

# Radial Basis Function Neural Network (RBFNN)Based Modeling in Liquefied Petroleum Gas (LPG)-Diesel Dual Fuel Engine with Exhaust Gas Recirculation (EGR)

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## Abstract

In this paper the application of Radial Basis Function Neural Network (RBFNN) for modeling the performance and emission parameters of a LPG-Diesel dual fuel engine with EGR is presented. Three different centre selection strategies namely fixed centre selected at random (FCSR), Fuzzy c-means Algorithm (FCM) and Conditional Fuzzy c-means Algorithm (CFCM) are used in the design of Radial Basis Function (RBF) network. The performance parameters included are Brake Power (BP), Brake Specific Energy Consumption (BSEC) and Brake Thermal Efficiency (BTE) and emission parameters included are Exhaust Gas Temperature (EGT), smoke, Hydro-Carbons (HC) and Nitrogen Oxides (NO<sub>x</sub>). The results showed that there is a good correlation between the RBFNN predicted values and the experimental values for various engine performance and emission parameters with R<sup>2</sup> value ranging from 0.90 to 0.99 and the Mean Relative Error (MRE) values are within 5% which is acceptable. A comparison with MLP modeling demonstrated that RBFNN is more efficient than Multi-Layer Perceptron Neural Network (MLPNN) for modeling in this application.

**Keyword:** LPG-Diesel, Dual Fuel, RBFNN, EGR

## 1. Introduction

Exhaust emissions from diesel engines are a great source of concern from an environmental standpoint, especially oxides of nitrogen, smoke and particulate matters. One way of reducing the pollutant emissions is to operate the diesel engine in dual fuel mode. This is mainly intended to reduce smoke, particulate matters and Nitrogen Oxides (NO<sub>x</sub>). In dual mode the gaseous fuel is inducted to the air intake manifold. The mixture of gaseous fuel and air is compressed during the compression stroke. A small quantity of pilot fuel (diesel) is used to initiate the combustion process. Pilot fuel is injected just before the end of compression stroke to initiate the burning process<sup>1</sup>. Another technique that has been adopted effectively to

reduce pollutant emissions, especially nitrogen oxides from diesel engines is Exhaust Gas Recirculation (EGR). In this a small fraction of the exhaust gas is diverted into the in-take manifold. The diverted exhaust gas combines with the fresh charge before entering into the engine cylinder. This mixing of exhaust gas with air results in the increase in the heat capacity of the air. This helps to lower the peak gas temperature during combustion, which in turn reduces the formation of No<sub>x</sub><sup>2,3</sup>. The light load performance of the engine is also improved because of the increased initial charge temperature<sup>4,5</sup>.

Engine design engineers are interested to know the performance of a LPG-Diesel dual fuel engine for various loads, LPG flow rates and load. This can be achieved either by conducting experiments for all possible combination

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of input parameters or by developing a model for the engine operation. Conducting experiments for all possible combination of test parameters requires more time and investment. Developing a precise and accurate model for the performance and emission parameters is also difficult because of the complex processes involved. As a solution to this problem the engine performance and emission parameters can be predicted using Artificial Neural Networks (ANNs) models. The developed model can be used to predict the output for a given set of inputs.

Artificial Neural Network techniques are used for modeling when the relationship between input and output is not known. Once trained the ANN model can be used to predict the output for a new input data set. ANNs mimic the learning process of the biological nervous system and the brain. ANNs are capable of processing non-linear and complex data sets including imprecise and noisy data<sup>6</sup>. Thus ANNs can be used to develop models to predict the engine performance and emission parameters when the exact relationships between input parameters and output parameters are not available. ANNs have been successfully used for developing prediction models for the performance and emission parameters of diesel and gasoline engines.

**LIU Zhen-tao et al (2004)** developed an RBFNN model for Compressed Natural Gas (CNG)-Diesel Dual Fuel Engine (DFE) to study the emission performance. The simulated results agreed very well with the traditional emission models with sum squared error being below the set value of 0.15%<sup>7</sup>. **Jerzy Kowalski (2009)** developed an ANN model to evaluate the concentration of NO<sub>x</sub> in the exhaust gas of a marine 2S diesel engine. The developed model successfully predicted the NO<sub>x</sub> concentration and fuel consumption with an average error of 1.83% and 1.12% respectively<sup>8</sup>. **M. I. Jahirul et al (2009)** applied ANN technique for the prediction of Brake Specific Fuel Consumption (BSFC) of retrofitted CNG engine. The ANN prediction model for the BSFC yielded a Mean Relative Error (MRE) of 0.224%, a Root Mean Square (RMS) error of 0.242gm/kWh and a correlation coefficient of 0.999931. The results demonstrated that the ANN model can effectively predict the BSFC despite wide range of operating conditions<sup>9</sup>. **O. Obodeh et al (2009)** developed an ANN model to predict the NO<sub>x</sub> emissions of a multi cylinder diesel engine. They obtained best results with 11 neurons for Levenberg-Marquardt (LM) algorithm. The Back Propagation (BP) algorithm with a learning rate and momentum of 0.05 and 0.05

respectively, gave the best-averaged accuracy<sup>10</sup>. **T. Hari Prasad et al (2010)** developed an ANN prediction model to predict the exhaust emissions from the diesel engine. The predicted values of the exhaust emissions of the model agreed quite well with experimental results with high correlation coefficients, with very low RMS errors<sup>11</sup>. **Mingvy Wang et al (2010)** developed an ANN based emission model for an intelligent marine diesel engine. They used back-propagation algorithm for learning and gradient descent rule for error minimization. Scaled Conjugate Gradient (SCG) and Levenberg-Marquardt (LM) algorithms have been used for training the model. The predicted results of the ANN were in good agreement with the experimental results<sup>12</sup>. **R. Manjunatha et al (2010)** demonstrated that ANN model based on Back Propagation (BP) using Levenberg Marquardt (LM) algorithm can successfully be used to predict the emissions from a C.I engine using biodiesels<sup>13</sup>. **Shiva Kumar et al (2010)** developed MLPNN and RBFNN models to predict the performance and emission parameters of a diesel engine fuelled with blend of diesel and waste cooking oil. The MLP model used different learning algorithms and RBF model used strategies like random initialization & fuzzy c-means (FCM) for center selection. RBF model resulted in lower MRE & high prediction accuracy when compared to MLP model<sup>14</sup>. **Mingvy Wang et al (2010)** used RBFNN for predicting the emissions of a marine two stroke diesel engine. The model yielded a R<sup>2</sup> value of 0.984 and the mean percentage errors were smaller. The complex nature of the combustion process and the error in the measurement has resulted in the high mean errors<sup>15</sup>. **Qiang Zhang et al (2010)** developed an emission prediction model based on RBFNN to predict emissions for Water Coal Slurry (WCS)/diesel dual fuel. The predicted results were in agreement with the traditional emission model, indicating applicability of the model for prediction<sup>16</sup>. **Yusaf et al (2010)** used ANN for developing models to predict the performance and emission parameters in a CNG-Diesel dual fuel engine. The model showed a good correlation between predicted and experimental values<sup>17</sup>. **Shiva Kumar et al (2011)** used ANN to develop performance and emission models of a variable compression ratio engine fuelled with waste cooking oil as a biodiesel at different injection timings. Good correlations between the predicted and experimental values were obtained for performance and emission parameters with Mean Relative Error (MRE) of less than 8%<sup>18</sup>. **J. Mohammad hassani et al (2012)** developed an

ANN model with LM algorithm for predicting the NOx emissions from a DI diesel engine. The developed model predicted the NOx emissions with a Mean Relative Error (MRE) of less than 8%<sup>19</sup>. **R. Manjunatha et al (2012)** used MLPNN and RBFNN model to predict the emissions from a diesel engine fuelled with biodiesel blends. The emission modeling of NOx, Carbon Monoxide (CO), Carbon Dioxide (CO<sub>2</sub>), Hydro Carbon(HC) and smoke proved that RBFNN is superior to MLP<sup>20</sup>. **Rashmi P. Shetty et al (2012)** developed a RBFNN model to predict performance and emission characteristics in a Dual Spark Plug S.I Engine and compared the results with MLPNN model. Two center selection strategies namely fixed center selected at random and Fuzzy c-means clustering techniques were used for RBFNN model. They demonstrated that RBFNN has better generalization capability in terms of higher prediction accuracy and is faster compared to MLPNN<sup>21</sup>. **Gorkem et al (2013)** developed a MLPNN and RBFNN model to predict the emission and exhaust gas temperature of a DI diesel engine with emulsified fuel. Both the developed models gave good results with RBF proving more reliable than MLP<sup>22</sup>. **Shivakumar et al (2013)** successfully developed an ANN model using BP algorithm to predict performance and emission parameters of 4 stroke diesel engine fuelled with Honge Methyl Ester. The prediction yielded good correlation with experimental values with R<sup>2</sup> value closer to 1 and MRE less than 9%<sup>23</sup>. **M. I. Jahirul et al (2013)** developed an ANN model for predicting the thermal efficiency, BSFC, EGT and air fuel ratio of a CNG/Diesel dual fuel engine. The predicted result gave RMS error smaller than 0.015, R<sup>2</sup> value of 0.999 and mean error smaller than 0.01% thus proving the effectiveness of ANN model in prediction capability<sup>24</sup>. **Mohammad H. Ahmadi et al (2014)** implemented an ANN model to predict the torque of a Stirling Heat engine. The predicted values of the model were very close to the experimental values<sup>25</sup>. **Syed Javed et al (2015)** used ANN modeling for prediction of BTE, BSEC, CO, O<sub>2</sub>, CO<sub>2</sub>, NOx, HC & EGT of a four stroke, single cylinder hydrogen-diesel dual fuel engine with Jatropa methyl ester biodiesel blends. Levenberg-Marquardt BP algorithm with logsig and tansig activation functions gave better prediction results<sup>26</sup>. The review presented establishes the effectiveness of ANN in modeling application related to engines. Various researchers have used RBFNN and MLP effectively in different applications.

This paper presents an application of RBFNN technique for predicting performance and emission

parameters of a LPG-Diesel dual fuel CI engine with EGR. The input parameters used to build the models include % Load, % Exhaust Gas recirculation (EGR) and LPG flow rate and output parameters included performance parameters namely BP, BSEC and BTE and emission parameters which include EGT, Smoke, HC and NOx. Three different center selection strategies have been studied in developing the RBFNN model. A comparison of their performance is discussed and the best possible strategy has been identified. This is compared with MLP model, which is a widely used ANN model in these applications.

## 2. Experimental Setup and Procedure

An existing single cylinder four stroke diesel engine is retrofitted with additional equipment so as to run as a LPG-Diesel dual fuel engine and facilitate Exhaust Gas Recirculation (EGR). The engine is fitted with data acquisition system, which in turn is interfaced with a personal computer. Experiments were conducted by operating the engine at a constant speed of 1500rpm. The engine was run at a specified load with various combinations of LPG flow rate and % EGR. At each combination of % Load, % EGR and LPG flow rate the performance parameters namely BP, BSEC, and BTE and emission parameters namely EGT, Smoke, HC, and NOx levels were measured under steady operating conditions. The experiments were conducted at 0%, 25%, 50%, 75% and 100% of the full load capacity. The LPG flow rates were 0, 0.3, 0.4, 0.5, 0.6, 0.7, and 0.8 kg/hr. The EGR was 0%, 5%, 10% and 15% of the total intake charge by mass.

## 3. Artificial Neural Networks

Artificial Neural networks are computational systems that attempt to mimic the way the human brain works. They are nonlinear information processing systems. ANNs are collection of individually interlinked basic processing units called neurons. They are capable of accommodating multiple input variables to predict multiple output variables. The ANN modeling differs from the conventional modeling approach in terms of its ability to learn about the system that needs to be modeled without the prior knowledge of the process relationships<sup>6</sup>.

Broomhead and Lowe (1988) and Moody and Darken (1989) exploited the use of RBFNN architecture. The RBF network differs from a MLP network trained by Back

Propagation algorithm in terms of the behaviour of the single hidden layer. In case of MLP the sigmoidal activation function (logistic sigmoid/hyper tangent sigmoid) is used whereas, the hidden units of RBF use Gaussian function or some other basis kernel function. The basic form of an RBFNN has single hidden layer. The MLPNN may consist of one or more number of hidden layers. The hidden unit activation function in an RBFNN calculates the Euclidean distance between the center and input vector of that unit. In an MLP the hidden unit activation function computes the inner product of the synaptic weight vector and input vector of that unit<sup>6</sup>.

## 4. Radial Basis Function Neural Networks

### 4.1 Introduction

The structure of RBFNN consists of three layers. The input layer consists of source nodes. The number of source nodes is equal to the number of input units. The second layer is the hidden layer. The hidden layer consists of nonlinear units. The nonlinear units are directly connected to each of the nodes in the input layer. The hidden unit contains a basic function characterized by center and width. Initially the radial distance  $d_i$  between the  $n$  - dimensional input vector  $x$  and the centre of the basis function  $c_i$  is computed for each unit  $i$  in the hidden layer as  $d_i = \|x - c_i\|$  using the Euclidian distance. The output of each hidden unit  $i$  is computed by using the Gaussian function;

$$\phi_j(x) = \exp\left(-\frac{\|x - c_i\|^2}{2\sigma_i^2}\right) \quad (1)$$

where,  $\Phi_j(x)$  is the output of the  $j^{\text{th}}$  basis function.  $\sigma$  is the width (standard deviation) of the basis function. Nonlinear transformation takes place between the input layer and the hidden layer and linear transformation between the hidden unit and the output space. Each of the basis function of RBFNN is connected to units in the output layer by the equation;

$$y_k(x) = \sum_{i=1}^M w_{k,i} \phi_j(x) + w_{k,o} \quad (2)$$

where,  $y_k(x)$  is the output of the  $k^{\text{th}}$  output neuron for the input vector  $x$ .  $M$  is the number of basis functions in the RBF network.  $w_{i,k}$  is the weight between  $k^{\text{th}}$  output neuron and  $i^{\text{th}}$  basis function and  $w_{k,o}$  is the bias constant for the  $k^{\text{th}}$  output neuron<sup>27,28</sup>.

A successful design of a Radial Basis Function Neural Network consists of three steps: the selection of basis function, the computation of proper kernel center and the learning of the weights between the hidden units and the output units. The most widely used basis function is the Gaussian kernel function. The key problem in RBFNN is the selection of basis function centers and its width, since they have a strong influence on the performance of the network. Once the kernel function and its centers are computed, the network can be considered synonymous to a single layer network with linear output units. Now the weights associated between the middle and the output layer can be computed by the Least Mean Square (LMS) error method<sup>29</sup>. In this work three different center selection strategies are used to develop the RBFNN model. The width of the RBF units can be selected by using heuristics as suggested by S. Haykin<sup>6</sup> or by trial and error method until the model results in maximum prediction accuracy on test data. The three strategies used are discussed below:

### 4.2 Learning Strategies for mModel Development

#### 4.2.1. Fixed Centers Selected at Random (FCSR)

The simplest and quickest approach to develop the RBFNN model is by selecting centers at random. Here the centers are selected at random from the given set of data points. The width of the each of these centers are set to be equal & fixed at an appropriate value based on the distribution of the data points<sup>30</sup>. These centers are used to train the model and the performance of the model is evaluated based on the Mean Relative Error (MRE). The MRE is the deviation of the predicted value from the experimental value.

The set of data points used for the selection of centers at random may not be evenly spread across the input space. Selecting the RBF centers at random from such a data set points and relying on that to be the best subset is always risky strategy. As a better approach, a suitable clustering technique can be used for selection of RBF centers, which are the better representative of the given set of the data points<sup>30</sup>.

#### 4.2.2. Fuzzy C-Means Algorithm (FCM)

Fuzzy c-means is a clustering algorithm proposed by Bezdek<sup>31</sup>. In this clusters are formed from the given set of data points based on the degree of membership grade. It

is an iterative algorithm which finds cluster centers based on minimization of an objective function;

$$J(U, c_1, c_2, \dots, c_n) = \sum_{i=1}^n \sum_{j=1}^v u_{ij}^m \|x_i - c_j\|^2, \quad 1 \leq m < \infty \quad (3)$$

Where  $u_{ij}^m$  is the degree of membership of  $x_i$  in cluster  $j$  and  $x_i$  is the  $i^{\text{th}}$  of the  $n$  dimensional measured data.  $c_j$  is the  $n$ -dimensional center of a cluster and  $\|x_i - c_j\|$  is the Euclidean distance measured between the data & the cluster center.

In the first step, the number of centers  $v$  ( $1 < v < n$ ) and fuzziness index  $m$  ( $1 < m < \infty$ ) are randomly selected. In the second step the algorithm starts by initializing the cluster centers  $c_j, j = 1, 2, 3, \dots, v$  to a random value from  $n$  data points  $\{x_1, x_2, \dots, x_n\}$ . In the third step the membership matrix  $u_{ij} = [U]$  is computed using the equation;

$$u_{ij} = \frac{1}{\sum_{k=1}^v \left( \frac{\|x_i - c_j\|}{\|x_i - c_k\|} \right)^{\frac{2}{m-1}}} \quad (4)$$

where  $\|x_i - c_j\|, \|x_i - c_k\|$  are the Euclidean distance between  $j^{\text{th}}$  and  $k^{\text{th}}$  cluster centers and the  $i^{\text{th}}$  data point. The objective function  $J$  is computed in the fourth step according to the equation (3). In the final step, the new fuzzy cluster centers  $c_j, j = 1, 2, \dots, v$  are computed by using equation (5).

$$c_j = \frac{\sum_{i=1}^n u_{ij}^m \cdot x_i}{\sum_{i=1}^n u_{ij}^m} \quad (5)$$

FCM is an iterative process to calculate new membership matrix and centers. The iteration process stops when the difference between two consecutive iterations is less than the threshold value<sup>31</sup>.

### 4.2.3. Conditional Fuzzy C-Means Algorithm (CFCM)

Generally clustering is considered as an unsupervised method where the information about the underlying structure of the membership function is not known. When some labeled membership functions are available, it is always advantageous to use them to influence the clustering process. This principle is used in conditional FCM. It is a FCM based clustering technique in which the outcome of the clustering process is guided by an auxiliary variable. For each of the labeled membership function, an auxiliary matrix condition  $f_i$  exist, where  $f_i$

ranges from (0, 1). The membership matrix  $u_{ij} = [U]$  is now modified as;

$$u_{ij} = \frac{f_i}{\sum_{k=1}^v \left( \frac{\|x_i - c_j\|}{\|x_i - c_k\|} \right)^{\frac{2}{m-1}}} \quad (6)$$

Here  $f_i$  describes the level of belongingness of  $x_i$  in the cluster formed. For the linguistic term defined in the output space expressed as a fuzzy set  $B, B: \mathbf{R} \rightarrow [0, 1]$ , then  $f_k = B(y_k), k = 1, 2, \dots, v$ , represents the degree of membership of  $y_k$  in  $B$ . The manner in which  $f_i$  can be associated with the computed membership values of  $x_i$ , say  $u_{1j}, u_{2j}, \dots, u_{nj}$ , is not unique. The  $f_i$  to be distributed additively across the entries of the  $i^{\text{th}}$  column of partition matrix meaning that

$$\sum_{i=1}^n u_{ij} = f_i, \quad i = 1, 2, \dots, n \quad (7)$$

The new constraint  $f_i$  captures the following.

- If the  $i^{\text{th}}$  element in the (pattern of) the data set is regarded as meaningless in the context imposed by  $B, B(y_k) = 0$ , then the pattern is excluded from any further involvement in the clustering procedure,  $u_{ij} = 0$ , for all  $i$ .
- On the other hand, the pattern for which  $B(y_k) = 1$  contributes maximum to the clustering process.
- All the elements with some intermediate context contribution i.e., between 0 and 1 are taken into account only on partial basis.

This is also an iterative process where iteration stops when the difference between two successive iterations is less than the threshold value<sup>32</sup>.

### 4.3 Performance Evaluation of the RBFNN Models

Computer codes for MLP and RBF models were developed in Matlab (version 2014)<sup>33</sup>. The models were trained until the best performance was obtained. In order to test the different models, two statistical tests, the correlation coefficient and Mean Relative Error (MRE) between the experimental and the predicted data were used.

The MRE is defined as:

$$MRE = \left| \frac{(t_j - o_j)}{t_j} * 100 \right| \quad (8)$$

where  $t_j$  = Experimental value  
 $o_j$  = Predicted value.

Out of 140 set of experimental data available, 119 set of data are used for training the model and remaining 21 set of data are used for testing. For testing only the input data is given to the model and is used to predict the output based on the weights generated during the training phase. The data used have been normalized between 0 and 1 using a suitable normalization procedure.

The schematic diagram of the developed RBFNN model is shown in Figure 1.

The centers selected by the above three center selection strategies are used to develop the RBFNN model. The linear weights are the only parameter that needs to be learned here. The weights are updated using simple Least Mean Square (LMS) algorithm. The maximum number of epochs is set to 2000, the simulation parameters  $\alpha = 0.5$  and  $\eta = 0.5$  were maintained constant for all the models developed. The width of the basis function in RBF has been fixed by trial and error method based on maximum prediction accuracy on the test data.

Trained network is tested for its performance. Prediction Accuracy, Mean Relative Error (MRE) and  $R^2$  value are the criteria used for testing the network performance. A MRE of 5% has been taken as a limit for performance parameters and 8% for the emission parameters<sup>18,34</sup>. The performance of the three models for BSEC and HC is given in Table 1 and Table 2 respectively with increasing number of centers. Results for other performance and emission parameters have not been included, as they are almost similar, with differences in the numerical values for different items in the table.

From the Table 1 it is clear that the maximum prediction accuracy of 80.59 on test data for BSEC is obtained for 100 centers and a width of 0.21 for FCSR-RBFN, whereas maximum prediction accuracy of 80.95 is obtained for 95 centers and a width of 0.21 for FCM -RBFN. The maximum prediction accuracy of 85.71 is obtained for 70 centers for a width of 0.24 for CFCM-RBFN. Thus the prediction

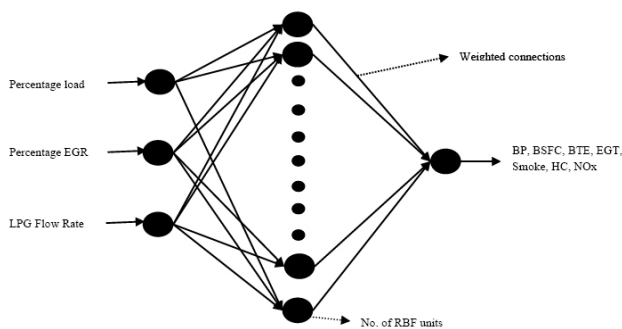


Figure 1. RBFNN model.

accuracy on training data and test data are more for CFCM, followed by FCM and FCSR method. This is also true for the emission parameter HC as seen from Table 2. This shows that CFCM is the effective clustering algorithm for center selection followed by FCM and FCSR and can be used for designing an optimal RBFNN architecture.

#### 4.4 Comparison of the Three RBFNN Models

In the present study three different center selection strategies namely Fixed Centre selected at Random, FCM and Conditional FCM have been used for center selection for the radial basis function. Table 3 shows the comparison of the performance of the three center selection strategies. From the Table it is clear that the RBFNN Model developed based on three different center selection strategies is able to predict the performance and emission parameters with  $R^2$  value well above 0.9 and MRE is well below the set value of 5% and 8% for performance and emission parameters respectively. The prediction accuracy of BP, BTE, EGT, and NO<sub>x</sub> are 100% where as for BSEC, Smoke and HC it is above 80.95%.

From the comparison of three strategies, it can be concluded that Conditional FCM is superior with higher prediction accuracy on test data, lowest MRE value and highest  $R^2$  value, followed by FCM and Random selection method. The value of MRE is less for performance parameters than the emission parameters for all the three models. This is because of cycle by cycle variation of the emission parameters with respect to combustion process, resulting in noise in the data. In case of Fixed Center Selected at Random (FCSR), the clusters and their centers are selected arbitrarily and randomly, whereas in FCM the input data points are partitioned into a number of clusters with the data point inside a cluster showing a degree of belongingness. FCM assigns each data point to a cluster with a degree of membership equal to one. With each iteration process it calculates new membership matrix and centres and hence better than FCSR. Conditional FCM is a better clustering algorithm than FCM, as it uses a supervised learning method, where the labeled membership function influences the clustering process through an auxiliary matrix condition.

### 5. MLPNN Modeling

Multi Layer Perceptron Neural Network models have been developed for both the performance and emission

**Table 1.** Performance of three models for BSEC

Parameter	Centre selection strategy	Width	Centres	Prediction Accuracy on Training data	Prediction Accuracy on Test data	Average MRE
BSEC	Fixed Centre	0.21	80	84.03	52.38	5.79
			85	84.03	61.90	5.56
			90	87.39	57.14	4.05
			95	91.53	76.19	3.89
			<b>100</b>	<b>92.13</b>	<b>80.59</b>	<b>4.46</b>
	FCM	0.21	80	82.35	80.95	5.54
			85	83.19	71.42	6.02
			90	73.10	71.42	7.10
			<b>95</b>	<b>90.75</b>	<b>80.95</b>	<b>4.28</b>
			100	91.59	76.19	4.05
	CFCM	0.24	60	72.26	38.09	8.31
			65	84.87	57.14	5.59
			<b>70</b>	<b>93.27</b>	<b>85.71</b>	<b>4.75</b>
			75	86.56	66.66	4.93
80			87.39	61.90	4.93	

**Table 2.** Performance of three models for HC

Parameter	Centre selection strategy	Width	Centres	Prediction Accuracy on Training data	Prediction Accuracy on Test data	Average MRE
HC	Fixed Centre	0.20	80	85.71	71.44	8.83
			85	86.55	66.66	6.86
			90	90.75	61.90	6.21
			95	91.59	71.42	6.12
			<b>100</b>	<b>89.91</b>	<b>76.19</b>	<b>5.29</b>
	FCM	0.23	80	73.31	71.42	6.92
			85	78.15	61.90	6.68
			90	81.51	71.42	6.06
			95	84.87	76.19	6.07
			<b>100</b>	<b>84.87</b>	<b>80.95</b>	<b>5.70</b>
	CFCM	0.25	80	75.63	61.90	5.96
			85	78.15	66.62	5.59
			90	82.35	76.19	4.04
			95	85.71	71.12	3.76
<b>100</b>			<b>86.55</b>	<b>80.95</b>	<b>4.68</b>	

**Table 3.** Performance of RBFNN Model for the Test Data for three different center selection strategies

Variable	RBF – FCSR			RBF – FCM			RBF – CFCM		
	Prediction Accuracy %	Average MRE	R <sup>2</sup> Value	Prediction Accuracy %	Average MRE	R <sup>2</sup> Value	Prediction Accuracy %	Average MRE	R <sup>2</sup> Value
BP	100	1.15	0.988	100	0.90	0.988	100	0.94	0.985
BSEC	80.59	4.75	0.976	80.95	3.82	0.984	85.71	2.02	0.992
BTE	100	1.96	0.991	100	1.93	0.994	100	0.98	0.995
EGT	100	2.16	0.885	100	1.79	0.913	100	1.74	0.904
Smoke	80.95	3.04	0.990	85.71	2.89	0.992	90.75	2.27	0.993
HC	76.19	4.28	0.929	80.95	5.70	0.931	80.95	4.00	0.949
NOx	100	1.21	0.976	100	2.35	0.967	100	1.17	0.973

parameters for comparison with RBFNN model. One hidden layer has been used and two different activation functions hyperbolic tangent sigmoid (tansig) and logistic sigmoid (logsig) have been compared. Linear function has been used in the output layer. For training the network, the Levenberg Marquardt back propagation algorithm has been used, since this is the fastest algorithm compared to all other back propagation algorithms<sup>18</sup>. The output of the network has been compared with the actual output at each presentation and the error has been computed. This error information is fed back to the system and the system adjusts the weights so that the error reduces with iterations. When the mean square error (MSE) or the number of epochs reaches the predefined value, the training is stopped.

For training the network, the simulation parameters of MSE, number of epochs and learning rate are set at  $1 \times 10^{-7}$ , 1000 and 0.4 respectively. The minimum MSE for tan sigmoid and log sigmoid transfer functions have been found to be 0.000655 and 0.000912 at 46 and 42 numbers of hidden neurons respectively. The variation of Mean Square Error with number of hidden neurons for tansig and logsig is shown in Figure 2 and Figure 3 respectively. Mean Relative Error (MRE) and  $R^2$  values are used to test the performance of the trained network. The architecture of the developed MLPNN model is shown in Figure 4.

## 6. Comparison of RBFNN with MLPNN Modeling

Table 4 shows the comparison of the performance for the developed MLPNN and RBFNN models. The comparison of the two models show that RBF performs better with low

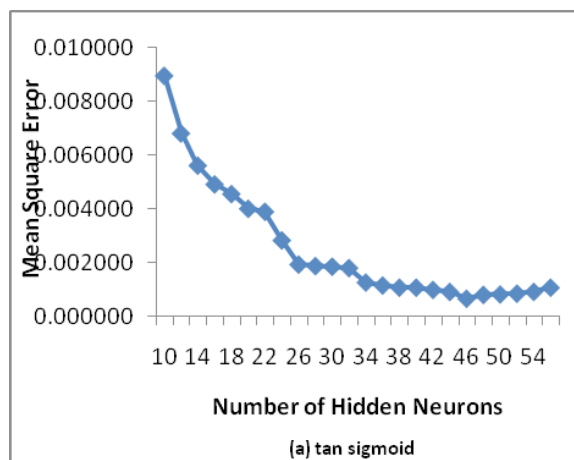


Figure 2. Variation of MSE with hidden neurons for tansig.

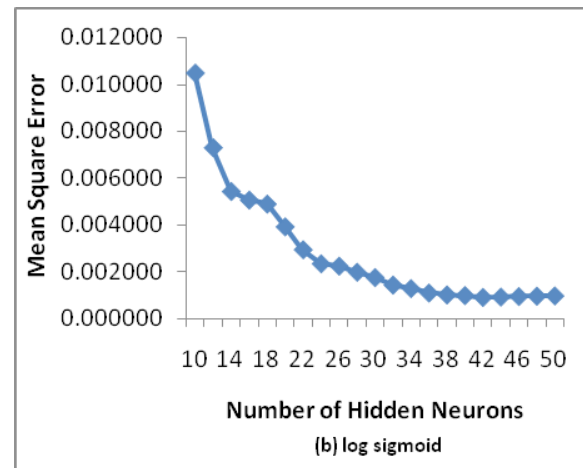


Figure 3. Variation of MSE with hidden neurons for logsig.

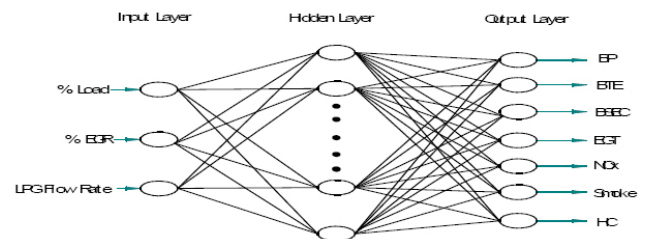


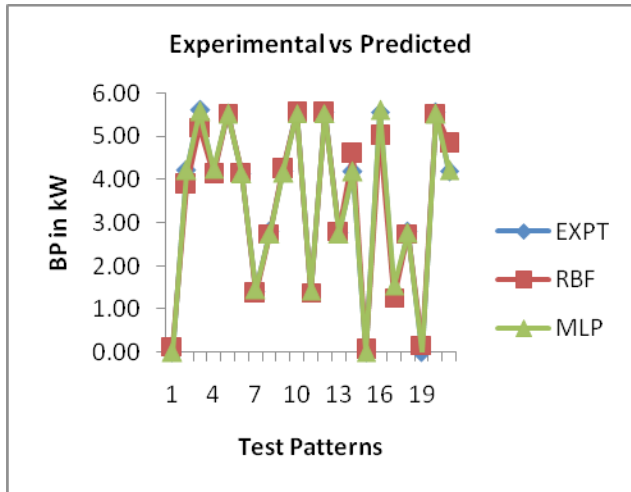
Figure 4. MLPNN Model.

Table 4. Comparison of performance of MLPNN & RBFNN MODELS for TEST DATA

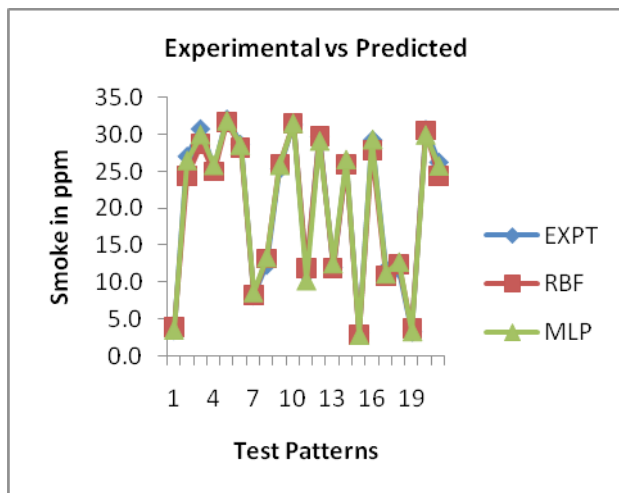
Variable	Prediction Accuracy %		Average MRE		R Square Value	
	MLPNN	RBFNN	MLPNN	RBFNN	MLPNN	RBFNN
BP	100	100	0.71	0.94	0.999	0.985
BSEC	100	86	4.08	4.75	0.994	0.992
BTE	100	100	2.74	0.98	0.995	0.995
EGT	100	100	2.44	1.74	0.946	0.904
Smoke	100	91	3.28	2.27	0.995	0.993
HC	76	81	6.82	5.15	0.970	0.949
NOx	95	100	4.06	1.17	0.985	0.973

MRE (less than 4%) values compared to MLP (less than 7%). The predictive ability of the RBF networks is decided on the basis of number of neurons in the hidden layer. Too few and too many basis functions results in poor predictive ability of the RBF networks. An optimum number of basis functions obtained by meeting the conflicting requirements of the two, results in better predictive ability for the RBFNN<sup>24</sup>. In this work it is achieved by the use of center selection strategies namely CFCM and FCM.





**Figure 5.** Experimental vs Predicted values of RBFNN and MLPNN for BP.



**Figure 6.** Experimental vs Predicted values of RBFNN and MLPNN for Smoke.

A graph of the experimental value vs predicted values for the test patterns by MLPNN and RBFNN has been plotted for BP and Smoke in Fig5 and 6 respectively. From the graph it is clear that the predicted data of RBF are closer to the experimental data compared to predicted values by MLP. The values predicted by RBFNN model are much closer to the experimental value resulting in low MRE than MLPNN model.

## 7. Conclusion

This paper discusses the application of RBFNN for modeling the performance and emission parameters in

a LPG-Diesel dual fuel engine with EGR. Three different center selection strategies have been used to model RBFNN. The comparison of performance of three strategies has proved that the conditional FCM has better generalization capability in terms of low MRE and high prediction accuracy on test data. Further the comparison between MLPNN and RBFNN shows that RBFNN models have better performance capability with low MRE and high prediction accuracy for both performance and emission parameters except for BSEC and Smoke. Thus RBFNN can be successfully used to model performance and emission parameters of LPG-Diesel dual fuel engine.

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