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Sensitivity Analysis and Optimal Production Scheduling as a Dual Phase Simplex Model

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Abstract

Background/Objectives: Production efficiency is mainly influenced by two factors – satisfying customer demands and profit maximization. Some methods for optimizing the production are proposed in the literature, which are computationally expensive. **Methods/Statistical Analysis**: Production scheduling problem is addressed by using a single objective function and suitable operational constraints. Dual-Phase Simplex method is used to determine the optimal schedule for production and sales. As compared to conventional methods, the current method is found to be computationally inexpensive and easy to implement. The sensitivity analysis is performed to study the effects of parametric variations on the production volume. **Findings:** The performance of a quality management system, in terms of production efficiency of an organization, is based on the quality of decisions taken at the shop floor. A good production manager must be able to foresee all decision-related discrepancies and adopt preventive measures appropriately. In order to resolve this contradictory requirement, an optimized production scheduling process is generally required, which deals with desired type and quantity of output at minimum cost. A typical case study of Tema (Ghana) based cable manufacturing firm is considered to optimize the production scheduling problem. In this paper, a mathematical model for the optimizing the production of different items having different costs is developed by considering the problem as Linear Programming Problem (LPP). Further, the formed LPP contains mixed type constraints, which are solved using the Dual-Phase Simplex method. It has been found from the literature that the Dual-Phase Simplex method is computationally inexpensive. Moreover, sensitivity analysis is done to observe the effect of break-downs on the production rate. **Application/Improvements:** The idea proposed in this paper has a wide scope in various physical processes to optimize their parameter values. However, for multi-objective problems the Dual-Phase Simplex method gives invalid results.

Keywords: Production Scheduling, Simplex Method

1. Introduction

Production scheduling is considered as one of the oldest and hardest problems of the manufacturing systems. Since the scheduling problem is dynamic in nature, finding the optimal solution is a tough task. However, with certain assumptions and given conditions, solution of the scheduling problem does not vary much from one system of operations to another. The main motive behind the production scheduling optimization is to reduce production costs, time and maximize the profit. An optimized production scheduling process is characterized by achieving the desired output of products, both in type and quantity, within the planned time at minimum costs[1].This paper seeks to discuss the application of the optimization concept at production scheduling with the help of single objective and typical operational constraints. In order to address the production scheduling process of various industries, a number of powerful models are reported in the literature, which are mainly based on linear programming models. For example, in yogurt producing industry, there is a wide variety of products according to different features like fat content, usage of whey to produce the mixture, the flavor, the size of the container or the label language. In this type of operations related to scheduling, constraints like satisfaction of multiple clients, variable processing times, setting up times according to sequence, production costs hikes and efforts to keep an eye inventory levels need to

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be deliberately handled in order to retain the required production objectives[2]. Optimal scheduling refers to a schedule, which yields the right quantity and type within the minimum expected time with the available resources to maximize the profit.

Optimal schedule consists of timely provision of input raw materials, identification of the appropriate machines for the job and provision of sufficient work instructions to operators. In this paper, the problem of a Ghana based cable manufacturing firm is considered for optimal scheduling. In order to meet market demands and the targeted sales as per the marketing strategy, an optimized production scheduling output model needs to be developed. Most of industrial problems, like production scheduling can be expressed in mathematical form in order to optimize it. It has been observed in the past that predominant production scheduling problems falls in one of the following types:

- To avoid long term delivery schedules, companies prefer to satisfy the demands instantly.
- After mapping of customer performance to product specification, shortage of one product would affect the sales pattern.
- Reference benchmark is needed to measure and analyze the performance at end of each month.

Production scheduling problem has gained the attention of many researchers in recent years. A fuzzy weighted averages approach[3]has been used to describe non-linear problems into simple extended algebric operations on fuzzy members. Fuzzy sets are represented using α-sets. This method generates discrete but exact solution using ‘ 2^{2n} ’ evaluations per α-cut. An efficient method[4] improved this existing approach using steepest-descent method to give solution in ‘$2^{2(n+1)}$’ iterations for each α-cut. The computational requirements are further optimized by transforming the given non-linear problem into linear one[5]using Charnes and Cooper transformation method[6].A detailed review[7] discusses approximately 90 research papers on multi-objective scheduling process involving Genetic Algorithm[8], Ant-Colony optimization[9], Particle Swarm algorithm[10], Heuristics approach[11]etc. Different approaches are categorized according to the application in scheduling e.g. considering processing time, breakdown, limited stock, job-shop scheduling, flow-shop scheduling problem etc. In this paper, a case study of a cable manufacturing company is discussed, so as to achieve a monthly target of *92* tons of Copper to be manufactured and sold. The target resulted from *14.6*% increase in production in *2012*. This study covers the use of linear programming to find the optimal solution for scheduling of production and sales. The study in this paper treats typical scheduling problems in manufacturing companies with special reference to a case study at Nexans Kabel metal Ghana Limited, a production firm engaged in the production of house wiring cables[12]. The constraints to the production and sales capacities were based on the company's activities over the period January-December, *2012*. The company manufactures more than one hundred and fifty types of Copper products on production program. However, an optimal scheduling solution need to be identified such that, at least *70* % of the total Copper sales can proceed from the six fast selling products.

In this paper, the Dual-phase Simplex method is used to optimize the production scheduling process, which is not reported earlier. As compared to the previous method like Simplex method, Big M method[12], the proposed method is computationally inexpensive and easy to implement. Furthermore, sensitivity analysis is also done to check the effect of parameter variations on the production output.

2. Structure of the Optimization Problem

Production scheduling problems can be expressed with one basic objective function subjected to a number of constraints. Therefore, these kinds of problems can be treated as linear optimization problem with all the variables taking real values. The structure of a general LPP problem consists of a linear objective function $f(x)$ of ‘*N*’ real variables $x_1, x_2, x_3 \dots . x_N$ is given as:

$$f(x) = k_1 x_1 + k_2 x_2 \dots \dots \dots \dots k_N x_N \quad (1)$$

where $k_1, k_2, k_3 \dots . k_N$ are ‘*N*’ real numbers. The constraints can be represented as:

$$g_i(x) \geq 0 \qquad \forall\ i = 1,2 \dots . . J \quad (2)$$

$$h_j(x) \geq 0 \qquad \forall\ j = 1,2 \dots . . K \quad (3)$$

$$\text{and } x_j(x) \geq 0 \qquad \forall\ j = 1,2 \dots . . N \quad (4)$$

where, $g_i(x)$ are inequality constraints and $h_j(x)$ are equality constraints on objective function $f(x)$. If $x_1, x_2, x_3 \dots . x_N$ satisfy all the constraints of formulated LPP, then these are treated as feasible solutions. Further, objective function and constraints are formulated in terms of the variables and finally this optimization problem can be solved using any linear programming method. Commercial solvers are available to solve optimization algorithms. However, a manual method, known as Dual-Phase Simplex method, is considered in this work.

Dual-phase Simplex method is generally the extension of Simplex method, which cannot be used for mixed constraint problems. For these kinds of problems, artificial variables are used along with the slack variables.

This method consists of following steps:

- Adding slack variables
- Addition of artificial variables
- Formulation of a dummy objective function to calculate the initial basic feasible solution (Phase-1)
- Using initial basic feasible solution to solve original objective function using Simplex method
- Obtaining the optimal solution

3. Problem Formulation

Dual-Phase Simplex method is considered in analyzing the data obtained[12] from a firm, Nexans Kabel metal Ghana Limited, manufacturer of electrical cable wires and conductors. With a 14.6% increased target of *2012*-year performance, in terms of production output and sales tonnage, an estimated target performance in 2013 is projected to 1100 tons of copper in *2013*. In this case study, the objective is to optimize the consumption of copper used for producing different pairs of cables. In short, optimizing Copper consumption gives an idea of quantity of different cables to be produced. If:

Z	=	Optimal Copper consumption to produce different sets of wires;
x_1	=	Total output for *NCY1.5 RS* and *NCY2.5 RS* in *tons/month*;
x_2	=	Total output for *NCY4 RS* and *NCY6 RS* in *tons/month*;
x_3	=	Total output for *NCY10 RS* and *NCY16 RS* in *tons/month*, as shown in (Table 1.)

Table 1. Details of Production

S. No.	Type	Purpose	Quantity
1	*NCY1.5 RS* and *NCY2.5 RS*	Lighting and socket wiring	x_1
2	*NCY4 RS* and *NCY6 RS*	Refrigeration and A.C.	x_2
3	*NCY10 RS* and *NCY16 RS*	High voltage wires	x_3

- Constraint 1

It has been observed by the firm that, doubling the production of *NCY1.5 RS* and *NCY2.5 RS*, provided all other production rates same, produces *10 tons* extra than expected target of *65 tons/month*. i.e.

$$2x_1 + x_2 + x_3 \leq 75$$

- Constraint 2

Data obtained from the firm illustrates that, the copper sales for *2012* was over *1000 tons*, when it was targeted near *960 tons*. Following the same trend, the expected sale till December end was decided to increase upto *1100 tonnes*. Monthly expected sales was *92 tons*. It was also observed that in *2012*, the house-wiring cables contribute *70 %* of the total sales. Therefore, expecting the same contribution for *2013* also, monthly sales due to house-wiring cables is *65 tons/month*. i.e.

$$x_1 + x_2 + x_3 = 65$$

- Constraint 3

From previous year's sales pattern, management deduced that, by targeting more hotels and institutional building projects in order to sale more *NCY4 RS* and *NCY6 RS*, additional *23 tons* are expected to be sold above previous *65 tons*. This implies:

$$x_1 + 2x_2 + x_3 \geq 88$$

Therefore, scheduling problem can be formulated as:

$$\begin{aligned} &\textit{Maximize} && Z = x_1 + x_2 + x_3 \qquad (5)\\ &\textit{Subject to:} && 2x_1 + x_2 + x_3 \leq 75\\ & && x_1 + x_2 + x_3 = 65\\ & && x_1 + 2x_2 + x_3 \geq 88\\ & && x_1, x_2, x_3 \geq 0 \end{aligned}$$

Now following the Dual-Phase Simplex Method, which states that:

3.1 Introduction of Slack/Surplus and Artificial Variables

First of all, the slack variables are added to all the lagging constraints. Similarly, the surplus variables and artificial variables are added to all the leading constraints. In addition to this, the artificial variables are added to equality constraints also.

$$Maximize \quad Z = x_1 + x_2 + x_3 - s_1 - s_2 - A_1 - A_2 \quad (6)$$

$$Subject\ to: \quad 2x_1 + x_2 + x_3 + s_1 = 75$$

$$x_1 + x_2 + x_3 + s_2 = 65$$

$$x_1 + 2x_2 + x_3 + s_3 = 88$$

$$x_1, x_2, x_3, s_1, s_2, A_1, A_2 \geq 0$$

3.2 Phase 1: Formulation of Dummy Objective Function

Due to the presence of both equality and inequality constraints, the dummy objective function (*W*) must be created for the initial basic feasible solution.

$$Maximize \quad W = 0x_1 + 0x_2 + 0x_3 - 0s_1 - 0s_2 - A_1 - A_2 \quad (7)$$

$$Subject\ to: \quad 2x_1 + x_2 + x_3 + s_1 = 75$$

$$x_1 + x_2 + x_3 + s_2 = 65$$

$$x_1 + 2x_2 + x_3 + s_3 = 88$$

$$x_1, x_2, x_3, s_1, s_2, A_1, A_2 \geq 0$$

Since, artificial variables cannot be treated as non-basic variables, choose:

$$x_1 = x_2 = x_3 = s_2 = 0$$

which leads to basic variables as:

$$s_1 = 75, \qquad A_1 = 65, \qquad A_2 = 88$$

This is the initial basic feasible solution for the *Phase 1* of the problem. It can be seen from the Table 2 that the function value for the initial basic feasible solution is *–153*. This much negative value of objective function is due to the presence of negative artificial variables. This function value is further optimized, using Simplex algorithm, where it is found to be *-21*. It can be clearly seen that, only after first iteration there is significant maximization in the function value. Going on with the same procedure, the function value is maximized to zero in Table 3. Since all the optimality values are zero, hence this is the final solution for *Phase 1*. According to the Dual-Phase Simplex method the solution of first phase can be treated as the initial basic feasible solution for the *Phase 2*.

3.3 Phase 2: Finding Optima of Actual Objective Function

From the Table 3 of *Phase 1*, it is clear that no artificial variable is now present in the system as basic variable. Therefore the obtained initial basic feasible solution is further analyzed using original objective function as shown in Table 4. The function value at the starting of the *Phase 2* is 55. Performing the calculations using Simplex algorithm, optimized function value comes out to be 65. Simplex method is again applied to find the optimal value for *Phase 2*. At optima:

$$x_1 = 10\frac{tons}{month}, \qquad x_2 = \frac{23tons}{month}, \qquad x_3 = \frac{32tons}{month}, \qquad Z_{max} = \frac{65tons}{month}$$

From the above calculations, it can be concluded that total optimum output for *NCY1.5 RS* and *NCY2.5 RS* is *10 tons/month*. Similarly, total optimum output for *NCY4 RS*

Table 2. First Step for Phase 1

C_j		0	0	0	0	0	–1	–1		
C_b	v_b	x_1	x_2	x_3	s_1	s_2	A_1	A_2	b	θ
0	s_1	2	1	1	1	0	0	0	75	75
–1	A_1	1	1	1	0	0	1	0	65	65
–1	A_2	1	2	1	0	–1	0	1	88	44
	Z_j	–2	–3	–2	0	1	–1	–1	**–153**	
	C_f	2	3	2	0	–1	0	0		

Table 3. Final Step for Phase 1

C_j		0	0	0	0	0	
C_b	v_b	x_1	x_2	x_3	s_1	s_2	b
0	s_1	0.8	0	0	1	0	2.5
0	x_3	1	0	1	0	1	42
0	x_2	0	1	0	0	–1	23
	Z_j	0	0	0	0	0	**0**
	C_f	0	0	0	0	0	

Table 4. First Step for Phase 2

C_j		0	0	0	0	0		
C_b	v_b	x_1	x_2	x_3	s_1	s_2	b	θ
–1	s_1	0.8	0	0	1	0	2.5	3.125
1	x_3	1	0	1	0	1	42	42
1	x_2	0	1	0	0	–1	23	–
	Z_j	0	1	1	–1	0	**62.5**	
	C_f	1	0	0	0	–1	0	

and *NCY6 RS* is *23 tons/month* and total output for *NCY10 RS* and *NCY16 RS* is *32 tons/month*. Corresponding to all these optimum outputs, the maximum value of total plant output comes out to *65 tons/months*. Moreover, the obtained optima also satisfy the given constraints. It was observed from the sales of *2012* that sales and production of *NCY1.5 RS* is 70 % of that of *NCY2.5 RS*, *NCY4 RS* is *54%* of *NCY6 RS* and *NCY10 RS* is 70 % of total sales and production of *NCY16 RS*[12].

Let, total output from *NCY1.5 RS = a; and* Total output from *NCY2.5 RS = b;* given,

$$\frac{a}{b} = 0.70$$

$$\Rightarrow \quad a = 4.12\frac{tons}{month}$$

$$b = 5.88\frac{tons}{month}$$

Similarly, if total output from *NCY4 RS= c; and* Total output from *NCY6 RS = d;*given,

$$\frac{c}{d} = 0.54$$

$$\text{Given} \quad c + d = 23\frac{tons}{month}$$

$$\Rightarrow \quad c = 8.06\frac{tons}{month}$$

$$d = 14.94\frac{tons}{month}$$

Let, total output from *NCY10 RS= e; and* Total output from *NCY16 RS = f;*given,

$$\frac{e}{f} = 0.70$$

$$\text{Given} \quad e + f = 32\frac{tons}{month}$$

$$\Rightarrow \quad e = 13.18\frac{tons}{month}$$

$$f = 18.82\frac{tons}{month}$$

The individual optimum output of all kinds of products is listed in Table 5. It is clear that this much output of each product, results in the optimal total output from the factory.

4. Sensitivity Analysis

It has been assumed that the design parameters that are used to develop the model of scheduling problem will remain constant throughout the year. The projected performance obtained from solving the mathematical model can however, be affected by changes depending upon a number of constraint factors. These operational limitations may be dynamic in nature, and can have direct bearing on scheduling outcomes of the management system. These include:

- Effect of time on production costs e.g. water, electricity and labour changes in tariffs overtime
- Stochastic nature of the expected sales volume over the time, due to changes in price and other economic developments
- Fluctuating costs of raw materials on the world market and likelihood of shipment delays coupled with management's obligations to maintain constant supply of appropriate raw materials for production
- Preventive maintenance and machine breakdown hours on production capacity

These factors may considerably influence the optimality of the linear model. To examine the extent of deviations of the actual scheduling output from the projected optimal performance with regard to the impact of the ignored limitations mentioned above, original data needs to be altered, in systematic order. This process is well known as sensitivity analysis. Sensitivity analysis is necessary for following reasons:

- To test the reliability of the results of the model in case of significant changes due to constraint factors
- To show the relationship that exist between input and output variables in the model
- To make recommendations more credible and persuasive to decision makers

Let the expected production schedules for *NCY1.5 RS* and *NCY2.5 RS* are affected by sudden machine

Table 5. Optimal solution of Original Problem (O.P.) and Sensitivity Analysis (S.A.)

S. No.	Product	Qty (O.P.)	Qty (S.A.)
1	*NCY1.5 RS*	4.12	1.287
2	*NCY2.5 RS*	5.88	1.838
3	*NCY4 RS*	8.06	8.06
4	*NCY6 RS*	14.94	14.94
5	*NCY10 RS*	13.18	16.008
6	*NCY16 RS*	18.82	22.867
Total		65	65

breakdown for several days beyond the promised repair duration. This circumstance can delay customer delivery schedules and reduce subsequent sales for the month. Assuming sales is reduced by *10%*(depending on breakdown hours), this will have effect on the constraints. Demand for the two products shall reduce to 90%. We can therefore alter the coefficient of 'x_1' and reduce also the expected sales tonnage by 10% to obtain *67.5 tons* as the right hand side constant for same constraint set. All other constraints remain the same as in the original problem.

$$\text{Maximize} \quad Z = x_1 + x_2 + x_3 \tag{5}$$

$$\text{Subject to:} \quad 1.8x_1 + x_2 + x_3 \leq 67.5$$

$$x_1 + x_2 + x_3 = 65$$

$$x_1 + 2x_2 + x_3 \geq 88$$

$$x_1, x_2, x_3 \geq 0$$

Proceeding in same manner as that of first case, adding slack, surplus, artificial variables and solving in two phases, solution comes out to be:

$$x_1 = 3.125\frac{tons}{month}, \quad x_2 = 23\frac{tons}{month}, \quad x_3 = 38.875\frac{tons}{month}, \quad Z_{max} = 65\frac{tons}{month}$$

The optimum results clearly depicts that the breakdown of the machine breakdown has almost zero effect on the output of 'x_2' i.e. total output for *NCY4 RS* and *NCY6 RS* as shown in Table 5, whereas, the total output for *NCY10 RS* and *NCY16 RS* is increased to *38.875* with decrease in total output for *NCY1.5 RS* and *NCY2.5 RS*. Hence, the machine breakdown has no effect on the total output.

5. Conclusion

This paper has presented a mathematical model for the optimization of the production scheduling of different items having different costs. The problem statements presented in this paper have been appropriately addressed, with reference to the case study recommendations and sensitivity analysis. The main objective of this study is to optimize the overall production of three sets of products using some constraints obtained from the factory data and availability of machines. The data used for the case study was the average values obtained from the previous year's production. A linear programming approach has been used for optimizing the output. This approach requires the production data to be quantified and to present in the form of linear relations. Although there are some limitations related to linear programming method like restrictive use and non-applicability to non-linear problems, it gives very close result to the actual optimal solution. In the last section of the paper i.e. sensitivity analysis, the effect of machine breakdown on output of each product has been observed.

5. References

1. Doganis, P, Sarimveis H. Optimal production scheduling for the dairy industry. Annals of Operations Research. 2008; 159(1):315–31.
2. Graves SC. A review of production scheduling. Operations Research. 1981; 29(4):646–75.
3. Dong WM, Wong FS. Fuzzy weighted averages and implementation of the extension principle. Fuzzy Sets and Systems. 1987; 21(2):183–99.
4. Liou TS, Wang MJ. Fuzzy weighted average: An improved algorithm. Fuzzy Sets and Systems. 1992; 49(3):307–15.
5. Guh YY, Hon CC, Lee ES. Fuzzy weighted average: The linear programming approach via Charnes and Cooper's rule. Fuzzy Sets and Systems. 2001; 117(1):157–60.
6. Charnes A, Cooper WW. Programming with linear fractional functionals. Naval Research logistics quarterly. 1962; 9(3–4):181–6.
7. Lei D. Multi-objective production scheduling: A survey. The International Journal of Advanced Manufacturing Technology. 2009; 43(9–10):926–38.
8. Jolai F, Rabbani M, Amalnick S, Dabaghi A, Dehghan M, Parast M Y. Genetic algorithm for bi-criteria single machine scheduling problem of minimizing maximum earliness and number of tardy jobs. Applied Mathematics and Computation. 2007; 194(2):552–60.
9. T'kindt V, Monmarche N, Tercinet F, Laugt D. An ant colony optimization algorithm to solve a 2-machine bicriteria flowshop scheduling problem. European Journal of Operational Research. 2002; 142(2):250–7.
10. Lie D. A pareto archive particle swarm optimization for multi-objective job shop scheduling. Computers and Industrial Engineering. 2008; 54(4):960–71.
11. Chen W. An Efficient Algorithm for Scheduling Jobs on a Machine with Periodic Maintenance. The International Journal of Advanced Manufacturing Technology. 2007; 34(11–12); 1173–82.
12. Addy I. optimization of production scheduling as linear programming model. Ph.D. Thesis; 2013.