

# Phased-MIMO Radar for Improvement in Target Detection

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## Abstract

**Objective:** Advancement in radar technology is moved from single antenna based system to multiple antenna system. From the viewpoint of the radar performance improvement, analysis of low Radar Cross Section (RCS) target detectability in rich scattering environment is utmost important. **Methods:** From the perspective of the advancement of the radar signal processing, phased array radar and MIMO radar have been already explored. This paper deals with the phased-MIMO radar, exploiting coherence signal processing gain and waveform diversity. Joint combination of coherency and diversity will improve the radar performance. **Improvements:** This paper deals with the analysis on the trade-off between the coherency and spatial diversity. The superiority of phased-MIMO radar, over other two systems, has been presented with respect to some selected parameters as in this paper.

**Keywords:** False Alarm, MIMO, Probability of Detection, RCS, Radar

## 1. Introduction

Improving the radar system performance is an ever green hot research<sup>1-2</sup>. Advancement of MIMO, motivate researchers to integrate MIMO in and it offers tremendous boost in radar performance improvement over the conventional single antenna system<sup>2</sup>. The performance of the radar is greatly influenced due to the presence of rich scattering environment and target scintillations<sup>3-5</sup>. This problem can be solved by enhancing the receiver signal processing gain and exploiting different kind of diversity techniques are the only way out.

In recent time various multiple antenna radar system has been proposed. Mainly, it is categorized into MIMO radar<sup>6-8</sup>, Phased array radar and hybrid phased-MIMO radar<sup>1,9</sup>. Phased array radar exploits coherency in signal whereas in MIMO radar transmit multiple orthogonal signal from each antenna elements to exploit waveform diversity<sup>10-13</sup>. As in MIMO radar antennas are widely

separated, the transmitted waveform are uncorrelated, helps to increase the diversity gain. But for the closeness of antennas in phased array system it does not suitable for exploiting the diversity gain, rather it rallies on coherence processing gain. As presented in<sup>4</sup>, severe fading condition, MIMO radar improve the target detectability by exploiting the angular spread of the target. But its performance degrades in case of low SNR condition, whereas phased-array radar improves the performance. In<sup>1</sup>, performances have been analyzed based on the overall beam forming. But there is no performance comparison based on the Probability of detection (Pd) and detector SNR level. This paper deals with analytical analysis to establish the utility of hybrid radar system.

In this paper, the details derivation of the probability of detection and detector output SNR have been presented. The performances of phased-MIMO have been analyzed based Pd and output SNR of the receiver system for each of the radar system.

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The paper is arranged as follows. Section 2 mathematical formulation for phased-MIMO radar. While Section 3 represents the simulated results to compare the system performances. Section 4 provides the conclusion remarks.

## 2. Mathematical Model for Performance Analysis

Phased MIMO radar is basically exploiting the sub-array concept. Here,  $M_{coh}$  and  $N_{coh}$  represents the number of elements per sub-array at transmitter and receiver side respectively.  $M_{div}$  and  $N_{div}$  represent the number of sub-array at the transmitter and receiver side respectively.

### 2.1 Probability of Detection

Let us consider the detection of a target at delay  $\tau$  as follows<sup>3-4</sup>,

- $H_0$  : Absence of target
- $H_1$  : Presence of target

As per Neyman-Pearson sense, the optimal detector Likelihood Ratio Test (LRT) can be given

$$T = \log \frac{f(r(t)|H_1)}{f(r(t)|H_0)} \underset{>H_0}{\underset{<H_1, \delta_{th}}{>}} \tag{1}$$

Where,  $f(r(t)|H_0)$  and  $f(r(t)|H_1)$  are Pd of the observation corresponding to the detection cases as presented above and the Probability of false alarm (Pfa) is calculated keeping threshold  $\delta_{th}$ .

For phased-MIMO radar, the distribution of the detected output can be written as

$$\|X^2\| \sim \begin{cases} \frac{N_{coh}\sigma_n^2}{2} X^2(2M_{div}N_{div}) & H_0 \\ \left( \frac{EN_{coh}^2M_{coh}^2}{2M_{coh}M_{div}} + \frac{N_{coh}\sigma_n^2}{2} \right) X^2(2M_{div}N_{div}) & H_1 \end{cases} \tag{2}$$

The Pd for phased-MIMO radar is given by

$$pr_D = 1 - F_{X^2(2M_{div}N_{div})} \left( \frac{\sigma_n^2}{\sigma_n^2 + \left( \frac{E}{M_{div}} \right) N_{coh}M_{com}} F^{-1}_{X^2(2M_{div}N_{div})}(1 - pr_{FA}) \right) \tag{3}$$

## 2.2 Output SNR of the Detector

As in<sup>4</sup>, the detector's output SNR,  $\beta$  is defined as given below,

$$\beta = \frac{|E(T/H_0) - E(T/H_1)|^2}{2[\text{var}(T|H_0) + \text{var}(T/H_1)]} \tag{4}$$

In case of phased-MIMO radar,

$$\begin{aligned} E(T/H_0) &= \sigma_n^2 N_{com} M_{div} N_{div} \text{ and} \\ E(T/H_1) &= \left( \sigma_n^2 N_{com} + \frac{EN_{com}^2 M_{com}}{M_{div}} \right) M_{com} \text{ and} \\ \text{var}(T/H_0) &= M_{div} N_{div} (\sigma_n^2 N_{com})^2 \text{ and} \\ \text{var}(T/H_1) &= M_{div} N_{div} \left( \sigma_n^2 N_{com} + \frac{EN_{com}^2 M_{com}}{M_{div}} \right)^2 \end{aligned}$$

Therefore,  $\beta$  can be calculated as,

$$\begin{aligned} \beta_{\text{phased-MIMO}} &= \frac{\left( \frac{EN_{coh}^2 M_{com}}{M_{div}} M_{div} N_{div} \right)^2}{\frac{1}{2} M_{div} N_{div} \left[ \sigma_n^4 N_{coh}^2 + \sigma_n^4 N_{coh}^2 + \frac{EN_{coh}^2 M_{coh}^2}{M_{div}} + 2 \frac{\sigma_n^2 N_{coh}^2 E}{M_{div}} \right]} \\ &= \frac{E^2 N_{coh}^4 M_{coh}^2 N_{div}^2}{\frac{1}{2} M_{div} N_{div} N_{coh}^2 \left[ 2\sigma_n^4 + \frac{E^2 N_{coh}^2 M_{coh}^2}{M_{div}^2} + 2 \frac{\sigma_n^2 E N_{coh} M_{coh}}{M_{div}} \right]} \\ &= \frac{E^2 N_{coh}^2 M_{coh}^2 M_{div}}{M_{div} \left[ \sigma_n^4 + \frac{E^2 N_{coh}^2 M_{coh}^2}{2M_{div}} + 2 \frac{\sigma_n^2 E N_{coh} M_{coh}}{M_{div}} \right]} \\ &= \frac{E^2 N_{coh}^2 M_{coh}^2 N_{div} / \sigma_n^4}{M_{div} \left[ 1 + \frac{N_{coh}^2 M_{coh}^2 E^2}{2M_{div}^2 \sigma_n^4} + \frac{E}{\sigma_n^2} \frac{N_{coh} M_{coh}}{M_{div}} \right]} \\ \beta_{\text{phased-MIMO}} &= \frac{p^2 N_{coh}^2 M_{coh}^2 N_{div}}{M_{div} \left[ 1 + \frac{N_{coh}^2 M_{coh}^2}{2M_{div}} p^2 + \frac{N_{coh} M_{coh}}{M_{div}} p \right]} \tag{5} \end{aligned}$$

## 3. Results

In Figure 1 and 2 Pd is calculated by varying SNR for different radar system. It represents the impact of the number of antennas system performance. As the number of antenna increases, Pd also increases.

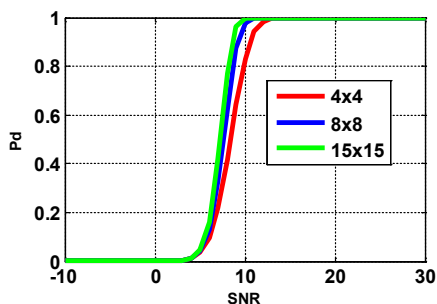


Figure 1. Variation of Pd with SNR for MIMO radar system.

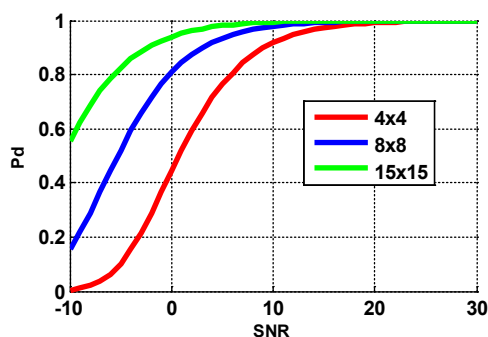


Figure 2. Variation of Pd with SNR for Phased array radar system.

In Figure 3, performances of phased array radar ( $M_t = 4, N_r = 4$ ), MIMO radar ( $M_t = 4, N_r = 4$ ) and phased-MIMO ( $(M_{coh} = 2, N_{coh} = 2, M_{div} = 2, N_{div} = 2)$ ) are presented. It is clear that in low SNR condition phased array radar performs better than other radar system whereas at high SNR MIMO radar is superior. But phased-MIMO radar performs moderately.

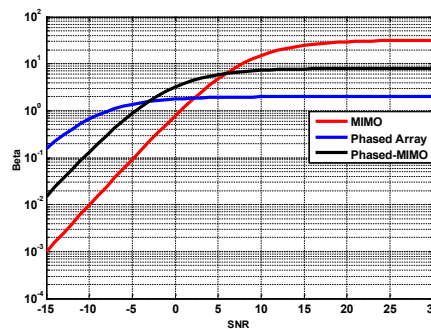


Figure 3. Output SNR variation.

Figure 4 and Table 1 present a comparative study on the performance of different radar system. As presented in Table, phased array radar provides highest SNR gain in comparison to other two radar system but its probability of detection in less in high SNR region in comparison to phased-MIMO radar. Therefore, phased-MIMO radar is a perfect tread-off between coherency and waveform diversity.

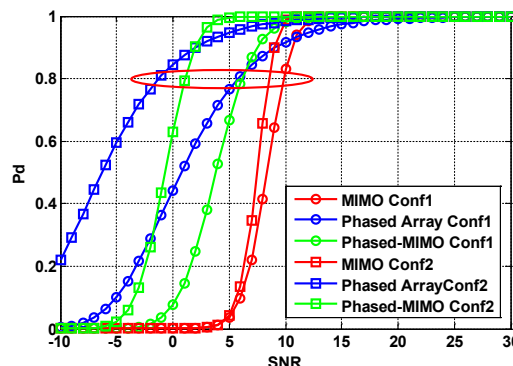


Figure 4. Output SNR variation.

Table 1. Pd comparison for different system (Pfa=10<sup>-6</sup>)

Configuration	MIMO radar	Phased Array Radar	Phased-MIMO Radar
Conf1	$M_t = 4$ $N_r = 4$	$M_t = 4$ $N_r = 4$	$M_{coh} = 2, N_{coh} = 2$ $M_{div} = 2, N_{div} = 2$
Conf2	$M_t = 9$ $N_r = 9$	$M_t = 9$ $N_r = 9$	$M_{coh} = 3, N_{coh} = 3$ $M_{div} = 3, N_{div} = 3$
SNR gain in Conf2 w.r.to Conf1 (dB)	1.6	7	5.2

Figure 5 represents the performance of phased array radar ( $M_t = 6, N_r = 6$ ), MIMO radar ( $M_t = 6, N_r = 6$ ) and phased-MIMO radar ( $M_{coh} = 3, N_{coh} = 3, M_{div} = 2, N_{div} = 2$ ) under SNR of -10dB.

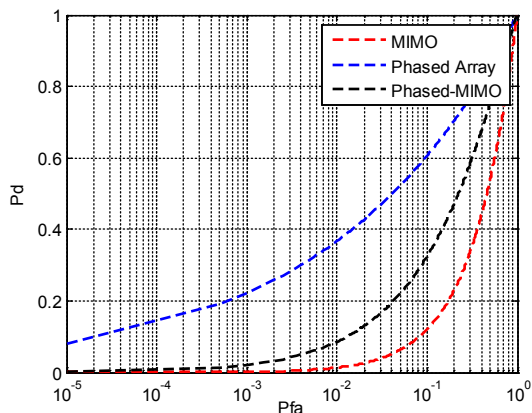


Figure 5. Output SNR variation.

### 4. Conclusion

As presented, phased-MIMO radar performance has been investigated and compared with its counterparts. The tradeoff between coherent processing and waveform diversity have also been analyzed. From the simulation, it is clear that phased-MIMO radar is a perfect choice under noisy environment.

### 5. References

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