more achieved with the maximum percentage deviation from any goal to be 0.689 lying within the range 0.58 to 1.25 and number of unsatisfied goals to be 4 lying within the range 3 to 5.

From this paper the decision-maker can now establish target values not only for the goals but also for relevant achievement functions which are assumed fuzzy in nature. Thus, Fuzzy Meta-Goal Programming model can be seen as providing a new dimension to programming by allowing the decision-maker to establish requirements on different achievement functions, rather than limiting his/her opinions to the requirements of a single variant. Thus, this approach is considered as a third stage to the second stage of Meta-Goal Programming after the traditional goal programming problem has been solved.

This approach is much more flexible than usual GP formulations and can be applied to more realistic problems with non-linear goals and/or non-continuous variables.

6. References

1. Simon HA. The new science of management decision: New Jersey, USA: Prentice-Hall, Inc.; 1977.
2. Charnes A, Cooper WW. Management models and industrial application of linear programming (Vol. I): New York: John Wiley and Sons Inc.; 1961.
3. Ignizio JP. Generalized goal programming, Computers and Operations Research. 1983; 10(4):277–89.
4. Romero C. Handbook of critical issues in Goal Programming: Oxford. Pergamon Press, 1991.

5. Gass S. The setting of weights in linear goal programming problems. Computers and Operations Research. 1987; 14(3):227–9.

6. Jones DF, Tamiz M. Goal Programming in the period 1990–2000, in multi-criteria optimization: State of the art annotated bibliographic surveys. Ehrgott M, Gandibleux X (Editors), Kluwer, Dordrecht. 2002. p. 129–70.

7. Romero C. A general structure of achievement function for a goal programming model. European Journal of Operational Research. 2004 Mar; 153(3):675–86.

8. Rad MF, Shirouyehzad H. Proposing an aggregate production planning model by Goal Programming Approach - A case study. Journal of Data Envelopment Analysis and Decision Science. 2014; 1-13.

9. Da Silva AF, Marins FAS. A Fuzzy Goal Programming model for solving aggregate production-planning problems under uncertainty: A case study in a Brazilian sugar mill. Energy Economics. 2014 Sep; 45:196–204.

10. Zadeh LA. Fuzzy Sets. Information and Control. 1965 Jun; 8(3):338–53.

11. Zadeh RE. Decision-making in a fuzzy environment. Journal of the Institute of Management Science. 1970; 17: B141–64.

12. Zimmermann HJ. Fuzzy Set Theory and its applications. USA, Kluwer Academic Publishers; 2001.

13. Mohandas SU, Phelps TA, Ragsdell KM. Structural optimization using a Fuzzy Goal Programming approach. Computers and Structures. 1990; 37(1):1–8.

14. Chanas S, Kuchta D. Fuzzy Goal programming - One notion, many meanings. Control and Cybernetics. 2002; 31(4):871–90.

15. Narasimhan R. Goal programming in a fuzzy environment, Decision Sciences. 1980 Apr; 11(2):325–36.

16. Narasimhan R. On Fuzzy Goal Programming – Some comments. Decision Sciences. 1981 Jul; 12(3):532–8.

17. Narasimhan R. A geometric averaging procedure for constructing supertransitive approximation to binary comparison matrices. Fuzzy Sets and Systems. 1982 Jun; 8(1):53–61.

18. Hannan EL. Linear programming with multiple fuzzy goals. Fuzzy Sets and Systems. 1981 Nov; 6(3):235–48.

19. Hannan EL. On Fuzzy Goal Programming. Decision Sciences. 1981 Jul; 12(3):522–31.

20. Hannan EL. Contrasting Fuzzy Goal Programming and fuzzy multicriteria programming. Decision Sciences. 1982 Apr; 13(2):337–9.

21. Ignizio JP. On the (re)discovery of Fuzzy Goal Programming. Decision Sciences. 1982 Apr; 13(2):331–6.

22. Rubin PA, Narasimhan R. Fuzzy Goal Programming with nested priorities. Fuzzy Sets and Systems. 1984 Nov; 14(2):115–29.

23. Tiwari RN, Dharmar S, Rao JR. Priority structure in Fuzzy Goal Programming. Fuzzy Sets and Systems. 1986 Jul; 19(3):251–9.

24. Tiwari RN, Rao JR, Dharmar S. Some aspects of Fuzzy Goal Programming. International Symposium on Mathematical Modelling of Ecological Environmental and Biological Systems; Kanpur, India. 1985.

25. Bhargava AK, Singh SR, Bansal D. A Fuzzy Goal Programming model for bakery production. International Journal of Computer and Mathematical Science. 2014 Jun; 3(4):1–11. ISSN: 2347-8527.

26. Bhargava AK, Singh SR, Bansal D. A Fuzzy Goal Programming approach for food product distribution of small and medium enterprises. Annals of Pure and Applied Mathematics. 2015; 9(2):157–66. ISSN: 2279-087X (P), 2279-0888(online).

27. Bhargava AK, Singh SR, Bansal D. Fuzzy Goal Programming techniques for production planning in industry. International Journal of Computer Applications Technology and Research. 2015; 4(2):92–6. ISSN: 2319–8656.

28. Yaghoobi MA, Jones DF, Tamiz M. Weighted additive models for solving Fuzzy Goal Programming problems. Asia Pacific Journal of Operational Research. 2008; 25(5):715–33.

29. Iskander MG. Exponential membership functions in Fuzzy Goal Programming: A computational application to a production problem in the textile industry. American Journal of Computational and Applied Mathematics. 2015; 5(1):1–6.

30. Taghizadeh H, Bazrkar A, Abedzadeh M. Optimization production planning using Fuzzy Goal Programming Techniques. Modern Applied Science. 2015 Jul; 9(9):68–77.

31. Hajikarimi A, Rahmani K, Farahmand NF. Designing a new model to improve productivity factors implementing the Fuzzy Goal Programming method. Indian Journal of Science and Technology. 2015 May; 8(S9):9´15.

32. Ighravwe DE, Oke SA. A combined Fuzzy Goal Programming and Big-Bang Big-Crunch algorithm for workforce optimisation with facility layout consideration. Engineering Journal. 2015; 19(2):7198.

33. Lin HW, Nagalingam S, Chiu M. Development of a dynamic task rescheduling approach for small and medium manufacturing enterprises. Eighth International Conference on Manufacturing and Management. Queensland, Australia: Pacific Congress on Manufacturing and Management; 2004 Feb.

34. Rodriguez-Uria MV, Caballero R, Ruiz F, Romero C. Meta-goal programming. European Journal of Operational Research. 2002 Jan; 136(2):4229.