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An Optimal Replenishment Policy for Non Instantaneous Deteriorating Items with Stock Dependent, Price Decreasing Demand and Partial Backlogging

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Abstract

This study is about mathematical modelling of deteriorating items in which demand pattern depends upon the current stock level and selling price of the product. The objective of study is to analyze profit in the business industry. We used optimization techniques and sensitivity analysis to maximize the profit as well as to find the optimal order quantity, optimal selling price and optimal order time to illustrate the behavior of the model. The developed model has wide applications for items such as electronics, high-tech product, automobile, cosmetics, garments, fashionable, commodities, fruits etc. in which stock and selling price play an important role for their demand.

Keywords: Demand, Deterioration, Inventory, Sensitivity Analysis

1. Introduction

Many researchers have taken demand of items as exponential increasing or decreasing, quadratic, time dependent, stock dependent, linearly increasing or decreasing and constant demand. After some time it has observed that the above demand does not reflect best for smooth running of business. The majority physical goods undergo deterioration over time such as volatile liquids, medicines, blood, etc. Rajoria et al.[1] have proposed an inventory model having ramp type demand pattern with partial backlogging. Tayal et al.[2] have studied economic production inventory model with trapezoidal type of demand in quantity elasticity. Ghare and Schrader[3] have done the first attempt to depict optimal ordering policies for decay item. After some time, Covert and Philip[4] explained the model with variable deteriorating rate of two-parameter Weibull distribution. Chung-Yung Dye[5] explained the rate of demand function which is instantaneous stock dependent demand by proposing a time-proportional backlogging rate to realistic theory good applicable in practice. Ramp type demand and time and time deteriorating rate of items with Heaviside's function are taken by S. K. Manna and K. S. Chaudhori[6]. Chun-Tao Chang et al.[7] established an EOQ model for perishable items under stock-dependent selling rate and time dependent partial backlogging with constant deterioration rate and unsatisfied demand in case of backlogging.

Nowadays in the super markets have come with varied items influenced by customer demand, pricing and deterioration factors. So we develop a inventory model in which demand is stock dependent with decreasing price. Our aim is to optimize the total profit for retailers. Numerical examples are explained using the Newton Rapshon method for solving the system of nonlinear equation with the help of mathematica software which is clarified through a sensitivity analysis and showing the graphical representation.

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2. Proposed Model

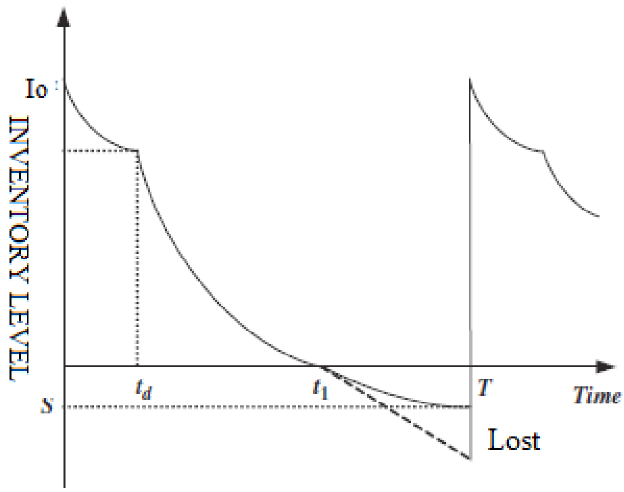

Figure 1. Graphical representation of inventory system.

3. Notations

To obtain the solution of the model are used by the following primary symbol.

$TP(p^*, t_1^*, T^*)$ The optimal total profit per unite time of the inventory system

$TP(p, t_1, T)$ The total profit per unite time of the inventory system

$PF(t_1, T, p)$ Average profit of the system

I_0 Maximum inventory level at initial time $t = 0$

$I_1(t)$ Inventory level at any time t, $0 < t < t_d$

$I_2(t)$ Inventory level at any time t, $t_d < t < t_1$

$I_3(t)$ Inventory level at any time t, $t_1 \le t \le T$

T The length of the replenishment cycle time

S The maximum amount of demand shortage level

p Selling price per unit of the item

p^* The optimal selling price per unit

t_1^* The optimal length of time in which there is no inventory shortage

T^* The optimal length of the length of the replenishment cycle time

Q^* Optimal ordering quantity

Q Ordering quantity

θ The constant deterioration rate, where $0 \le \theta < 1$

t_d The length of time in which the product exhibits no deterioration

t_1 Time when inventory level reaches to zero

c_s Shortage cost per unit per unit time

c_l Lost sale cost per unit

c_h Holding cost per unit per unit time

c_d Cost of deteriorated unit

c_p Purchase cost per unit

A Ordering cost per order

4. Assumptions

The following fundamental notations are used to derive the solution of the model.

- There is no deterioration up to the time t_d and end of this period the inventory item deteriorates with constant rate θ.
- The demand function $D(t)$ rate at time t which is assumed as stock and time dependent, is expressed as:
- $$D(t) = \begin{cases} a + bI(t) - cp, & \mathrm{I}(t) \ge 0 \\ a - cp, & \mathrm{I}(t) \le 0 \end{cases}$$

  Where a, b, c are positive constant and p is selling price per unit item and I(t) is inventory level at any time t.

- Shortages are allowed and partially backlogging rate is inversely proportional to waiting time smaller the backlogging rate and vice versa and it is defined as: $B(T-t) = \dfrac{1}{1+\delta(T-t)}$ where $\delta(\delta > 0)$ is any arbitrary constant and $(T - t)$ is waiting time up to next replenishment.
- Lead time is zero and replenishment rate is infinite.

5. Mathematical Analysis

The variation of inventory level $I_1(t)$ changes with respect to t due to effects of demand as well as there is no deterioration. Hence the variation of inventory levels can be described by the differential equation as follow:

$$\frac{dI_1(t)}{dt} = -D(t) \qquad 0 \le \mathrm{t} \le \mathrm{t_d} \tag{1}$$

With boundary condition $I_1(t) = I_0$ when at $t = 0$ (2)

Solution of the differential Equation (1) with boundary conditions, the inventory level with decay

$I_1(t)$ at any time t is described as follow:

$$I_1(t)=\frac{(-a+cp)}{(\theta+b)}\left[1-e^{-bt}\right]+I_0e^{-bt} \quad (3)$$

The variation of inventory level $I_2(t)$ changes with respect to t due to effects of demand as well as there is deterioration. Hence the variation of inventory levels can be described by the differential equation as follow:

$$\frac{dI_2(t)}{dt}+\theta I_2(t)=-D(t) \qquad t_d\le t\le t_1 \quad (4)$$

With boundary condition $I_2(t)=0$ when at $t=t_1$ (5)

Solution of the differential Equation (4) with boundary conditions, the inventory level with decay $I_2(t)$ at any time t is described as follow: $I_2(t)=\left(\frac{-a+cp}{\theta+b}\right)\left[1-e^{(\theta+b)(t_1-t)}\right]$ (6)

Again boundary condition $I_1(t) = I_2(t)$ when at $t = t_d$ then the maximum inventory level is given by:

$$I_0=\left(\frac{-a+cp}{b}\right)\left[1-e^{bt_d}\right]+\left(\frac{-a+cp}{\theta+b}\right)\left[e^{bt_d}-e^{(\theta+b)t_1-\theta t_d}\right] \quad (7)$$

Using (3) and (7) we get

$$I_1(t)=\frac{(-a+cp)}{(\theta+b)}\left[1-e^{bt_d-t)}\right]+\left(\frac{-a+cp}{\theta+b}\right)\left[e^{b(t_d-t)}-e^{(\theta+b)t_1-\theta t_d-bt}\right] \quad (8)$$

The variation of shortage level as t during the inventory level $[t_1$ T] is described by the differential equation:

$$\frac{dI_3(t)}{dt}=-\frac{D(t)}{1+\delta(T-t)}, \quad t_1\le t\le T \quad (9)$$

With boundary condition $I_3(t)=0$ when *at* $t=t_1$ then the solution of the Equation (9) is represented by:

$$I_3(t)=\left(\frac{-a+cp}{\delta}\right)\left[\log(1+\delta(T-t_1)-\log(1+\delta(T-t)\right] \quad (10)$$

Now at the time $t = T$ in 10 we get the maximum of demand backlogged per period is given by:

$$S\equiv -I_3(T)=\left(\frac{a-cp}{\delta}\right)\left(\log(1+\delta(T-t_1)\right) \quad (11)$$

The order quantity can be obtained from Equations (7) and 11 that is represented by:

$$Q=I_0+S$$

Or

$$Q=\left(\frac{-a+cp}{b}\right)\left[1-e^{bt_d}\right]+\left(\frac{-a+cp}{\theta+b}\right)\left[e^{bt_d}-e^{(\theta+b)t_1-\theta t_d}\right]+\left(\frac{a-cp}{\delta}\right)\left(\log(1+\delta(T-t_1)\right) \quad (12)$$

The total relevant inventory cost per cycle including following factors:

- A is the Setup cost per cycle.
- The purchase cost for the total order quantity is expressed as:

$$PC=C_pQ$$

$$=c_p\left(\frac{-a+cp}{b}\right)\left[1-e^{bt_d}\right]+\left(\frac{-a+cp}{\theta+b}\right)\left[e^{bt_d}-e^{(\theta+b)t_1-\theta t_d}\right]+\left(\frac{a-cp}{\delta}\right)\left(\log(1+\delta(T-t_1)\right)$$

- Inventory holding/storage cost of the system is given by:

$$HC=c_h\left[\int_0^{t_d}I_1(t)dt+\int_{t_d}^{t_1}I_2(t)dt\right]$$

$$\begin{aligned}\mathrm{HC}=c_h\Bigg[&\left(\frac{-a+cp}{b}\right)t_d+\left(\frac{-a+cp}{b^2}\right)\left(1-e^{bt_d}\right)-\left(\frac{-a+cp}{b^2+b\theta}\right)\left(1-e^{bt_d}\right)\\&+\left(\frac{-a+cp}{b^2+b\theta}\right)e^{(b+\theta)(t_1-t_d)}\\&-\left(\frac{-a+cp}{b^2+b\theta}\right)e^{(b+\theta)t_1-\theta t_d}+\left(\frac{-a+cp}{b+\theta}\right)\left(t_1-t_d\right)\\&+\left(\frac{-a+cp}{(b+\theta)^2}\right)\left(1-e^{(b+\theta)(t_1-t_d)}\right)\Bigg]\end{aligned}$$

- The deterioration cost per cycle is represented by:

$$DC=c_d\left[I_0-\int_0^{t_1}D(t)dt\right]$$

$$\mathrm{DC} = c_d\Bigg[-\left(\frac{-a+cp}{b+\theta}\right)\left(e^{bt_d} - e^{(b+\theta)t_1-\theta t_d}\right) + \left(\frac{-a+cp}{b^2+b\theta}\right)e^{(b+\theta)(t_1-t_d)} - (-a+cp)t_d - \left(\frac{-a+cp}{b+\theta}\right)\left(e^{bt_d}-1\right) - at_1 + ct_1 - \left(\frac{-a+cp}{b+\theta}\right)\left(e^{(b+\theta)(t_1-t_d)} - e^{(b+\theta)t_1-\theta t_d}\right) - \left(\frac{-ab+bcp}{b+\theta}\right)(t_1-t_d) - \left(\frac{-ab+bcp}{b+\theta}\right)\left(1-e^{(b+\theta)(t_1-t_d)}\right)\Bigg]$$

- The cost due to lost sales is given by:

$$LC = c_l\int_{t_1}^{T} D(t)\left(1-\frac{1}{1+\delta(T-t)}\right)dt$$

$$LC = c_l(-a+cp)\left[(T-t_1) - \frac{\log\left[1+\delta(T-t_1)\right]}{\delta}\right]$$

- Total sales revenue is represented by:

$$\mathrm{SR} = p\left[\int_0^{t_1} D(t)dt - I_3(T)\right]$$

$$SR = \Bigg[(-ab+bcp)t_d + \left(\frac{-a+cp}{b}\right)\left(1-e^{bt_d}\right) - \left(\frac{-a+cp}{b+\theta}\right)\left(1-e^{bt_d}\right) + at_1 - cpt_1 + \left(e^{(b+\theta)(t_1-t_d)} - e^{(b+\theta)t_1-\theta t_d}\right) + \left(\frac{-ab+bcp}{b+\theta}\right)(t_1-t_d) + \left(\frac{-ab+bcp}{(b+\theta)^2}\right)\left(1-e^{(b+\theta)(t_1-t_d)}\right) - \left(\frac{a-cp}{\delta}\right)\left(\log(1+\delta(T-t_1))\right)\Bigg]$$

- The shortage cost in the entire cycle is described by:

$$SC = -c_s\int_{t_1}^{T} I_3(t)dt$$

$$\Rightarrow = c_s\left(\frac{-a+cp}{\delta}\right)\left[(T-t_1) - \frac{1}{\delta}\log\left(1+\delta(T-t_1)\right)\right]$$

Total profit of the system per unit time per cycle is given by:

$$PF(t_1,T,p) = \frac{1}{T}\left[SR-(A+HC+DC+SC+LC+PC)\right] \tag{13}$$

5.1 Optimality

In our expression to determine the optimal value of t_1, T and p which maximizes *PF(t1, T, p)*. The necessary conditions for maximization of the total profit function given by Equation (13) are:

$$\frac{\partial PF_N(t_1,T,p)}{\partial t_1} = 0,\quad \frac{\partial PF_N(t_1,T,p)}{\partial T} = 0 \text{ and } \frac{\partial PF_N(t_1,T,p)}{\partial p} = 0 \text{ for N=1, 2} \tag{14}$$

Equation (14) can be solved simultaneously for the optimal value for t_1, T and p. The sufficient condition for maximizing $PF_N(t_1,T,p)$ using the Hessian matrix H, which is the matrix of second order partial derivatives are:

$$H = \begin{pmatrix} \frac{\partial^2 PF_N(t_1,T,p)}{\partial t_1^2} & \frac{\partial^2 PF_N(t_1,T,p)}{\partial t_1\partial T} & \frac{\partial^2 PF_N(t_1,T,p)}{\partial t_1\partial p} \\ \frac{\partial^2 PF_N(t_1,T,p)}{\partial T\partial t_1} & \frac{\partial^2 PF_N(t_1,T,p)}{\partial T^2} & \frac{\partial^2 PF_N(t_1,T,p)}{\partial T\partial p} \\ \frac{\partial^2 PF_N(t_1,T,p)}{\partial p\partial t_1} & \frac{\partial^2 PF_N(t_1,T,p)}{\partial p\partial T} & \frac{\partial^2 PF_N(t_1,T,p)}{\partial t_1^2} \end{pmatrix}$$

$$H_1 = \frac{\partial^2 PF_N(t_1,T,p)}{\partial t_1^2} < 0,$$

$$H_2 = \begin{pmatrix} \frac{\partial^2 PF_N(t_1,T,p)}{\partial t_1^2} & \frac{\partial^2 PF_N(t_1,T,p)}{\partial t_1\partial T} \\ \frac{\partial^2 PF_N(t_1,T,p)}{\partial T\partial t_1} & \frac{\partial^2 PF_N(t_1,T,p)}{\partial T^2} \end{pmatrix} > 0$$

$$\text{And } H_3 = \det \text{ H=}\begin{pmatrix} \frac{\partial^2 PF_N(t_1,T,p)}{\partial t_1^2} & \frac{\partial^2 PF_N(t_1,T,p)}{\partial t_1\partial T} & \frac{\partial^2 PF_N(t_1,T,p)}{\partial t_1\partial p} \\ \frac{\partial^2 PF_N(t_1,T,p)}{\partial T\partial t_1} & \frac{\partial^2 PF_N(t_1,T,p)}{\partial T^2} & \frac{\partial^2 PF_N(t_1,T,p)}{\partial T\partial p} \\ \frac{\partial^2 PF_N(t_1,T,p)}{\partial p\partial t_1} & \frac{\partial^2 PF_N(t_1,T,p)}{\partial p\partial T} & \frac{\partial^2 PF_N(t_1,T,p)}{\partial t_1^2} \end{pmatrix} < 0$$

Where H_1, H_2, and H_3 are the minors of the Hessian matrix H. Using these optimal values of t_1, T and p, the optimal values of Q and maximum average profit can be obtained.

6. Profit Function for Some Special Cases

Case 1: Model with completely backlogging.

This case $\delta = 0$ or $B(T-t) = 1$ hence, no lost sale or $LC = 0$

And

$$Q = \left[\left(\frac{-a+cp}{b}\right)\left(1-e^{bt_d}\right)+\left(\frac{-a+cp}{\theta+b}\right)\left(e^{bt_d}-e^{(\theta+b)t_1-\theta t_d}\right)-(a-cp)(t_1-T)\right]$$

The relevant profit per unit time represented by:

$$PF = \frac{1}{T}\left[SR - (A+HC+DC+SC+LC+PC)\right]$$

Where

$$\begin{aligned} SR = \Big[&(-ab+bcp)t_d+\left(\frac{-a+cp}{b}\right)\left(1-e^{bt_d}\right) \\ &-\left(\frac{-a+cp}{b+\theta}\right)\left(1-e^{bt_d}\right)+at_1-cpt_1 \\ &+\left(e^{(b+\theta)(t_1-t_d)}-e^{(b+\theta)t_1-\theta t_d}\right) \\ &+\left(\frac{-ab+bcp}{b+\theta}\right)(t_1-t_d) \\ &+\left(\frac{-ab+bcp}{(b+\theta)^2}\right)\left(1-e^{(b+\theta)(t_1-t_d)}\right) \\ &+(a-cp)(T-t_1)\Big] \end{aligned}$$

$$\begin{aligned} HC = c_h\Bigg[&\left(\frac{-a+cp}{b}\right)t_d+\left(\frac{-a+cp}{b^2}\right)\left(1-e^{bt_d}\right) \\ &-\left(\frac{-a+cp}{b^2+b\theta}\right)\left(1-e^{bt_d}\right)+\left(\frac{-a+cp}{b^2+b\theta}\right)e^{(b+\theta)(t_1-t_d)} \\ &-\left(\frac{-a+cp}{b^2+b\theta}\right)e^{(b+\theta)t_1-\theta t_d}+\left(\frac{-a+cp}{b+\theta}\right)(t_1-t_d) \\ &+\left(\frac{-a+cp}{(b+\theta)^2}\right)\left(1-e^{(b+\theta)(t_1-t_d)}\right)\Bigg] \end{aligned}$$

$$\begin{aligned} DC = c_d\Bigg[&-\left(\frac{-a+cp}{b+\theta}\right)\left(e^{bt_d}-e^{(b+\theta)t_1-\theta t_d}\right) \\ &+\left(\frac{-a+cp}{b^2+b\theta}\right)e^{(b+\theta)(t_1-t_d)}-(-a+cp)t_d \\ &-\left(\frac{-a+cp}{b+\theta}\right)\left(e^{bt_d}-1\right)-at_1+ct_1 \\ &-\left(\frac{-a+cp}{b+\theta}\right)\left(e^{(b+\theta)(t_1-t_d)}-e^{(b+\theta)t_1-\theta t_d}\right) \\ &-\left(\frac{-ab+bcp}{b+\theta}\right)(t_1-t_d) \\ &-\left(\frac{-ab+bcp}{b+\theta}\right)\left(1-e^{(b+\theta)(t_1-t_d)}\right)\Bigg] \end{aligned}$$

$$PC = c_p\left[\left(\frac{-a+cp}{b}\right)\left(1-e^{bt_d}\right)+\left(\frac{-a+cp)}{\theta+b}\right)\left(e^{bt_d}-e^{(\theta+b)t_1-\theta t_d}\right)-(a-cp)(t_1-T)\right]$$

$$SC = (-a+cp)\, c_s\left[Tt_1-\frac{T^2}{2}-\frac{t_1^2}{2}\right]$$

Case 2: Model without shortage

Since in this model there is no shortage that is $\delta \to \infty$ then $T = t_1$, hence shortage cost and lost sale cost vanishes or $SC = 0$, $LC = 0$

And

$$Q = \left(\frac{a-cp}{\theta+b}\right)\left(-e^{bt_d}+e^{(\theta+b)t_1-\theta t_d}\right)-\left(\frac{a-cp}{b}\right)\left(b-e^{bt_d}\right)$$

$$PF = \frac{1}{T}\left[SR - (A+HC+DC+SC+LC+PC)\right]$$

Where

$$\begin{aligned} SR = p\Bigg[&\left\{\left(\frac{a-cp}{\theta+b}\right)\left(-e^{bt_d}+e^{(\theta+b)t_1-\theta t_d}\right)-\left(\frac{a-cp}{b}\right)\left(b-e^{bt_d}\right)\right\} \\ &\left(\frac{1-e^{bt_d}}{b}\right)+\left(\frac{a-cp}{b}\right)\left(1-e^{bt_d}-t_d\right)\Big) \\ &+\left(\frac{a-cp}{\theta+b}\right)\left\{\left(-T-\left(\frac{1}{\theta+b}\right)\right)-\left(t_d-\left(\frac{e^{(\theta+b)(T-t_d)}}{\theta+b}\right)\right)\right\}\Bigg] \end{aligned}$$

$$HC = c_h\left[\left\{\left(\frac{a-cp}{\theta+b}\right)\left(-e^{bt_d}+e^{(\theta+b)t_1-\theta t_d}\right)-\left(\frac{a-cp}{b}\right)\left(b-e^{bt_d}\right)\right\}\right.$$
$$\left(\frac{1-e^{bt_d}}{b}\right)+\left(\frac{a-cp}{b}\right)\left(1-e^{bt_d}-t_d\right)$$
$$\left.+\left(\frac{a-cp}{\theta+b}\right)\left\{\left(-T-\left(\frac{1}{\theta+b}\right)\right)-\left(t_d-\left(\frac{e^{(\theta+b)(T-t_d)}}{\theta+b}\right)\right)\right\}\right]$$

$$DC = c_d\left[\left\{\left(\frac{a-cp}{\theta+b}\right)\left(-e^{bt_d}+e^{(\theta+b)t_1-\theta t_d}\right)-\left(\frac{a-cp}{b}\right)\left(b-e^{bt_d}\right)\right\}\right.$$
$$e^{-bt_d}-T(a-cp)+(a-cp)\left(1-e^{bt_d}-t_d\right)$$
$$\left.+\left(\frac{ab-bcp}{\theta+b}\right)\left\{\left(-T-\left(\frac{1}{\theta+b}\right)\right)-\left(t_d-\left(\frac{e^{(\theta+b)(T-t_d)}}{\theta+b}\right)\right)\right\}\right]$$

$$PC = C_p Q$$

Case 3: Model with instantaneous deterioration and completely backlogging.

In this situation $t_d = 0$ and $\delta = 0$ then lost sale cost vanishes or $LC = 0$ and

$$Q = \left(\frac{a-cp}{\theta+b}\right)\left(1-e^{(\theta+b)t_1}\right)-(a-cp)(t_1-T)$$

And

$$PF = \frac{1}{T}\left[SR - (A+HC+DC+SC+LC+PC)\right]$$

Where

$$SR = p\left[\left(\frac{a-cp}{(\theta+b)^2}\right)\left(3-2e^{-(\theta+b)t_1}-e^{(\theta+b)t_1}\right)-(a-cp)(t_1-T)\right]$$

$$HC = c_h\left[\left(\frac{a-cp}{(\theta+b)^2}\right)\left(3-2e^{-(\theta+b)t_1}-e^{(\theta+b)t_1}\right)\right]$$

$$DC = c_d\left[\left(\frac{a-cp}{\theta+b}\right)\left(1-e^{(\theta+b)t_1}\right)-(a-cp)t_1\right.$$
$$\left.-\left(\frac{ab-bcp}{(\theta+b)^2}\right)\left(3-2e^{-(\theta+b)t_1}-e^{(\theta+b)t_1}\right)\right]$$

$$SC = (-a+cp)c_s\left[Tt_1-\frac{T^2}{2}-\frac{t_1^2}{2}\right]$$

$$PC = c_P Q$$

Case 4: Model with instantaneous deterioration and without shortage.

In this situation $t_d = 0$, $\delta \to \infty$ then $T = t_1$, hence shortage cost and lost sale cost vanishes or $SC = 0$, $LC = 0$ and the order amount of quantity and profit are respectively given by: $Q = \left(\frac{a-cp}{\theta+b}\right)\left(e^{(\theta+b)t_1}-1\right)$

And

$$PF = \frac{1}{T}\left[SR - (A+HC+DC+SC+LC+PC)\right]$$

Where

$$SR = p\left[\left(\frac{a-cp}{(\theta+b)}\right)\left\{\frac{\left(e^{(\theta+b)T}-1\right)}{(\theta+b)}-T\right\}\right]$$

$$HC = c_h\left[\left(\frac{a-cp}{(\theta+b)}\right)\left\{\frac{\left(e^{(\theta+b)T}-1\right)}{(\theta+b)}-T\right\}\right]$$

$$DC = c_d\left[\left(\frac{a-cp}{\theta+b}\right)\left(e^{(\theta+b)t_1}-1\right)-(a-cp)T\right.$$
$$\left.-\left(\frac{ab-bcp}{(\theta+b)}\right)\left\{\frac{\left(e^{(\theta+b)T}-1\right)}{(\theta+b)}-T\right\}\right]$$

$$PC = c_P Q$$

7. Numerical Examples

Example 1.

For the numerical illustration, we consider an inventory system with the following parameter in proper unit. Where for T and t_1 take fixed value.

c_d = \$1 / *unit*, c_l = \$1 / *per unit*, c_h = \$1 / *per unit / per unit time*, $c = 1$,

c_s = \$1 / *per unit / per unit time*, c_p = \$1 / *per unit*, $\theta = 0.04$, $\delta = 0.04$

$t_d = 0.2$ year, $t_1 = 0.5$ year, $T = 0.791$ year, $a = 100$ *units / year*,

$b = 0.06$, $A = 40$ / *per order*,

Then using Equation (13), on solving system of non linear equation by Newton Rapshon method by we get:

The optimal price $p^* = \$50.609$

Optimal order quantity $Q^* = 39.487$,

Optimal Total average profit of the system $TPF^* = \$2407.16$,

8. Graphical Analysis

Using the derivation of Example 1 and taken the numerical values with the help of Mathematica software the graphical representation of the effect of selling price p when on hand inventory reduces to zero, on profit is done in Figure 2 as follows:

Example 2.

For the numerical illustration we consider an inventory system with the following parameter in proper unit, where T takes fixed value.

$c_d = \$1$ / *unit*, $c_l = \$1$ / *per unit*, $c_h = \$1$ / *per unit* / *per unit time*, $c = 10$,

$c_s = \$1$ / *per unit time*, $c_p = \$1$ / *per unit*, $\theta = 0.6$, $\delta = 1$ $t_d = 0.3$ *year*,

$T = 2$ *year*, $a = 50$ *units* / *year*, $b = 0.6$, $A = 1$/ *per order*,

Then using Equation (13) on solving system of non linear equation by Newton Rapshon method by:

The optimal time $t_1^* = 1.737$ *year*,

The optimal price $p^* = \$3.729$,

Optimal order quantity $Q^* = 65.58$,

Optimal Total average profit of the system $TPF = \$31.44$,

9. Sensitivity Analysis

Sensitivity analysis is carried out by changing the specified parameter a, b, c, θ, δ and t_d by –50%, –25%, +25%, +50% keeping the remaining parameter at their standard value.

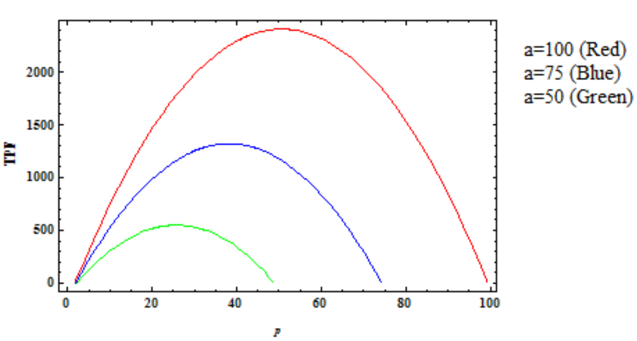

Figure 2. Variation of profit with respect to price p.

The study manifested the Table 1 following facts:

- Optimal average profit PF^* is highly sensitive to demand when the change in the value of its parameters a, b and c is done. It is slightly sensitive to the change in the backlogging parameter δ. It is moderately sensitive to changes in t_d and θ.
- The study reflects that optimal price P^* is highly sensitive to the change in demand parameter a and c where as moderately sensitive of p^* is observed to the b while it is slightly sensitive to change in t_d, θ and backlogging parameter δ.
- Q^* is slightly sensitive to the change in the parameter δ also it is observed that it is highly sensitive to the change in the parameter a, b and c where as it is moderately sensitive to t_d and θ.
- t_1^* is and highly sensitive to the change in parameter a, b and c where as moderately sensitive to change in θ, δ and t_d.

10. Graphical Analysis

Using the derivation of Example 2 and taken the numerical values with the help of Mathematica software the graphical representation of the effect of selling price p and time t_1, when on hand inventory reduces to zero on profit is done in Figure 3 as follows:

Example 3.

For the numerical illustration we consider an inventory system with the following parameter in proper unit, where for t_1 take fixed value.

$c_d = \$1$ / *unit*, $c_l = \$1$ / *per unit*, $c_h = \$1$ / *per unit* / *per unit time*, $c = 1$,

$c_s = \$1$ / *per unit time*, $c_p = \$1$ / *per unit*, $\theta = 0.04$, $\delta = 0.04$ $t_d = 0.2$ *year*,

$t_l = 0.5$ *year*, $a = 100$ *units* / *year*, $b = 0.06$, $A = 40$/ *per order*,

Then using Equation (13), on solving system of non linear equation by Newton Rapshon method by:

The optimal time $T^* = 0.791$ *year*

The optimal price $p^* = \$50.609$

Optimal order quantity $Q^* = 39.487$,

Optimal Total average profit of the system $TPF^* = \$2407.16$

Table 1. Sensitivity analysis changing the specified parameter *a*, *b*, *θ*, *δ* and t_d by percentage

Parameters	% changes	t^*	p^*	Q^*	PF^*
a	−50	+72.67	−30.29	−100.00	−101.61
	−25	−30.41	−23.775	−60.16	068.78
	+25	+48.37	+30.23	+187.44	+163.10
	+50	+58.09	+50.76	+379.49	+489.06
b	−50	−33.33	−6.63	−45.91	−26.58
	−25	−23.38	−5.08	−34.21	−17.40
	+25	+59.49	+10.56	+225.90	+100.17
	+50	+67.27	+7.54	+508.76	+341.36
c	−50	+72.49	+84.67	+410.80	+192.73
	−25	+53.33	+37.83	+13.80	+169.11
	+25	−26.13	−19.62	−40.43	−48.28
	+50	−36.59	−30.27	−56.73	−72.03
θ	−50	+28.23	+0.07	+21.40	+31.11
	−25	+17.28	+1.18	+16.28	+14.14
	+25	−15.30	−1.67	−15.10	−10.14
	+50	−26.10	−2.82	−25.30	−17.31
δ	−50	−9.71	−2.79	−10.01	+1.57
	−25	−3.97	−1.14	−4.30	+0.58
	+25	+2.80	+0.79	+3.20	−0.40
	+50	4.80	+1.36	+5.60	−0.61
t_d	−50	−12.50	−0.88	−13.42	−12.44
	−25	−6.35	−0.50	−6.95	−6.24
	+25	+6.44	+0.57	+7.38	+6.26
	+50	+12.72	+1.15	+14.87	+12.50

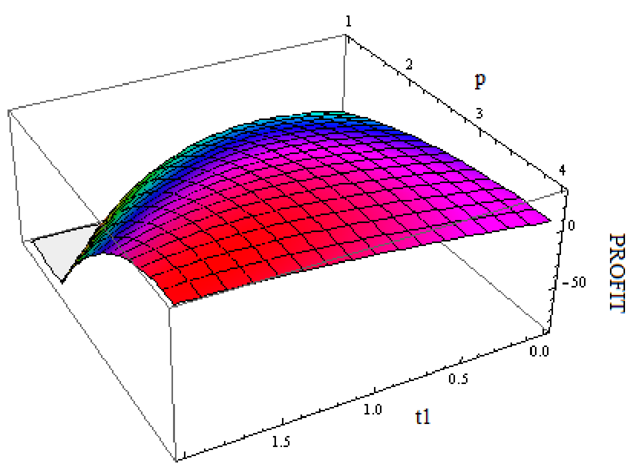

Figure 3. Variation of profit with respect to time t_1 and price *p*.

11. Sensitivity Analysis

Sensitivity analysis is carried out by changing the specified parameter *a*, *b*, *c*, *θ*, *δ* and t_d by −50%, −25%, +25% +50% keeping the remaining parameter at their standard value.

The study manifested the Table 2 following facts:

- Optimal average profit PF^* is highly sensitive to demand when the change in the value of its parameters *a* and *c* is done. It is slightly sensitive to the change in parameter *θ*, t_d and backlogging parameter *δ*. It is moderately sensitive to changes in *b*.
- Study reflects that optimal price P^* is highly sensitive to the change in demand parameter *a*, *c* and *θ* where as slight sensitiveness of P^* is observed to the parameter *b*, t_d and *δ*.

Table 2. Sensitivity analysis changing the specified parameter *a*, *b*, *θ*, *δ* and t_d by percentage

Parameters	% changes	T^*	P^*	Q^*	PF^*
a	−50	+73.48	−49.24	−15.93	−76.79
	−25	+26.79	−24.66	−5.93	−45.04
	+25	−17.96	+24.70	+3.32	+58.34
	+50	−31.16	+49.42	+4.63	+130.01
b	−50	+10.05	+0.01	+9.28	+36.91
	−25	+5.14	+0.004	+4.76	−0.25
	+25	−5.43	−0.003	−5.05	+0.26
	+50	−11.21	−0.004	−10.46	+0.54
c	−50	−28.97	+98.79	−27.92	+104.99
	−25	−12.06	+32.93	−11.50	+34.93
	+25	+9.26	−19.75	+8.66	−20.91
	+50	+16.68	−32.91	15.47	−34.83
θ	−50	−0.04	−100.00	−0.35	+0.002
	−25	−0.03	−0.43	−0.18	+0.001
	+25	+0.01	+0.43	+0.16	−0.0008
	+50	+0.04	−100.00	+0.36	−0.002
δ	−50	+13.97	+0.01	+13.77	+0.14
	−25	+5.81	+.005	+5.73	+0.06
	+25	−4.37	−0.001	−4.30	−0.05
	+50	−7.80	−0.002	−7.66	−0.09
t_d	−50	−0.08	−0.002	−0.07	+0.004
	−25	−0.04	−0.02	−0.51	+0.002
	+25	−0.10	+0.03	+0.68	+0.005
	+50	−0.49	+0.07	+1.41	+−023

- Q^* is slightly sensitive to the change in the parameter θ and t_d also it is observed that it is moderately sensitive to the change in the parameter *a*, *b*, *c* and *δ*
- Optimal cycle T^* is highly sensitive to parameter *a* whereas, might sensitiveness is exhibited due to change θ, t_d and it is moderately sensitive to changes in *b*, *c* and *δ*.

12. Graphical Analysis

Using derivation of Example 3 and taken the numerical values with the help of Mathematica software the graphical representation of the effect of selling price p and time *T*, when on hand inventory reduce to zero, on profit is done in Figure 4 as follows:

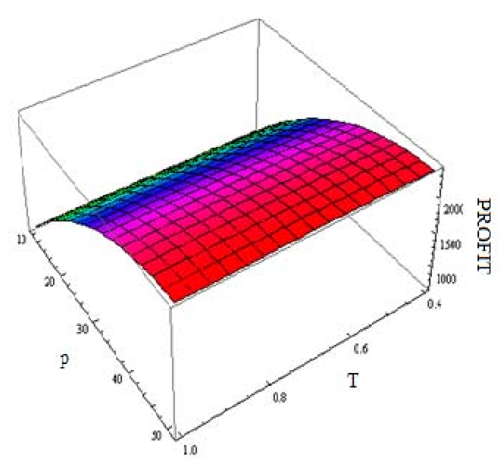

Figure 4. Variation of profit with respect to time *T* and price *p*.

Table 3. Sensitivity analysis changing the specified parameter a, b, θ, δ and t_d by percentage

Parameters	% Changes	t^*	T^*	P^*	Q^*	PF^*
a	−50	−228.88	−106.69	+49.80	−128.79	−842.79
	−25	−24.33	+17.57	−16.47	−51.29	−84.67
	+25	+346.31	−146.29	+105.96	−2414.33	−8466.13
	+50	−896.48	−160.13	+90.85	−91.11	−92.26
b	−50	−411.75	−113.61	+165.79	−267.24	−2835.38
	−25	−30.46	−16.08	−0.71	−19.87	−2.18
	+25	221.47	−222.64	−4.96	207.29	+184393.00
	+50	+568.72	−1404.51	+75.09	+22629.20	−74556.90
θ	−50	+2207.62	+4893.74	19422.00	−100.00	−100.00
	−25	−208.90	−213.05	−0.43	−202.31	+18.91
	+25	−21.44	−12.52	−0.39	−14.40	−0.71
	+50	−149.23	−179.77	−0.40	−176.08	+23.54
δ	−50	+4210.20	+15360.00	−885.89	−100.00	−100.00
	−25	−178.20	−196.60	−3.83	−189.68	20.31
	+25	+0.58	−0.65	+0.03	−0.55	−0.05
	+50	+1.09	−1.25	+0.05	−1.04	−0.09
t_d	−50	−6.92	−1.78	−0.05	−2.81	−1.09
	−25	−3.52	0.95	−0.03	−1.48	−0.54
	+25	+3.71	+1.11	+0.03	+1.65	+0.53
	+50	+7.56	+2.36	+0.07	+3.47	+1.06

Example 4.

For the numerical illustration we consider an inventory system with the following parameter in proper unit:

c_d = \$1 / *unit*, c_l = \$1 / *per unit*, c_h = \$1 / *per unit / per unit time*,

c = 50, c_s = \$1 / *per unit time*, c_p = \$4 / *per unit*, $\theta = 0.19$, $\delta = 0.10$

t_d = 0.05 year, a = 300 *units / year* b = 0.80, A = 3 / *per order*,

Then using Equation (13) on solving system of non linear equation by Newton Rapshon method by:

The optimal time $t_1^* = 0.2538$ *year*
The optimal time $T^* = 0.4678$ *year*
The optimal price p^* = \$5.0820
Optimal order quantity $Q^* = 22.85$,
Optimal Total average profit of the system TPF^* = \$38.039,

13. Sensitivity Analysis

Sensitivity analysis is carried out by changing the specified parameter a, b, θ, δ and t_d by −50%, −25%, +25% +50% keeping the remaining parameter at their standard value.

The study manifested the Table 3 following facts:

- Optimal average profit PF^* is highly sensitive to demand when the change in the value of its parameters a and b is done. It is slightly sensitive to the change in the deterioration parameter t_d. It is moderately sensitive to changes in θ and backlogging parameter δ.
- Study reflects that optimal price P^* is highly sensitive to the change in demand parameter θ and b, backlogging parameter δ but again in backlogging parameter δ only slightly sensitive change where as low sensitive-

ness of P^* is observed to the parameter t_d while it is moderately sensitive to change in t_d.

- Q^* is slightly sensitive to the change in the parameter t_d also it is observed that it is highly sensitive to the change in the parameter a and b where as it is moderately sensitive to backlogging parameter δ and θ
- Optimal cycle T^* is highly sensitive to parameter b, θ and backlogging parameter δ where as slight sensitiveness exhibited due to change in t_d and it is moderately sensitive to changes in a.
- t_1^* is slightly sensitive to the change in parameter a and t_d where as moderately sensitive to change in b and α and highly sensitive to a.

14. Conclusions

An optimal price on unit selling price and the optimal time to maximizing the total profit per unit are determined and profit is presented by both analytically and graphically. Sensitivity analysis with respect to various parameters has been carried out. This model incorporates some realistic feature with some kind of inventory and real market. It can be used for clothes, vegetable and fruit, fashionable goods, electronic component and other product which have more likely the characteristic are given example 1, 2, 3, 4. The present study can be further extended by adding some extra realistic factors associated with the inventory system.

15. References

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