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Assessment of Optimal Combination of Operating Parameters using Graph Theory Matrix Approach

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Abstract

Background/Objectives: Graph theory matrix approach is a logical and systematical approach originated from combinatorial mathematics. Graph theory matrix approach is adopted to find the optimal combination of operating parameters. **Methods/Statistical Analysis**: Graph theory matrix approach helps to analyze and understand the system as a whole by identifying system and sub-system up to the component level. Attributes digraph is developed to represent the inheritance and the interdependencies of the subsystems. Matrix method is adopted to convert the digraph into mathematical form. Permanent function is deduced to determine the parameter index to find the optimal combination of operating parameters on a diesel engine. **Findings**: The combination of 18 Ampere load, 270 BTDC Injection timing and 200 bar Injection pressure forms the optimal combination of operating parameters having the highest value of Permanent index. **Applications/Improvements:** Graph theory matrix approach offers simple, generic, easy and convenient computation. It finds applications in the fields of education, neural networks, automotive industry, manufacturing, electronic devices, total quality management, location of plants, supply chain management, information technology, human resource selection etc.

Keywords: Engine, Graph Theory, Matrix Approach, Operating Parameter, Permanent Index

1. Introduction

The art of making a choice from a number of alternatives to achieve a desired result is decision making[1]. The decision making process is applied in all aspects of individual and organizational level. Decision makers are often forced with several conflicting alternatives. Various multiple attribute decision making methods used by researchers and experts are ANP, AHP, SAW, WPM, VIKOR, ELECTRE, PROMETHEE, TOPSIS, GTMA etc. Out of these methodologies, Graph theory matrix approach is a logical and systematical approach. It synthesizes the interrelationship among the different parameters and systems to evaluate score for the entire system. Graph theory matrix approach offers simple, generic, easy and convenient computation[2] and have wide range of applications in science & technology and in numerous other areas. GTMA is adopted in neural stochastic coupled oscillators to check the exponential stability. Delay-dependent criteria are developed to ensure moment exponential stability[3]. GTMA is adopted into power distribution network for different topologies of power distribution. The optimal

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radial path while minimizing the cost was found using GTMA[4]. Graph theory was used for analyzing group decision-making regarding renewable energy policy selection. Compared to the conventional deterministic method, the stochastic graphical matrix approach provides more reliable estimation accuracy[5]. Graph theory matrix approach is adopted in optimizing leader-follower multi-agent systems. It is shown that the convergence rate depends on a leader-induced distance[6]. GTMA is used in Patch definition in meta population analysis to find the relationships between patch characteristics such as area, connectivity and the demographic processes of colonization's and extinctions[7]. Graph theory approach is applied to find the intensity of barriers in the implementation of total productive maintenance[8]. GTMA was adopted to develop an integrated system model for the structure of water resources development and management system which consists of five subsystems along with interactions between them[9].

This paper presents a combinatorial model using graph theory matrix approach to find the optimal combination of operating parameters of a diesel engine considering various subsystems and interactions between them.

2. Graph Theory Matrix Approach

Graph theory matrix approach consists of following three steps[10]:

- Digraph representation
- Matrix representation
- Permanent function representation

2.1 Digraph Representation

Graph theoretical models have adaptability to model the attributes and their interdependencies in the form of digraph. This digraph G(V,E) consists of set of nodes V= $\{V_i\}$ with i=1,2,3,……M and a set of directed edges D= $\{d_{ij}\}$. A node V_i represents the i^{th} attribute and the edge represents the relative importance between the attributes. The number of nodes M considered is equal to the number of attributes. If a node 'i' has a relative importance over another node 'j', then a directed edge is drawn from node i to j (d_{ij}). If the node j has a relative importance over i, then a directed edge is drawn from node j to node i (d_{ji}). In the present work, the system parameters considered for generating the digraph are load (L), injection timing (IT) and injection pressure (IP). The subsystems, brake power (BP), brake specific fuel consumption (BSFC), Nitric oxide (NO) and Carbon monoxide (CO). The parameter digraph of nodes and their interdependencies is shown in Figure 1.

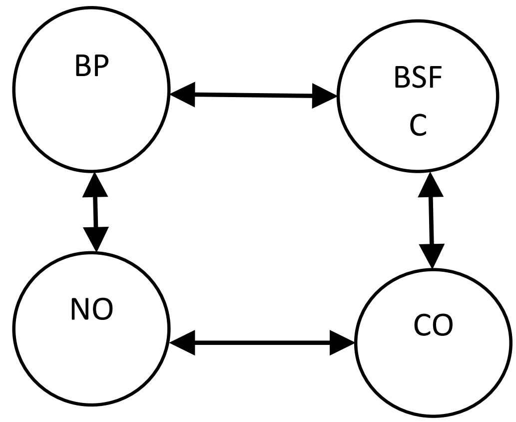

Figure 1. Attributes Digraph.

As the number of nodes and their relative importance increase, the digraph becomes complex. As such the visual analysis of digraph is more difficult and complex. To overcome this, the digraph is represented in matrix form.

2.2 Matrix Representation

Matrix representation of parameter digraph presents a one-to-one representation which is useful in analyzing the digraph expeditiously and also it is useful in representing a digraph to a computer. This is a MXM matrix and considers all of the attributes. (D_i) and their relative importance (a_{ij}). The attributes matrix is shown in Equation1.

$$A = \begin{matrix} D_i & a_{12} & a_{13} & a_{14} \\ a_{21} & D_2 & a_{23} & a_{24} \\ a_{31} & a_{32} & D_3 & a_{34} \\ a_{41} & a_{42} & a_{43} & D_4 \end{matrix} \qquad (1)$$

Where D_i is the value of the i^{th} attribute represented by node V_i and a_{ij} is the relative importance of the i^{th} attribute over the j^{th} represented by the edge D_{ij}. The values of D_i are taken as the experimental results which are normal-

Table 1. Quantitative and normalized value

Exp No.	Factors			Quantitative Values				Normalized Values			
	Load (A)	IT (^{0}BTDC)	IP (bar)	BP (kW)	BSFC (kg/h kW)	NO_x (ppm)	CO (%)	BP (kW)	BSFC (kg/h kW)	NO_x (ppm)	CO (%)
1	9	19	200	2.422	0.410	239	0.03	0.547	1.000	0.311	0.75
2	9	19	220	2.364	0.404	378	0.02	0.533	0.985	0.492	0.50
3	9	19	240	2.401	0.399	372	0.02	0.542	0.973	0.484	0.50
4	9	23	200	2.422	0.396	234	0.03	0.547	0.965	0.304	0.75
5	9	23	220	2.373	0.403	310	0.02	0.535	0.982	0.403	0.50
6	9	23	240	2.401	0.399	515	0.02	0.542	0.973	0.670	0.50
7	9	27	200	2.404	0.399	472	0.03	0.542	0.973	0.614	0.75
8	9	27	220	2.364	0.404	478	0.02	0.533	0.985	0.622	0.50
9	9	27	240	2.401	0.399	515	0.02	0.542	0.973	0.670	0.50
10	13	19	200	3.492	0.354	317	0.04	0.788	0.863	0.412	1.00
11	13	19	220	3.369	0.349	469	0.02	0.760	0.851	0.610	0.50
12	13	19	240	3.430	0.332	420	0.02	0.774	0.809	0.546	0.50
13	13	23	200	3.477	0.348	278	0.03	0.785	0.848	0.362	0.75

14	13	23	220	3.395	0.347	344	0.02	0.766	0.846	0.447	0.50
15	13	23	240	3.430	0.332	595	0.02	0.774	0.809	0.774	0.50
16	13	27	200	3.426	0.352	615	0.03	0.773	0.858	0.800	0.75
17	13	27	220	3.369	0.349	630	0.02	0.760	0.851	0.820	0.50
18	13	27	240	3.430	0.332	595	0.02	0.774	0.809	0.774	0.50
19	18	19	200	4.428	0.338	419	0.04	1.000	0.824	0.545	1.00
20	18	19	220	4.340	0.321	523	0.02	0.980	0.782	0.681	0.50
21	18	19	240	4.387	0.319	482	0.02	0.990	0.778	0.627	0.50
22	18	23	200	4.428	0.347	310	0.03	1.000	0.846	0.403	0.75
23	18	23	220	4.364	0.320	382	0.02	0.985	0.780	0.497	0.50
24	18	23	240	4.387	0.319	610	0.02	0.990	0.778	0.794	0.50
25	18	27	200	4.384	0.333	768	0.03	0.990	0.812	1.000	0.75
26	18	27	220	4.340	0.303	753	0.02	0.980	0.739	0.980	0.50
27	18	27	240	4.387	0.306	666	0.02	0.990	0.746	0.867	0.50

ized on the scale of 0 and 1. The attribute's quantitative and normalized values are shown in Table 1.

2.3 Permanent Function

The permanent of the attributes matrix is a universal

$$
\begin{aligned}
\text{Per(A)=} \; & \prod_{i=1}^{M} Si + \sum_i \sum_j \sum_k \ldots \sum_m (a_{ij} a_{ji}) R_k R_l \ldots R_m + \sum_i \sum_j \sum_k \ldots \sum_m (a_{ij} a_{ji} a_{ki} + a_{ik} a_{kj} a_{ji}) R_l R_m \ldots R_m \\
& + \left(\sum \sum \sum \ldots \sum (a_{ij} a_{ji})(a_{kl} a_{lk}) R_m R_n \ldots R_m + \sum_i \sum_j \sum_k \ldots \sum_m (a_{ij} a_{jk} a_{kl} a_{li} + a_{il} a_{lk} a_{kj} a_{ji}) R_m R_n \ldots R_m \right) \\
& + \left(\sum_i \sum_j \sum_k \ldots \sum_m (a_{ij} a_{ji})(a_{kl} a_{lm} a_{mk} + a_{km} a_{ml} a_{lk}) R_n R_o \ldots R_m + \sum_i \sum_j \sum_k \ldots \sum_m (a_{ij} a_{jk} a_{kl} a_{lm} a_{mi} + a_{im} a_{ml} a_{lk} a_{kj} a_{ji}) R_n R_o \ldots R_m + \ldots\ldots \right)
\end{aligned} \tag{2}
$$

function[11]. It is a standard matrix function used in combinatorial mathematics[12]. Application of the permanent concept will lead to a better appreciation as no negative sign will appear in the expression and hence no information will be lost[13]. The permanent function is written in Equation (2).

A computer program is developed to evaluate the values of permanent index. Substituting the values of D_i and a_{ij} for all the experiments, the permanent index values are evaluated for all experiments and tabulated in the descending order to rank them. The permanent index values are shown in Table 2.

The experiment no. with maximum permanent index is found to be the optimal combination of operating parameters[14]. In the present investigation, the experiment

Table 2. Permanent index values

Exp No.	Parameters			Permanent Index	Rank
	Load	IT	IP		
25	18	27	200	3.168	1
19	18	19	200	2.840	2
16	13	27	200	2.656	3
26	18	27	220	2.613	4
27	18	27	240	2.506	5
24	18	23	240	2.461	6
10	13	19	200	2.413	7
22	18	23	200	2.349	8
20	18	19	220	2.324	9
17	13	27	220	2.322	10

21	18	19	240	2.267	11
7	9	27	200	2.248	12
15	13	23	240	2.244	13
18	13	27	240	2.244	14
23	18	23	220	2.114	15
11	13	19	220	2.097	16
13	13	23	200	2.082	17
6	9	23	240	2.039	18
9	9	27	240	2.039	19
12	13	19	240	2.004	20
8	9	27	220	1.993	21
14	13	23	220	1.924	22
1	9	19	200	1.912	23
4	9	23	200	1.878	24
2	9	19	220	1.863	25
3	9	19	240	1.854	26
5	9	23	220	1.775	27

no. 25 has the highest value of permanent index (3.168). The final decision is taken keeping the practical considerations in mind.

3. Conclusion

In this paper, Graph theory matrix approach is adopted to find the optimal combination of operating parameters by evaluating the permanent index. Nodes of the digraph represent the attributes and their inter-dependencies represent the edges. The attributes digraph is translated into attributes matrix. The permanent index was evaluated from the permanent function. The higher value of permanent index indicates the best combination of operating parameters. Graph theory matrix approach was adopted as it offers a generic, simple, easy and convenient decision making method that involves less computation.

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