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Doing Numerical Calculus using Microsoft EXCEL

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Abstract

Objectives: Numerical calculus can be done using calculators, readymade packages or programming languages. The spreadsheets are a middle course between the programming languages and the readymade packages. **Methods/Statistical Analysis:** Spreadsheet programs serve as a powerful tool for graphical analysis and programming. **Findings:** We shall describe how the ubiquitous Microsoft EXCEL can be used to do numerical calculus, with appropriate examples. Use of EXCEL does not require any prior knowledge of computer programming and are straightforward to use. **Applications/ Improvements:** We apply the Microsoft EXCEL to do the numerical integration and to obtain numerical solutions of equations using the Newton-Raphson method.

Keywords: Microsoft EXCEL, Newton-Raphson Method, Numerical Calculus, Numerical Integration, Spreadsheets, Trapezoidal Rule.

1. Introduction

Numerical calculus can be done either using a calculator, readymade packages or programming languages. Most calculators are not designed for exhaustive numerical methods. This necessitates the use of computers. There are several possible approaches in using computing technology such as programmable calculators, readymade packages and the programming languages. The readymade packages yield results but the underlying mechanism (the techniques of numerical methods) remains largely invisible. Consequently the learning is usually minimal. Moreover some knowledge of programming specific to that package may be required. In order to write effective algorithms in a given programming language, it requires training and some expertise. The required expertise may not be available with the students, particularly in their introductory science courses. In such courses the emphasis is more on the topics of the course rather than on the programming. Spreadsheet programs serve as a powerful tool for graphical analysis and programming. They can be applied to solve a wide range of problems in mathematics and science. Thus the spreadsheets are a middle course between the programming languages (say for example, FORTRAN or C++) and the readymade packages. This approach is more suited for learning[1–3]. The ubiquitous Microsoft EXCEL can be easily used for numerical calculus by writing a few formulae and clicking and dragging. The fact that the Microsoft EXCEL is already installed on many computers means that one of the barriers to using computers for numerical methods, namely the price of acquiring the software is removed. In this article, we shall cover the interesting historical aspects of the spreadsheets and their distinct advantages. To illustrate this point, we shall consider the examples of numerical integration and the solutions of equations using the Newton-Raphson method.

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2. Spreadsheets and their Advantages

The spreadsheets have a very interesting history[4]. Accountants have been using the paper spreadsheets for centuries. In the world of accounting and commerce, the spreadsheet was originally a huge sheet of papers with numerous rows and columns. These paper spreadsheets could be conveniently used to organize the data. The organized data was manually examined and the corresponding results such as totals or averages noted at the end of the rows and columns. The data could be from the business transactions, tax collections, or data related to manpower. The data displayed in these paper spreadsheets could be quickly read and analyzed by the concerned. These sheets helped in making decisions based on the data. An electronic spreadsheet essentially consists of columns and rows which are software-defined. It is possible to employ the inbuilt formulae into the spreadsheets. This saves the task of calculations such as totals or averages. In fact the spreadsheets have many statistical functions (such as average. median, mean, mode, standard deviation and many more). Any of these numerous functions can be applied to any part of the digital spreadsheets with ease.

It was in the year 1961, when Richard Mattessich pioneered the digital spreadsheets, particularly for accounting and business applications. These spreadsheets were installed in the mainframe computers and beyond the reach of masses.

The first appearance of the spreadsheets was in 1979, in the form of *VisiCalc*, operating on the Apple-II platform. The *VisiCalc* was the creation of Dan Bricklin and Bob Frankston at Harvard Business School[5]. The *VisiCalc* was designed to perform repetitive computations. The name VisiCalc is derived from phrase *visible calculator*. The basic idea was to do simple computations, which were done manually in paper spreadsheets. With passage of time the basic idea has been strengthened with a library of large inbuilt statistical, mathematical and scientific functions along with a graphic interface. The trend started by VisiCalc, was continued with the Lotus 1-2-3, which had much more capabilities including: graphs and databases. Lotus 1-2-3 could also perform complex computations and present the results graphically. The easy to read visual output made Lotus 1-2-3 a good choice for the office automation in the 1980s. In 1984, Microsoft devised the spreadsheets now known by the very familiar name EXCEL[6]. It used the graphical interface accompanied with *pull down menus*. Moreover, it incorporated the *point and click capability* using a device now known as the *mouse*. With these features, it became very easy to use the Microsoft EXCEL, requiring no prior knowledge of computers or programming.

The universally available spreadsheets such as the MS EXCEL are being used as a powerful tool in science and education. The MS spreadsheets are easy to use enabling an interactive way of doing numerical problems. The graphical interface of MS EXCEL is very advanced accompanied with a powerful help module. It can be used to do a comprehensive statistical analysis with ease. It has numerous inbuilt functions for computing any conceivable quantity including: total, average, mean, median, mode and standard deviation. The analysis can be presented graphically using pie charts, bar charts, among others. MS EXCEL can also be used for complex mathematical computations on wide range of topics from geometry to calculus. All this is within the reach of individuals with no prior background in computers or programming. The graphical interface when used appropriately enhances the student learning outcomes. It is possible to visualize concepts using different types of graphs. The data and the resulting analysis can be stored for reuse and also easily shared with the modern communication technologies. A major advantage about MS EXCEL is its universal availability. Persons with different backgrounds (such as business, mathematics, science and arts) are using it. So, in a classroom it does not appear alien unlike the programming languages as everybody would have heard about it or used it sometime[7–9].

3. Numerical Calculus with Microsoft EXCEL

Mathematics is the language of nature[10] and calculus is central to understanding many of the phenomena. Concepts from calculus find applications in sciences, engineering and social sciences. In many applications it is difficult to proceed analytically. Consequently, numerical calculus forms a significant component in any science/engineering curriculum. Most practical problems are either very difficult or in many cases impossible to do analytically. At times one has to analyze the data obtained from experiments and field measurements. This necessitates the use of numerical methods. We shall briefly describe two techniques from numerical calculus, which are very widely used namely numerical integration by the trapezoidal rule and the solutions of equations using the Newton-Raphson method.

3.1 Numerical Integration

Many integrals *cannot* be evaluated exactly; for example, $\int_1^2 e^{-ax^2}dx$. Such integrals can be done approximately using a calculator. Since, it involves a lot of calculations, it is best to use a computer program, where it is easy to change the interval of integration and other parameters (such as a in the above example). The integral, $\int_a^b f(x)dx$, where a and b are numerical parameters, can be evaluated numerically by dividing the function $f(x)$ into n trapeziums of equal width, $h = (b-a)/n$, using the well-known result[11],

$$\int_a^b f(x)dx \approx h\left\{\frac{1}{2}f(a) + f(a+h) + f(a+2h)\cdots + f\left(a+(n-1)h\right) + \frac{1}{2}f(b)\right\}.$$

Larger the n better the result is. The desired degree of accuracy is achieved by increasing the value of n.

As an example, we evaluate $\int_a^b e^{cx}dx$. We choose this example because it is possible to evaluate it analytically for the purpose of comparison. In the enclosed screenshot, we have shown the evaluation of the above integration for the interval, $a = 1$, $b = 3$ and $c = 1$; with $n = 10$. In this example, the numerically evaluated integral differs only in the decimal with the exact result obtained analytically. In the EXCEL spreadsheet it is straightforward to change the interval of integration $[a,\ b]$ and the numerical parameter c as often as required. If the value of n is increased to 20 and 40 the corresponding results are found to be 17.3817254 and 17.37087312 respectively. In fact it is required to keep the value of n large, which is straightforward. We choose it to be ten just for demonstration. These are to be compared with the exact value 17.36725509 obtained analytically. We have found EXCEL to give very accurate results for a variety of integrands (such as exponential, trigonometric; algebraic and their combinations).

	A	B	C	D
1	$\int_a^b e^{cx}dx$		**n**	**10**
2			**a**	**1**
3			**b**	**3**
4			**c**	**1**
5			**h**	**0.2**
6	i	x_i	$f(x_i) = e^{cx_i}$	Sum
7	0	1	2.718281828	1.359140914
8	1	1.2	3.320116923	3.320116923
9	2	1.4	4.055199967	4.055199967
10	3	1.6	4.953032424	4.953032424
11	4	1.8	6.049647464	6.049647464
12	5	2	7.389056099	7.389056099
13	6	2.2	9.025013499	9.025013499
14	7	2.4	11.02317638	11.02317638
15	8	2.6	13.46373804	13.46373804
16	9	2.8	16.44464677	16.44464677
17	10	3	20.08553692	10.04276846
18			**Sum**	**87.12553694**
19			**Integral**	**17.42510739**
20			**Exact Answer**	**17.36725509**

Figure 1. Numerical Integration using Microsoft EXCEL.

3.2 Newton-Raphson Method

We often encounter equation which cannot be solved analytically. In such situations, one needs to use some numerical method. One of the most commonly used methods is the Newton-Raphson Method. This powerful iterative process follows a set guideline to approximate one root, considering the function, its derivative, and an approximate initial value x_0. The initial value is obtained by deduction (such as the change in the sign of the function) or even at times by a random guess! The Newton-Raphson method is an iterative process to compute the roots of the given function, $f(x) = 0$. The specific root that the process locates depends on the initial value x_0. Starting with x_0, we obtain after the first iteration, $x_1 = x_0 - f(x_0)/f'(x_0)$, where, $f'(x_0)$ is the derivative of $f(x)$ at the point x_0. The above procedure is repeated with x_1 in place of x_0 to obtain $x_2 = x_1 - f(x_1)/f'(x_1)$. Then using x_2 in place of x_1, we obtain x_3. The iterations are repeated several times to obtain the root to the desired degree of accuracy. The general expression is

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}.$$

The Newton-Raphson method generally converges rapidly. Since, the procedure involves lot of calculations it is useful to use a computer programme rather than a calculator.

As an example, we solve the equation $f(x) = xe^{ax} - b = 0$. In the enclosed screenshot of the EXCEL spreadsheet, we have chosen $a = 1$ and $b = 3$. For these choices of a and b, it is straightforward to see that the above function changes sign between 1 and 2. So, we choose x_0 as 2. The first iteration gives the value for the root 1.46 and the second 1.15. After successive iterations, we are also able to see the value of $f(x_n)$ approaching zero, as it should. After just six iterations the root of the above equation is obtained to ten decimal places. If we had started with x_0 as 1, the root would have been obtained after four iterations. With the random choices of 5 and 10 for x_0 the root is obtained after ten and sixteen iterations respectively. The procedure is generally not very sensitive to the choice of x_0.

	A	B	C	D	E
3	$xe^{ax} - b = 0$			a	1
4				b	3
5					
6	n	x_n	$f(x_n)$	$f'(x_n)$	x_{n+1}
7	0	2	11.7781122	22.1671683	1.468668617
8	1	1.46866862	3.379086491	10.72253499	1.153529846
9	2	1.15352985	0.65595198	6.825312523	1.057424067
10	3	1.05742407	0.044266214	5.923211672	1.04995072
11	4	1.04995072	0.000244994	5.85775529	1.049908896
12	5	1.0499089	7.62236E-09	5.857390795	1.049908895
13	6	1.04990889	0	5.857390784	1.049908895
14	7	1.04990889	0	5.857390784	1.049908895

Figure 2. Newton-Raphson Method using Microsoft EXCEL

4. Concluding Remarks

Microsoft EXCEL is a great tool for doing calculations and making presentations. We have seen how it can be used for doing numerical integrations and to find the roots of transcendental equations. The spreadsheet oriented techniques can be used to tackle the numerical solutions of a wide spectrum of problems such as, differential equations; integral equations and Fourier analysis. MS EXCEL is also useful in physics[2–3, 12–13]. It has also found applications in specific problems such as the study of quadratic surfaces in the laboratory[14–17]; chemical physics[18]; resistor networks[19–24]; and number theory[25].

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