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Intuitionistic Fuzzy Genetic Weighted Averaging Operator and its Application for Multiple Attribute Decision Making in E-Learning

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Abstract:

Background/Objectives: Adaptation and personalization of E-learning systems require efficient learner modeling. Attributes of learner are evaluated to classify their knowledge without considering the weight difference with respect to their similarity level in E-learning environment for intuitionistic fuzzy data. Methods/Statistical analysis: This paper proposes an Intuitionistic Fuzzy Weighted Averaging (IFWA) operator. The IFWA operator is combined with Genetic Algorithm (GA) to tune the weight of the attributes of learners with respect to their similarity level. The proposed model tests and evaluates the IFWA algorithm on user knowledge modeling data set taken from UC irvine machine learning repository. Findings: The algorithm measures the performance in terms of the best weight values corresponding to the classification. Intuitionistic fuzzy data set is compared based on mean error for different run of generations' with best weight values. The mean square error .002349 proves the consistent performance of the algorithm to allocate weight to the attributes in intuitionistic fuzzy domain. Applications/Improvements: The proposed Intuitionistic Fuzzy Genetic Weighted Averaging Algorithm (IFGWA) can play an efficient role in various decision making problems.

Keywords: Domain Dependent Data Classifier, Intuitionistic Fuzzy Genetic Weighted Averaging Operator, Multiple Attribute Decision Making

1. Introduction

1.1 E-Learning

Personalization in E-learning system could be through goals, knowledge, background, hyperspace experience, and preferences which are stored in user model. The content of user model can be divided into two categories viz. domain specific information and domain independent information. Domain specific information reflects the status and degree of a knowledge which user achieves while learning a domain concept. Domain independent information includes goals, interests, backgrounds, hyperspace experiences, and individual traits. These feature based modeling is used to attempt the specific feature of the individual user in an E-learning systems such as knowledge, interest, goal etc[1]. During user's work with the system, these features may change, so goal of feature based models is to track and represent an up- to date state for modeled features. There are some limitations of automatic user modeling. Some components of user modeling such as background and preferences of the user, cannot be deduced at all and have to be provided directly by the user. In such kind of systems, watching the user's action like what user is doing in E-learning systems etc, provides insufficient

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information. The user information can be tracked using the path through the hyperspace and time spent on each node. The fact that user has visited a page several times or spent reasonable amount of time on it does not guarantee that the user attentively read the contents[2]. Due to imprecise nature of human being, user modeling faces the challenges to capture the user features and its evaluation. The vagueness and fuzziness of the user modeling can be handled through good decision making. Various decision making and selection problems can be solved through Multiple Attribute Decision Making (MADM).

1.2 Multi Attributes Decision Making (MADM)

Decision making takes place in an environment where goal, constraints, and consequence of actions are generally unknown[3]. Multiple attribute decision making problems are adapted where data and information are vague, imprecise and uncertain by nature. It is also used in analysis- based decisison making where big data plays an important role[4]. Decisions making in multiple attributes consist of two types of goal: first to select an alternative score from a set of scores. These scores are based on values and importance of attribute of each alternative. Second to classify alternatives in the form of defined characteristics. Both types of goals require preferences among instances of an attribute-preference. It can be provided directly by the experts or by the past choices. It can also be defined as weight of the criteria. Analytical hierarchy process, fuzzy theory, utility preference are the most common form for expressing the importance of the criteria. According to Liu and Wang[5] evaluation funtion defines decision making problems to measure the degree of satisfaction and dissatisfaction of decision maker's requirement. Due to complexity of socio economic environment and subjective nature of human mind, information provided by the decision makers can possibly be uncertain.

Conventional techniques such as linguitic quantifier, langrage multiplier, linear objective programming model, normal distribution methods etc. also known as calculas based methods, determine the weights for multitple attribute decision making. These calculus based methods are subdivided into two main classes namely indirect and direct methods. Indirect methods seek local extrema by solving the non-linear set of equations resulting from setting the gradient of the objective function equal to zero, for a given smooth and unconstrained function, finding a possibile peak starts by restricting the search to those points which have zero slopes in all directions. Direct search methods seek local optima by hopping on the function and moving in a direction related to the local gradient. Both these methods are local in scope i.e. the optima they seek are the best in a neighbourhood of the current point. Another problem with calculus based method is that they depend upon the existence of the derivatives.

Enumerative schemes require a finite search space, the search algorithm look at objective function values at every point in space, one at a time. Genetic algorithm uses a random search procedure through a coding of parameter space. It searches from a population of points, not from a single point[6]. It uses objective function statistics over many generations. It works from a rich database of points simultaneously (a population of strings) climbing many peaks in parallel. Thus the probability of finding a false peak is reduced over methods that go point to point.

1.3 Genetic Algorithm

To simulate evolution in a computer algorithm a genetic algorithm uses the principles of evolution, natural selection and genetics from natural biological systems[7]. These algorithms are used as optimization techniques to evolve the fittest population. In this algorithm, a better solution is generated for an optimization problem through a population of candidate solutions. Each individual has set of attributes (its chromosome or genotype) which can be mutated and altered. Chromosomes are represented in binary strings (0s and 1s) but other encoding are also possible. Each iteration of randomly generated individual from the population is called generation. The fitness of every individual of the population is evaluated in each generation. The fit individuals are randomly selected from the current population, and each individual genome is modified to form a new generation. The new generation candidate solution is then used in the next iteration of the algorithm. The algorithm terminates when either a maximum number of generations has been produced or a satisfactory fitness level has been reached for the populations.

In E-learning system, the genetic algorithm can be used to allocate the weight to the attributes of learner since it can artificially evolve an appropriate decision that meets the performance specification to the greatest extent possible. So it is required to decide the knowledge of a learner considering multiple attributes due to complex socio-economic environment as well as uncertainty of user's mind. So far researchers have evaluated the user models without taking into account the weight difference among the attributes of the user or by allocating adhoc weight to the user attributes. Kahraman et al.[8] applied the genetic to tune the weight conbined with the intuitive knowledege classifier, based on domain dependent data of user. Hence this paper modifies the genetic algorithm for intuitionistic fuzzy data and assign the weight to the multiple attributes in domain dependent data. It proposes the allocation of weight to the multiple attributes using genetic weighted averaging operator in intuitionistic fuzzy environment.

The organization of this paper is as follows: remaining parts of the current section presents the related works. Section 2 presents the intuitionistic fuzzy genetic approach to tune the different weight of the attributes classifying knowledge of a student proportional to their respective similarity levels. The results of the experimental study are presented in section 3. An intuitionistic fuzzy genetic algorithm is tested and their performances are compared with each other. In last, conclusion and future work is discussed.

Zadeh[9] membership function which deals with the fuzziness, is applied to many decision making problems. Decision making takes place in an environment where goals, constraints and consequences of possible actions are not precisely known. Analytical hierarchy process, utility preference function and a fuzzy version of the classical linear weighted average are the most common forms for expressing the importance of criteria. In multiple attribute decision problems, weighting method is applied to weight the specified attribute on a set of alternatives. The most commonly used objective function ($D(A_i)$) is termed weighted average rating, used to aggregate the attribute values to a single or fuzzy number for each alternative, given as follows.

$$D(A_i) = \frac{\sum_{j=1}^{n} W_j * r_{ij}}{\sum_{j=1}^{n} W_j}$$

Where A_i represents alternative i, w_j represents the importance of criteria j and r_{ij} represent the relative merit of criteria j for alternative i. Statistical weighted average algorithm is used to allocate the weight to the attributes and classifies each case in to their respective fuzzy objects. Yager[10] introduced the concept of ordered weighted aggregation operator to determine the crisp weight for Fuzzy Multiple Attribute Decision Making (FMADM). It is used in the form of AND and OR operators. It gives weight either to all criteria or to none.

Definition 1: An ordered weighted averaging operator of dimension n is a mapping $OWA : R^n \rightarrow R$ that has an associated vector$_n$ $\omega = \omega_1, \omega_2, \dots \dots .. \omega_n)^T$ such that $\omega_{\bar{1}} \in [0,1]$ and $\sum_{j=1} \omega j = \mathbf{1}$. Furthermore

$$OWA_W(a_1, a_2, . \quad a_n = \sum_{j=1}^{n} \omega_j b_j .$$

In this approach weight is associated with a particular ordered position of aggregation. Gong[11] presented a crisp standard deviation and mean deviation of interval type -2 fuzzy sets to introduce the optimization model to determine the completely unknown weight of the attribute. Later Atanassov[12,13] generalized this concept as membership and non-membership functions which is more suitable to deal with the fuzziness and uncertainty. Many Researchers' have been applying Intuitionistic Fuzzy Set (IFS) multi attribute decision making in different situations. Chen and Tan[14] define an evaluation function based on max- min operator and the score function that evaluate the degree of satisfiabilty and non- satisfiabilty of each alternatives with respect to set of criteria for vague values.

Definition 2: A Multi-criteria fuzzy decision making problem defined on a set of alternatives M and a set of criteria C, where

$$M = \{M_1, M_{2, \dots \dots \dots ,} M_{m\},} C = \{C_1, C_2, \dots \dots \dots, C_n\}$$

IFS theory for the characteristics of the alternative is

$M_i = \{(C_1, \mu_{i1}, \vartheta_{i1}), (C_2, \mu_{i2}, \vartheta_{i2}), \dots (C_2, \mu_m, \vartheta_m)\}, i = 1, 2, \dots \dots \dots .. m$ where M_{ij}, indicates the degree to which the alternative M_i satisfies the criterian C_j, ϑ_{ij} indicates the degree to which the alternative does not satisfy the criterion.$C_j(\mu_{ij}, \vartheta_{ij}) \in L$ $(j = 1, \dots \dots \dots n; i = 1, \dots \dots \dots m)$.

Chen and Tan[14] define an evaluation function E to measure the degrees to which the alternatives M_i satisfy and does not satisfy the expert's requirement as:

$$\mu_{Mi} = \max(\min(\mu_{ij}, \mu_{ik}, \dots \dots \dots .. \mu_{ip}), \mu_{is})$$
$$\vartheta_{Mi} = \min(\max(\vartheta_{ij}, \vartheta_{ik}, \dots \dots \dots .. \vartheta_{ip}), \vartheta_{is})$$

They defined the weighted score function as

$$W_c(M_1) = \max(w_j S((\mu_{ij}, \vartheta_{ij})) + w_k S(\mu_{ik}, \vartheta_{ik}) + \dots + w_p S((\mu_{ip}, \vartheta_{ip})), S((\mu_{is}, \vartheta_{is})))$$

This function denotes the degree of suitability to which the alternative satisfies the experts requirement, where w_j,

w_k, … … … … . w_p are the degrees of importance of the criteriaC_j, C_k, … … … … … … . C_p respectively. Here w_j, w_k, … . $w_p \in [0,1]$, $w_j + w_k + \cdots \ldots + w_p = 1$ are the weights of different criteria.

Later these operators were improved by Hong and Chai[15] through adding an accuracy function H which is similar to the relationship between mean and variance.

Definition 3: Let α̃ = (ɱ, ʋ) be an intuitionistic fuzzy number then an accuracy function H of an intuitionistic fuzzy value can be represented as: H (α̃) = + ʋ (α̃) ∈ [0,1], to evaluate the degree of accuracy of the intuitionistic fuzzy value ã = (ɱ, ʋ), where H (ã) ∈ [0, 1] The larger the value of H (ã), more the degree of accuracy of the intuitionistic fuzzy value ấ. Later Xu[16] construct an order relation between two intuitionistic fuzzy values which is defined as in definition 4.

Definition 4: Let $a_1 = (t_1, f_1)$ and $a_2 = (t_2, f_2)$ be two intuitionistic fuzzy numbers, and and $S(a_2) = t_2 - f_2$ be the score function of a_1 and a_2 respectively, and let $H(a_1)\ t_1 + f_1$ and $H(a_2)\ t_2 + f_2$ be the accuracy functions of a_1 and a_2 , respectively, then

If $S(a_1) < S(a_2)$, then a_1 is smaller than a_2, denoted by $a_1 < a_2$

If $[S(a]_1) = S(a_2)$, then

1) If $H(a_1) = H(a_2)$ then a_1 and a_2 represent the same information denoted by $a_1 = a_2$

2) If $H(a_1) < H(a_2)$, then a_1 is smaller than a_2, denoted by $a_1 < a_2$;

3) If $H(a_1) < H(a_2)$, then a_1 is greater than a_2, denoted by $a_1 > a_2$;

Liu and Wang[5] presented an evaluation function using Intuitionistic Fuzzy (IF) point operator to reduce the degree of uncertainty of the element in a universe for MADM based on intuitionistic fuzzy sets. Some score function were also introduced based on IF point operator. Yager and Xu[10,16] has proposed some geometric mean operators for aggregating intuitionistic information to reduce the loss of decision information using minimum and maximum operations. Some of the intuitionistic fuzzy geometric mean operators are intuitionistic fuzzy weighted geometric aggregation operator, the intuitionistic fuzzy ordered weighted geometric aggregation operator.

Definition 5: Let $\tilde{\alpha}_j$ = (ɱ$_j$, ʋ$_j$) (j=1, 2…n) be a collection of intuitionistic fuzzy values, and let IFWG:

$$Q^n \to Q, if\ IFWG_w(\tilde{a}_1, \tilde{a}_2, \ldots \ldots, \tilde{a}_n)$$

$$= \prod_{j=1}^{n} \tilde{a}_j^{\,w_j} = \left[\prod_{j=1}^{n} [u_j^{\,w_j}, 1 - \prod_{j=1}^{n} (1-\upsilon_j] \right)^{w_j} \right]$$

Where $w = (w_1, w_2, \ldots w_n)^T$ be the weight vector of $\tilde{\alpha}_j$ (j = 1, 2, … n), and $w_j > 0, \sum_{j=1}^{n} w_j = 1,$ then IFWG is called the intuitionistic fuzzy weighted geometric (IFWG) operator.

Definition 6: Let $\tilde{\alpha}_j$ = (ɱ$_{i,\ \upsilon j}$) (j = 1, 2, …… .. n) be a collection of intuitionistic fuzzy values. An Intuitionistic Fuzzy Ordered Weighted Geometric (IFOWG) operator of dimension n is a mapping IFOWG: $Q^n \to Q$, that has an associated weight vector $w = (w_1, w_2, \ldots \ldots ..w_n)^T$ such that $w_j > 0$ and $\sum_{j=1}^{n} w_j = 1$, furthermore,

$$\mathrm{IFOWG_w}(\tilde{a}_1, \tilde{a}_2, \ldots \ldots \tilde{a}_n) = \prod_{j=1}^{n} \tilde{a}_{\sigma(j)}^{\,w_j}$$

$$= \left[\prod_{j=1}^{n} u_{\sigma(j)}^{\,w_j} 1 - \prod_{j=1}^{n} (1-\upsilon_{\sigma(j)})^{w_j} \right]$$

where (Л(1), Л(2), … … Л(*n*) is a permutation of (1,2,…n), such that $\tilde{\alpha}_{Л(j-1)} \geq \tilde{\alpha}_{Л(j)}$ *forallj = 2, … … n.* The extension of Induced Ordered Weighted Averaging (IOWG) operator is Induced Intuitionistic Fuzzy Ordered Weighted Geometric (I-IFOWG).

Definition 7: An induced Intuitionistic Fuzzy Ordered Weighted Geometric (I-IFOWG) operator is defined as follows:

$$1 - \mathrm{IFOWG_w}(<u_1, \tilde{a}_1>, <u_2, \tilde{a}_2>, \ldots \ldots, <u_n, \tilde{a}_n>)$$

$$\left[\prod_{j=1}^{n} \tilde{u}_j^{\,w_j}, 1 - \prod_{j=1}^{n} (1-\tilde{\upsilon}_j)^{w_j} \right]$$

Definition 8: (Intuitionistic fuzzy sets) According to Atanassov's[12,13] Intuitionistic Fuzzy Set (IFS) theory generalizes the Zadeh[9] fuzzy set theory and hence all fuzzy sets are IFS but the converse is not necessarily true. IFS theory is beneficial in handling exact and incomplete information and has been proved to be useful in various application areas of science and technology.

An IFS A in X is an object having the following form:

$$A = \{x, t_1A\ (x), f_1A\ (x) \dashv \ | x \in X\}$$

which is characterized by a membership function t_A and a non-membership function f_A, where

$$f_A\text{:X} \rightarrow [0,1], x \in \text{X} \rightarrow f_A(x)\ [0,1]$$

with the condition:
$t_A(x) + f_A(x) \leq 1$ for all $x \in X$ for each IFS A in X, if

$$\pi_A(x) = 1 - t_A(x) - f_A(x)$$

for all $x \in X$ then $\pi_A(x)$ is called the degree of indeterminacy of x to A. It is a hesitancy degree of x to A which is equal to $0 \leq \pi_A(x) \leq 1$ *for all* $x \in X$.

In literature there are many approaches to allocate the weight of the attributes to classify each case into their fuzzy objects. In some approaches equal weight age are given to all attributes whereas in other approaches weight are given only to a single attribute. Moreover most of the techniques assign weights through experts or normal distribution[17,18]. Different weights for user attributes proportional to their similarity level have not been discussed so far. Main approaches to solving multi attribute decision making problem are as shown in Table 1.

1.4 Role of Intuitionistic Fuzzy Genetic in Application Domain

In an E-learning system the knowledge of user can be accessed on the basis of score for a domain concept and

Table 1. Approaches to solve multi attribute decision making

Author	Criteria	Weights	Aggregation Rule	Solution
Kahne(1975) [3]	Stochastic	Intervals	Monte Carlo Simulation	Crisp
Saaty(1978) [3]	Pair wise Comparison	Crisp	Hierarchical aggregation	Crisp
Laarhiven and pedrycz(1983) [3]	Crisp	Crisp	Hierarchical aggregation	Fuzzy
Ballman and Zadeh(1970) [3]	Fuzzy	Crisp	Max- Min	Crisp
Yager(1977,1978,1981) [3]	Fuzzy	Crisp/ Fuzzy	Max-Min	Crisp
Baas and Kwakernaak(1977) [3]	Fuzzy ratings	Fuzzy	Weighted Average	Fuzzy
Dong, Shah and Wong (1985,1987) [3]	Fuzzy rating	Fuzzy	Extension Principle + α- cuts+ intervals	Fuzzy
Dubois and Prade(1980) [3]	Fuzzy	Fuzzy	Approximate Extension Principle	Fuzzy
Schmucker(1985) [3]	Fuzzy	Fuzzy	Extension Principle	Fuzzy
Tseng and Klein(1992) [3]	Linguistic Variables	Fuzzy	Weighted Average	Crisp
Yager(1998) [3]	Fuzzy	Crisp	OWA Operators	Crisp
Baldwin(1992) [3]	Fuzzy	Crisp	Evidential Logic rule	Crisp
Chen, tan(1994) [14], Hong and Choi(2000) [15]	Vague set Theory	Fuzzy	Max- Min, Max-Max, Max-Centre	Crisp
Xu (2007g) [16], Li (2005) [17], Wei (2010b) [19]	Interval ranges,	Partially known, IFS	Linear Programming model	IFS
Liu , Wang (2007) [5]	Intuitionistic Fuzzy	-	IF point operator	IFS
Juchi Hou(2010) [20]	IFS	Crisp, completely known	Grey Relational Analysis	Crisp
Lin, Yuan, Xia(2006)[21]	IFS	Fuzzy	Simplex Method	Fuzzy
Xu , Yager (2008) [22]	IFS	IFS	BUM function based method, normal distribution based method, Exponential distribution based method	IFS
Zeshui Xu(2010) [23]	IFS	Fuzzy	Choquet Integral	IFS
Kahraman et.al. (2013) [8]	real	real	Genetic	real

time spent on that concept. Domain concepts, which are decomposed from the knowledge, are connected by different kind of relationship that forms a complex network. In an E-learning system, these links are considered as prerequisite and related concepts to achieve the goal. These links store individual user's knowledge about domain concepts. At present, most of the web learning systems determines learners' knowledge through their crisp responses to the tests taken during learning process and interaction history with E-learning systems. Crisp responses are not always appropriate to judge their knowledge learning level. Due to uncertainty of user's mind few researchers have considered knowledge acquisition using fuzzy inference systems as well as neuro- fuzzy systems[24,25,26]. These systems consider only degree of memberships while ignoring the degree of non-membership of elements. Neural network and SOM techniques were also used by some researchers to classify the learner's knowledge[27]. The limitation of neural network and SOM is that it takes time to train the networks when the input data set is large. AEEC approach[11] applied the genetic approach to allocate the weight of the attributes and to classify the data with different K- value and distance metrics. The limitation of this approach is that the sums of the attribute weights are greater than one.

Atanassov's intuitionistic fuzzy approach handles inaccurate information about the learner in the assessment process using both, degree of membership and degree of non-membership of the elements. Due to imprecise nature of human being the data is stored in the form of intuitionistic fuzzy. Since most of the E-learning systems classify the knowledge level without considering the weight difference among the domain dependent data of the user and limitations of calculus based methods, this paper proposes a genetic algorithm to determine the weight of the domain dependent data in intuitionistic fuzzy environment.

2. Weight Tuning by Genetic Algorithm (GA) for Intuitionistic Fuzzy Data in E-Learning Domain

This section primarily describes the weight tuning for intuitionistic fuzzy data set using genetic algorithm and classifies the intuitionistic fuzzy dataset through genetic approach in E-learning environment.

2.1 User's Attribute in E-Learning Domain

Depending on the object model of AEEC, attributes in a learning domain are degree of study time (STG), degree of repetition number (SCG), and the performance in exams (PEG). The attributes related to prerequisite concepts are considered as the degree of study time (STR), and the knowledge level (Learning status (LPR)). The current knowledge of student (UNS) is determined using real values of STC, SCG, PEG, STR, LPR. This data is collected for users from their learning activities/ feedbacks/answers/navigation paths about the learning concepts and prerequisite concepts in model of AEEC. Reading texts, solving problems/ exercises/tests, navigations in the different pages in E-learning systems are other behavior of user to collect the knowledge level in user model. This collected data is evaluated by rule based systems. In AEEC, the knowledge of the user is classified depending on the real values of the attributes of the related concepts and prerequisite concepts corresponding to a goal.

An intuitionistic fuzzy synthetic data set is generated by using quasi random generator, written in python considering each dimension of user knowledge using AEEC approach (Table 2). Each domain has a data set having values between 0 and 1 for 94 observations. In the cases where membership value is greater than 0.7, it has been checked that non-membership value is less than 0.3. Rule based system is used to classify the learner's knowledge (Table 1). An Intuitionistic fuzzy weighted averaging operator is proposed to fuse with the genetic algorithm to tune the weight of the attributes in E-learning system.

2.2 Intuitionistic Fuzzy Weighted Averaging Operator

Fuzzy sets introduced by Zadeh handle uncertainty and vagueness. In his work he has shown meaningful applications in many areas like mathematical subjects, information theory, aggregation operators, and cluster analysis and so on. Motivated by classical operators like weighted averaging or ordered weighted averaging operators, Xu has introduced intuitionistic fuzzy aggregation operators for aggregating intuitionistic fuzzy information. Atannssov defined the operational law for intuitionistic fuzzy values, given in definition 2.2.1

Table 2. Membership and Non- membership value of user attribute in E- learning domain

S.No	STG(M)	STG(N)	SCG(M)	SCG(N)	STR(M)	STR(N)	LPR(M)	LPR(N)	PEG(M)	PEG(N)
1	0.14	0.63	0.38	0.4	0.59	0.18	0.11	0.66	0.32	0.45
2	0.22	0.55	0.48	0.3	0.47	0.3	0.78	0.07	0.81	0.04
3	0.1	0.73	0.33	0.5	0.02	0.81	0	0.83	0.25	0.58
4	0.18	0.8	0.31	0.7	0.81	0.17	0.7	0.28	0.4	0.58
5	0.2	0.56	0.25	0.5	0.7	0.06	0.25	0.51	0.03	0.73
6	0.1	0.87	0.27	0.7	0.35	0.62	0.45	0.52	0.05	0.92
7	0.13	0.86	0.28	0.7	0.18	0.81	0.75	0.24	0.32	0.67
8	0.11	0.85	0.29	0.7	0.2	0.76	0.05	0.91	0.66	0.3
9	0.16	0.65	0.25	0.6	0.01	0.8	0.1	0.71	0.07	0.74
10	0.05	0.73	0.05	0.7	0.55	0.23	0.6	0.18	0.14	0.64
11	0.66	0.09	0.9	0.1	0.76	0.2	0.87	0.09	0.74	0.22
12	0.54	0.31	0.82	0	0.71	0.14	0.29	0.56	0.77	0.08
13	0.61	0.36	0.78	0.2	0.69	0.28	0.92	0.05	0.58	0.39
14	0.89	0.04	0.68	0.3	0.49	0.44	0.65	0.28	0.9	0.03
15	0.91	0.06	0.58	0.4	0.26	0.71	0.89	0.08	0.88	0.09
16	0.71	0.13	0.46	0.4	0.95	0.04	0.78	0.21	0.86	0.13
17	0.66	0.12	0.36	0.4	0.56	0.22	0.4	0.38	0.83	0.05
18	0.83	0.01	0.44	0.4	0.49	0.35	0.91	0.02	0.66	0.18
19	0.99	0	0.49	0.5	0.07	0.92	0.7	0.29	0.69	0.3
20	0.88	0.03	0.335	0.6	0.19	0.72	0.55	0.36	0.78	0.13
21	0.77	0.15	0.29	0.6	0.74	0.18	0.82	0.1	0.68	0.24
22	0.6	0.29	0.31	0.6	0.31	0.58	0.87	0.02	0.58	0.31
23	0.9	0.06	0.26	0.7	0.19	0.77	0.58	0.38	0.79	0.17
24	0.85	0.1	0.05	0.9	0.91	0.04	0.8	0.15	0.68	0.27
25	0.78	0.16	0.21	0.7	0.68	0.26	0.65	0.29	0.75	0.19
26	0.4	0.44	0.61	0.2	0.71	0.13	0.88	0.05	0.67	0.26
27	0.42	0.31	0.7	0	0.72	0.01	0.3	0.43	0.8	0.07
28	0.49	0.49	0.9	0.1	0.52	0.46	0.9	0.08	0.47	0.51
29	0.495	0.21	0.82	0.1	0.67	0.04	0.01	0.7	0.93	0.02
30	0.46	0.39	0.78	0.1	0.38	0.47	0.24	0.61	0.89	0.03
31	0.58	0.22	0.4	0.4	0.32	0.48	0.22	0.58	0.24	0.56
32	0.58	0.22	0.4	0.4	0.32	0.48	0.22	0.58	0.24	0.56
33	0.56	0.43	0.27	0.7	0.11	0.88	0.59	0.4	0.22	0.77
34	0.9	0.09	0.31	0.7	0.24	0.75	0.3	0.69	0.97	0.02
35	0.54	0.27	0.25	0.6	0.07	0.74	0.09	0.72	0.11	0.7
36	0.75	0.17	0.22	0.7	0.24	0.68	0.96	0.04	0.62	0.38
37	0.65	0.25	0.09	0.8	0.16	0.74	0.49	0.41	0.31	0.59
38	0.4	0.54	0.59	0.3	0.77	0.17	0.99	0.01	0.24	0.76
39	0.36	0.62	0.52	0.5	0.07	0.91	0.1	0.88	0.15	0.83
40	0.47	0.33	0.47	0.3	0.25	0.55	0.96	0.04	0.61	0.39

S.No	STG(M)	STG(N)	SCG(M)	SCG(N)	STR(M)	STR(N)	LPR(M)	LPR(N)	PEG(M)	PEG(N)
41	0.37	0.39	0.07	0.7	0.1	0.66	0.41	0.35	0.3	0.46
42	0.28	0.57	0.55	0.3	0.38	0.47	0.9	0.1	0.22	0.78
43	0.29	0.46	0.66	0.1	0.35	0.4	0.28	0.47	0.31	0.44
44	0.3	0.6	0.32	0.6	0.43	0.47	0.87	0.03	0.83	0.07
45	0.28	0.54	0.06	0.8	0.7	0.12	0.27	0.55	0.32	0.5
46	0.28	0.61	0.08	0.8	0.52	0.37	0.25	0.64	0.08	0.81
47	0.22	0.5	0.86	0.1	0.83	0.16	0.89	0.1	0.65	0.34
48	0.08	0.75	0.56	0.3	0.7	0.13	0.14	0.69	0.1	0.73
49	0.22	0.65	0.8	0.1	0.44	0.43	0.78	0.09	0.88	0.03
50	0.18	0.72	0.7	0.2	0.41	0.49	0.67	0.23	0.33	0.57
51	0.12	0.79	0.56	0.4	0.13	0.78	0.48	0.43	0.32	0.59
52	0.45	0.42	0.65	0.2	0.19	0.68	0.99	0.01	0.55	0.45
53	0.44	0.29	0.55	0.2	0.11	0.62	0.26	0.47	0.83	0.09
54	0.49	0.37	0.34	0.5	0.88	0.08	0.75	0.21	0.71	0.25
55	0.44	0.27	0.33	0.4	0.59	0.12	0.53	0.18	0.85	0.09
56	0.41	0.39	0.49	0.3	0.34	0.46	0.21	0.59	0.92	0.03
57	0.38	0.37	0.36	0.4	0.46	0.29	0.49	0.26	0.78	0.17
58	0.4	0.54	0.33	0.6	0.12	0.82	0.3	0.64	0.9	0.04
59	0.43	0.42	0.45	0.4	0.27	0.58	0.27	0.58	0.89	0.09
60	0.445	0.38	0.39	0.4	0.02	0.8	0.24	0.58	0.88	0.03
61	0.495	0.36	0.276	0.6	0.58	0.27	0.77	0.08	0.83	0.02
62	0.49	0.5	0.245	0.7	0.38	0.61	0.14	0.85	0.86	0.13
63	0.48	0.42	0.3	0.6	0.15	0.75	0.65	0.25	0.77	0.13
64	0.46	0.5	0.2	0.8	0.76	0.2	0.95	0.01	0.65	0.31
65	0.48	0.45	0.12	0.8	0.28	0.65	0.7	0.23	0.71	0.22
66	0.299	0.68	0.7	0.3	0.95	0.03	0.22	0.76	0.66	0.32
67	0.312	0.53	0.8	0	0.67	0.17	0.92	0.06	0.5	0.48
68	0.325	0.53	0.9	0	0.52	0.34	0.49	0.37	0.76	0.1
69	0.325	0.63	0.61	0.3	0.46	0.49	0.32	0.63	0.81	0.14
70	0.315	0.64	0.69	0.3	0.28	0.68	0.8	0.16	0.7	0.26
71	0.329	0.65	0.55	0.4	0.02	0.96	0.4	0.58	0.79	0.19
72	0.32	0.55	0.28	0.6	0.72	0.15	0.89	0.09	0.58	0.4
73	0.299	0.54	0.295	0.5	0.8	0.03	0.37	0.46	0.84	0.07
74	0.299	0.69	0.32	0.7	0.31	0.68	0.33	0.66	0.87	0.12
75	0.258	0.47	0.25	0.5	0.295	0.43	0.33	0.4	0.77	0.07
76	0.323	0.4	0.32	0.4	0.89	0.11	0.32	0.68	0.8	0.2
77	0.325	0.51	0.25	0.6	0.38	0.45	0.31	0.52	0.79	0.04
78	0.28	0.5	0.16	0.6	0.69	0.09	0.33	0.45	0.78	0.11
79	0.31	0.55	0.1	0.8	0.41	0.45	0.42	0.44	0.75	0.11
80	0.18	0.53	0.51	0.2	0.58	0.13	0.33	0.38	0.82	0.01
81	0.24	0.75	0.75	0.2	0.32	0.67	0.18	0.81	0.86	0.13

S.No	STG(M)	STG(N)	SCG(M)	SCG(N)	STR(M)	STR(N)	LPR(M)	LPR(N)	PEG(M)	PEG(N)
82	0.18	0.73	0.34	0.6	0.71	0.2	0.71	0.2	0.9	0.01
83	0.13	0.87	0.39	0.6	0.85	0.15	0.38	0.62	0.77	0.23
84	0.2	0.58	0.49	0.3	0.6	0.18	0.2	0.58	0.78	0
85	0.2	0.58	0.45	0.3	0.28	0.5	0.31	0.47	0.78	0.17
86	0.18	0.7	0.32	0.6	0.04	0.84	0.19	0.69	0.82	0.06
87	0.15	0.75	0.275	0.6	0.8	0.1	0.21	0.69	0.81	0.09
88	0.08	0.85	0.325	0.6	0.62	0.31	0.94	0.01	0.56	0.39
89	0.09	0.69	0.3	0.5	0.68	0.1	0.18	0.6	0.85	0.04
90	0.12	0.63	0.12	0.6	0.75	0.1	0.35	0.4	0.8	0.1
91	0.1	0.67	0.1	0.7	0.7	0.07	0.15	0.62	0.9	0.1
92	0.18	0.76	0.18	0.8	0.55	0.39	0.3	0.64	0.81	0.13
93	0	0.88	0	0.9	0.5	0.38	0.2	0.68	0.85	0.03
94	0.08	0.83	0.08	0.8	0.1	0.81	0.24	0.67	0.9	0.01

2.2.1 Definition (Xu, 2007)

Let $\tilde{a} = [t_{\tilde{a}}, f_{\tilde{a}}]$ and $\tilde{b} = \left[t_{\tilde{b}}, f_{\tilde{b}}\right]$ be two intuitionistic fuzzy values; then

1) $A \cup B = \{< x, \max(t_{\tilde{a}}(x), t_{\tilde{b}}(x)), \min(f_{\tilde{a}}(x), f_{\tilde{b}}(x))\}$ (1)

2) $A \cap B = \{< x, \min(t_{\tilde{a}}(x), t_{\tilde{b}}(x)), \max[(f_{\tilde{a}}(x), f_{\tilde{b}}(x))\}]$; (2)

3) $\lambda\tilde{a} = [1 - (1 - t_{\tilde{a}})^{\lambda}, f_{\tilde{a}}^{\lambda}], \lambda > 0.$ (3)

4) $\tilde{a}^{\lambda} = [(t_{\tilde{a}})^{\lambda}, 1 - (1 - f_{\tilde{a}})^{\lambda}].$ (4)

Let $S_P(t_{\tilde{a}}, t_{\tilde{b}}) = \max(t_{\tilde{a}}, t_{\tilde{b}})$ and $T_{\downarrow}p\ ((t_{\tilde{a}}, t_{\tilde{b}}) = \min(t_{\tilde{a}}, t_{\tilde{b}})$ then the operational law (1) in Definition 2.2.1 can be rewritten as:

$\tilde{a} \cup b^{\sim} = [S_P((t_{\tilde{a}}, t_{\tilde{b}}), T_p(t_{\tilde{a}}, t_{\tilde{b}})]$, and

Operational law (2) can be rewritten as:

$\tilde{a} \cap \tilde{b} = [T_p((t_{\tilde{a}}, t_{\tilde{b}}), S_P(t_{\tilde{a}}, t_{\tilde{b}})]$

In this paper we propose a weighted Intuitionistic fuzzy aggregation operator by using t-norm and t-conorm.

2.2.2 Definition

Let $(a_j)^{\sim} = [t_{(a_j)^{\sim}}, f_{(a_j)^{\sim}}]$("j = 1,2 … n)" be a collection of intuitionistic fuzzy values and let intuitionistic fuzzy weighted aggregating (IFWA): $\Omega^n \to \Omega$, if

$$IFWA_w(\tilde{a}_1, \tilde{a}_2, \ldots\ldots\ldots \tilde{a}_n) = w_1\tilde{a}_1 \cap w_2\tilde{a}_2 \cap \ldots\ldots \cap w_n\tilde{a}_n$$

Then IFWA is called a weighted Intuitionistic fuzzy aggregation operator of dimension n, where $w = (w_1, w_2, \ldots\ldots\ldots\ldots w_n)^T$ is the weight vector of

$\tilde{a}_j(j = 1, 2, \ldots .\ n)$, with $w_{j\in[0,1]}$ and $\sum^{n} w_i = 1$

$\tilde{a}_j(j = 1, 2, \ldots .\ n)$, with $w_{j\in[0,1]}$ and $\sum_{j=1}^{n} w_j = 1$

2.2.3 Theorem

Intuitionistic Fuzzy Weighted Aggregating (IFWA) operator of dimension n is defined as:

Let $(a_j)^{\sim} = [t_{(a_j)^{\sim}}, f_{(a_j)^{\sim}}]$("j = 1,2 … n)" be a collection of Intuitionistic fuzzy values then their aggregated value by using the IFWA operator is also an Intuitionistic fuzzy value and defined as:

$$IFWA_w(\tilde{a}_1, \tilde{a}_2, \ldots\ldots \tilde{a}_n) = \left\{\min_{j=1}^{n}\left(t_{\tilde{a}_j}\right)^{w_j}, 1 - \min_{j=1}^{n}\left(1 - f_{\tilde{a}_j}\right)^{w_j},\right\} \quad (5)$$

where $w = (w_1, w_2, \ldots w_n)^T$ is the weight vector of $\tilde{a}_j(j = 1, 2, \ldots .\ n)$, with $w_{j\in[0,1]}$ and $\sum_{j=1}^{n} w_j = 1$

2.2.4 Proof 2.2.3

Equation 5 can be proved by using mathematical induction on n. let n = 2

Since,

$\tilde{a}_1^{w_1} - [t_{\tilde{a}_1}^{w_1}, 1 - (1 - f_{\tilde{a}_1})^{w_1}]$

$\tilde{a}_2^{w_2} - [t_{\tilde{a}_2}^{w_2}, 1 - (1 - f_{\tilde{a}_z}))^{w_z}]$

Then

$IFWA_w(\tilde{a}_1, \tilde{a}_2) = \tilde{a}_1^{w_1} \cap \tilde{a}_2^{w_2}$

$= [\min((t_{(\tilde{a}_1)})^{(w_1)}, (t_{(\tilde{a}_2)})^{(w_2)})\dashv, \max(1-(1-f_{(\tilde{a}_1)})^{(w_1)}\dashv, 1(1-f_{(\tilde{a}_2)})^{(w_2)})]$

$1 - \min((1-f_{\tilde{a}_1})^{w_1} (1-f_{\tilde{a}_2})^{w_2}))$

If Equation 5 holds for n = k, that is

$IFWA_w (\tilde{a}_1, \tilde{a}_2, \ldots \ldots \tilde{a}_k) = \{\min_{i=1}^{k} (t_i)^{w_i}, 1-\min_{i=1}^{k} (1-f_1)^{w_1}\}$,

$IFWA_w (\tilde{a}_1, \tilde{a}_2, \ldots \ldots \tilde{a}_{k+1},) =$

$$\left\{\min_{j=1}^{k}(t_{\tilde{a}_j})^{w_j}, 1-\min_{j=1}^{k}[\left(\left(1-f_{\tilde{a}_j}\right)^{w_j}\right], \left(1-f_{\tilde{a}_{k+1}}\right)^{w_{k+1}}\right)\right\}$$

$IFWA_w (\tilde{a}_1, \tilde{a}_2, \ldots \ldots \tilde{a}_{k+1},) =$

$$\left\{\min_{j=1}^{k+1}(t_{\tilde{a}_j})^{w_j}, 1-\min_{j=1}^{k+1}\left(1-f_{\tilde{a}_j}\right)^{w_j}\right\}$$

Then, when n= k+1, by the operational laws in definitions 2.2.1 we have

i.e. (5) hold for n=k+1

Therefore (5) holds for all n, which completes the proof of theorem.

2.3 Intuitionistic Fuzzy Genetic Weighted Averaging Algorithm (IFGWA)

A genetic algorithm requires the genetic representation of the solution domain and a fitness function to evaluate the solution domain. Next sections describe the processing of genetic algorithm in intuitionistic fuzzy domain.

2.3.1 Definition of Chromosome Strings

In this study a serial number from 1 to n is assigned to the user's. The whole individuals are represented by chromosomes. Chromosomes are represented by weight parameter for genetic algorithm.(Figure 1)

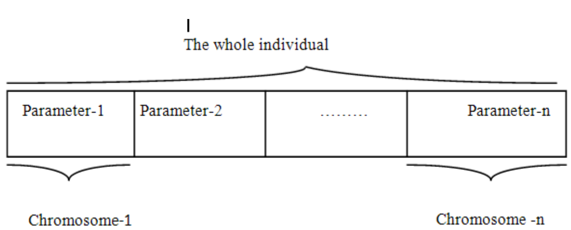

Figure 1. Individual string combined with the parameters of the student for the GA.

2.3.2 Initial Population Size

A large population size increases the probability of finding a high quality solution, though it reduces the search speed of the GA. To construct a high quality learning path for an individual learner, the initial population size of 200 is chosen for generation of weight tuning.

2.3.3 Selection of Fitness Function

The fitness function is a performance index that is applied to judge the quality of learning path generated using GA, as discussed earlier the parameters designed to generate a weight for user modeling of an individual learner are: degree of study time(STG) to goal object material, degree of repetition number(SCG), degree of study time(STR) to related object, knowledge level(LPR) of related object with goal object, performance in the exam(PEG) for goal object. While applying GA each of parameters are represented as a gene in the chromosome (Figure1). Fitness of a particular chromosome is computed using fitness function, shown in Equation 5.

2.3.4 Reproduction and Crossover Operation

In the reproduction operation the chromosome with the minimum fitness function value will have a higher probability to reproduce the next generation. In the crossover operation, the two minimum randomly selected values of the chromosomes in two individuals, exchange the entire chromosome by probability decision. This operation combines to two parent chromosomes to generate better child chromosome as shown in Figure 2. The crossover probability for the given approach is selected as 0.8%.

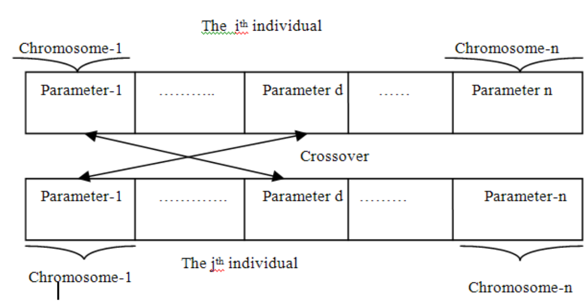

Figure 2. Crossover operation.

2.3.5 Mutation

Mutation is performed by swapping the parameter weights as shown in Figure 3. Mutation probability of 0.01 % worked well for the given approach.

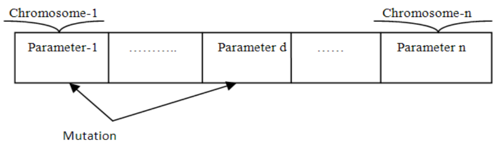

Figure 3. Mutation Operation.

2.3.6 Stopping Criteria

Figure 4 shows the architecture of intuitionistic fuzzy genetic algorithm for tuning the weight of student's attributes in E-learning system. The status of knowledge acquisition of student (Table 1) is based on rule base (Table 2).

After collecting data in an information table, an optimum weight array is constructed in the form of w_1, w_2, w_3, w_4, w_5 which refers to attribute of a student STG, SCG, STR, LPR, PEG respectively. An Intuitionistic Fuzzy Genetic Weighted Average(IFGWA) algorithm(Figure 5) is used to process the data and weigh the attributes. We finalize the searching process and save the attributes weight value of the most suitable individuals with respect to their similarity level.

Algorithm: Intuitionistic Fuzzy Genetic Weighted Averaging Algorithm (IFGWA)

Initialize the table entry T(s ∈ S, a ∈ A) à D

Here 's' is the student list and 'a' is the intuitionistic fuzzy value of the student, 'a' belong to the attribute list and is equal to 5, D is the decision based on rule base.

Let $a_1 = (t_i, f_i)$ (i = 1… n) be a collection of intuitionistic fuzzy numbers, then IFGWA fitness function (refer Equation 5

$$W(x) = (\min((t_i)^{wi}, 1 - \min(1 - f_i)^{wi}$$

where $w = (w_1, w_2 \ldots . . w_n)$ is the weight vector of $a_i = (1, 2, \ldots \ldots . n)$ and $w_i > 0$ *and* $\sum_{i=1}^{n} w_i = 1$

Step 1: For j = 0 to n (choose an initial random population of individuals)

Step 2: For i = 0 to m (evaluate the fitness (W(x)) of each individuals)

Repeat

Step 3: Select the minimum value of individual obtained by the fitness function (W(x))

Step 4: Generate new individual using random single point crossover.

Step 5: Apply mutations 0.01

Step 6: Evaluate the fitness of new individuals

Step 7: Select the most suitable individual

Step 8: Stop searching process until final value = D

Steps 9: Assign weight to attributes corresponding to the decision value.

Figure 5. Intuitionistic fuzzy genetic weighted averaging algorithm.

3. Experimental Results

In this section, the best weight values corresponding to the classification is discussed. Accuracy of tuning the weight corresponding to the classification of the algorithm over the validation or intuitionistic fuzzy data set is comapared based on mean errorfor different runs of generations' weight values.

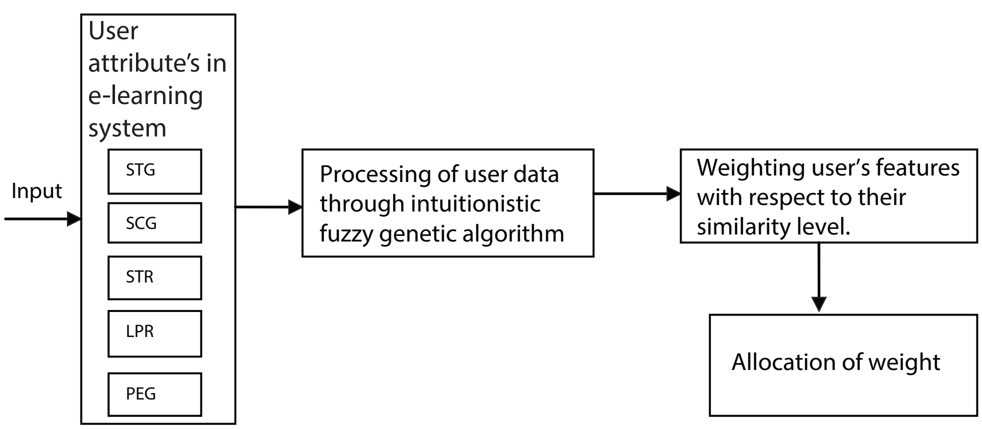

Figure 4. Architecture of intuitionistic fuzzy genetic algorithm for tuning the weight of student's attribute in e-learning system.

Table 3. Membership and Non Membership Values corresponding to each attribute for knowledge acquisition

Criteria	Very low	low	Middle	high
STG	(<0.25,>0.75)	(>=0.25and <0.33,>0.66)	(>=0.33 and <0.5,<0.5)	(>= 0.5,<0.5)
SCG	(<0.3,>0.7)	(>=0.3 and <0.5, >0.7)	(>=0.5 and <0.7,>0.3 and <0.5)	(>=0.7,<0.3)
STR	(<0.25,>0.75)	(>=0.25 and <0.33,>0.66)	(>=0.33 and <0.66,<0.33)	(>=0.66,<0.3)
LPR	(<0.25,>0.75)	(>=0.25 and <0.33,>0.66)	(>=0.33 and < 0.66,<0.33)	(>=0.66,<0.3)
PEG	(<0.25,>0.75)	(>=0.25 and <0.33,>0.66)	(>=0.33 and <0.5,>0.3 and <0.5)	(>=0.5,<0.5)

The procedure to identify the learner‘s learning feature is difficult in E-learninng system[2]. Moreover, it is difficult to identify the learners‘ attribute and to process these attributes in learner‘s model as it require lot of time. In this work the real data set of the object model of AEEC has been used to convert them into intuitionistic fuzzy data set. The membership value of each attribute is same as the value of the object model of AEEC where as the non- membership value is created randomly using python programming language. According to Atanassov, the intuitionistic fuzzy number is defined as $t_A(x) + f_A(x) \leq 1$ for all $x \in X$ i.e. It has to be taken care that sum of memebership value and non-membership value should be less than or equal to 1. To classfy the status of the student membership function for intuitionistic fuzzy (t_i ,f_i) following values were applied, where t_i represent membership value and f_i represent non membership value

- Very low = [0--0.4, 0.7--1]
- Low = [0.3--0.6, 0.5--0.7]
- Middle = [0.5—0.8,0.3—0.6]
- High = [0.7—1.0, 0.—0.3]

To assess the correctness of algorithm, we carried a series of experiments on synthetic intuitionistic fuzzy data set. This intuitionistic fuzzy data set has 94 instances i.e. users, with five attributes values mentioned in section 3.1, Table 2 shows the synthetic intuitionistic fuzzy data basefor the linguistic variable „*high*“. Corresponding to each attribute, the membership and non membership value for knowledge acquisition used is as shown in Table 3.

Intuitionistic fuzzy genetic weight tuning method search the optimum weight values of the domain dependent data of users, corresponding to their membership and non-membership fitness value. The experiment have been performed by the IFGWA algorithm for different runs of generations as shown in Table 4, Table 5, Table 6, Table 7, Table 8 and Table 9 respectively for 50, 100, 200, 300, 400 and 500 generations. These tables show the genes of the best individual. The genes present the best membership and non-membership weight values and fitness values of the best individuals for a specific generation for different runs.

After analyzing the weight values of Table 4, 5, 6, 7, 8, and 9, it is observed that the measure of error between membership value and non-membership value is very

Table 4. Genes of the best individuals for different run at generation 50

Run	Membership fitness value	Non-Membership fitness value	weight(M)	weight(N)	Error in weight
1	0.826026223	0.147972732	0.493934025	0.493173919	0.000760106
2	0.8463538	0.142123278	0.541024559	0.542470735	0.001446176
3	0.8463538	0.142123278	0.541024559	0.542470735	0.001446176
4	0.792952981	0.11069652	0.50481302	0.503339206	0.001473814
5	0.869066552	0.120990345	0.405419604	0.403132429	0.002287175
6	0.82884415	0.101971637	0.495294872	0.500175735	0.004880863
7	0.827596414	0.102746432	0.478370061	0.488963177	0.010593117
8	0.816860559	0.098857069	0.469631069	0.450213489	0.01941758
9	0.937049964	0.052596605	0.319673698	0.293435779	0.026237919
10	0.815369964	0.110789019	0.566501368	0.533649181	0.032852187

Table 5. Genes of the best individuals for different run at generation 100

Run	Membership fitness value	Non-Membership fitness value	weight(M)	weight(N)	Error in weight
1	0.695204872	0.099152836	0.386187075	0.382509721	0.003677354
2	0.803827209	0.134182142	0.373399647	0.379081129	0.005681482
3	0.802302875	0.119015031	0.479044406	0.485093905	0.006049499
4	0.785919295	0.129409688	0.690697311	0.683475127	0.007222185
5	0.781863063	0.131997359	0.386582432	0.378604622	0.00797781
6	0.948589796	0.047342999	0.156063613	0.145856774	0.010206838
7	0.866677906	0.105908107	0.369764273	0.382140637	0.012376364
8	0.892370886	0.085237139	0.226196683	0.210086917	0.016109766
9	0.859642596	0.105976387	0.469260765	0.45284134	0.016419425
10	0.829102796	0.08944918	0.459046547	0.480031406	0.020984859

Table 6. Genes of the best individuals for different run at generation 200

Run	Membership fitness value	Non-Membership fitness value	weight(M)	weight(N)	Error in weight
1	0.82542798	0.096227788	0.126708771	0.169239991	0.04253122
2	0.834880628	0.140327195	0.618934635	0.608677188	0.010257447
3	0.916453692	0.016708168	0.05253333	0.047239999	0.005293331
4	0.854668742	0.086380894	0.402645037	0.443032789	0.040387752
5	0.940017677	0.056715761	0.385993447	0.382835243	0.003158204
6	0.814587576	0.171757467	0.671119571	0.657887647	0.013231924
7	0.760626523	0.190291158	0.674648251	0.641877556	0.032770695
8	0.83314395	0.09968925	0.356859158	0.399431498	0.04257234
9	0.827016732	0.131234705	0.369717637	0.364533261	0.005184376
10	0.759525631	0.190093877	0.720019399	0.705245157	0.014774242

Table 7. Genes of the best individuals for different run at generation 300

Run	Membership fitness value	Non-Membership fitness value	weight(M)	weight(N)	Error in weight
1	0.800015735	0.158130703	0.579276176	0.567715097	0.011561079
2	0.841723088	0.094367636	0.484621498	0.417125457	0.067496041
3	0.915502147	0.042993492	0.077479299	0.073506803	0.003972496
4	0.904662249	0.09203934	0.410119944	0.418698862	0.008578918
5	0.894575093	0.105424907	0.45566361	0.440128801	0.015534809
6	0.796037274	0.192727336	0.592714337	0.51744936	0.075264977
7	0.853196904	0.11527669	0.413043538	0.411782556	0.001260981
8	0.779111731	0.133546531	0.417506468	0.40189779	0.015608678
9	0.858065647	0.081798166	0.29589912	0.205450176	0.090448944
10	0.912418595	0.071791894	0.216826507	0.267913422	0.051086915

Table 8. Genes of the best individuals for different run at generation 400

Run	Membership fitness value	Non-Membership fitness value	weight(M)	weight(N)	Error in weight
1	0.883987725	0.073856901	0.114303855	0.110692972	0.003610883
2	0.7797446	0.203563506	0.967538555	0.905049476	0.062489079
3	0.918069008	0.077519247	0.119832806	0.113112469	0.006720337
4	0.787714863	0.167151668	0.999797021	0.922433372	0.077363649
5	0.777195446	0.111390171	0.325827144	0.387173796	0.061346652
6	0.818992548	0.029953068	0.241283779	0.2183712	0.022912578
7	0.844126787	0.082122389	0.141706802	0.185464593	0.043757791
8	0.815692436	0.102002393	0.248295473	0.225758043	0.022537429
9	0.78189118	0.190604389	0.730003853	0.712769285	0.017234568
10	0.790643051	0.18282378	0.23937841	0.267409903	0.028031492

Table 9. Genes of the best individuals for different run at generation 500

Run	Membership fitness value	Non-Membership fitness value	weight(M)	weight(N)	Error in weight
1	0.702358226	0.285069976	0.648116577	0.665939476	0.0178229
2	0.908554965	0.04019017	0.289802002	0.201633684	0.088168318
3	0.924469428	0.038936563	0.070579328	0.085956304	0.015376976
4	0.712063767	0.279098518	0.370611422	0.398612112	0.02800069
5	0.732181384	0.241344722	0.340205344	0.336436452	0.003768892
6	0.969659075	0.028664007	0.06465045	0.041751296	0.022899153
7	0.760549567	0.112752349	0.240219201	0.200106608	0.040112592
8	0.78171917	0.111568253	0.207665839	0.295390959	0.087725121
9	0.767586935	0.140619928	0.643987877	0.632903473	0.011084404
10	0.817151049	0.142454502	0.324536521	0.300848344	0.023688178

Table 10. Mean Error for various generations

	No of Generations					
Errors	**50**	**100**	**200**	**300**	**400**	**500**
Mean	0.01013951	0.010671	0.021016	0.034081	0.034600446	0.033864722
Median	0.00358402	0.009092	0.014003	0.015572	0.025472035	0.023293665
stdev	0.01185487	0.005641	0.016577	0.033509	0.025320578	0.030130992
Best	0.00076011	0.003677	0.003158	0.001261	0.003610883	0.003768892
Worst	0.03285219	0.020985	0.042572	0.090449	0.077363649	0.088168318

small in each run for different generations. Smaller the values of these measures, proves the performance of the algorithm. Table 10 shows the mean error for various generations.

When the mean error is analyzed for runs of different generations, it is seen that the algorithm has a consistent performance. The mean square error .002349 also proves the consistent performance of the algorithm. The best and worst error values are also shown in the Table 10, which remains almost constant after 50 generations. Hence, the proposed operator tuned with the genetic algorithm, helps to assign the weight more efficiently to the attributes in intuitionistic fuzzy data set for E-learning domain. Thus it will reduce the computation time when problem size is large.

4. Conclusion

This paper presents an efficient and simple Intuitionistic Fuzzy Genetic Weighted Average (IFGWA) algorithm to assign weights corresponding to user's similarity level. The proposed method learns the attribute weight on intuitionistic fuzzy data. So far researchers have used expert weight or adhoc weight to assign the attribute weight corresponding to their similarity level. The proposed aggregation operator in combination with population based searching approach can play an efficient role in various decision making problems. In future this algorithm may further be enhanced to deal with sequencing of content based on hybrid approach like intuitionistic fuzzy ant colony optimization for an E-learning system.

5. References

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