Perfect Vertex (Edge) Domination in Fuzzy Graphs

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Abstract

The objective of this paper is to generalize and improve the domination variant called perfect domination. In this paper we have defined and derived some results related to the two new parameters perfect k-domination and perfect edge domination. The same can be applied in the networks arising from system of parallel computers.

Keywords: Fuzzy Graph, Fuzzy Dominating Set, Perfect K-Dominating Set, Perfect Edge Dominating Set, Perfect K-Edge Dominating Set

1. Introduction

Graph theory is one of the most flourishing branches of mathematics with applications to wide variety of subjects. In many real world problem we get only partial information about the problem, the vagueness in the description and uncertainty has led to the growth of fuzzy graph theory.

A mathematical frame work to describe uncertainty in real life situation was first suggested by L. A. Zadeh. In 1975 Rosenfeld introduced fuzzy graph theory. Fuzzy line graphs was discussed by Mordeson on his paper on fuzzy graphs and hyper graphs[1]. After the pioneering work of Rosenfeld several authors has been finding deeper results, and fuzzy analogs of several graph theoretic concepts. One such interesting graph theoretic concept is domination in graphs.

The study of dominating set in graphs was begun by Ore and Berge. The edge domination was introduced by Mitchell and Hedetniemi. Also in[2] studied about the edge domination in graphs. The concept of Perfect domination was introduced by CoCkayne et al., Perfect edge domination in graphs was studied in[3]. Perfect k-domination in graphs was studied in[4].

A remarkable beginning in fuzzy graphs for the concept domination was made in[5,6]. They obtained several bounds for domination number. Edge domination in fuzzy graphs was defined in[7]. A work on Fuzzy Multiple domination was done in[8]. In[9] authors studied about the concept of perfect domination in fuzzy graphs. They defined the perfect domination number of a fuzzy graph and obtained the relation between perfect domination number and independent domination number of a fuzzy graph.

In this paper we generalise the perfect domination number of a Fuzzy graph and introduce a new graph theoretic parameter known as perfect edge domination in fuzzy graphs and obtained some results related to these new parameter. Also we give an application of these graph theoretic parameters.

2. Preliminaries

In this section we briefly list out few basic definitions in fuzzy graphs which are presented in[5–9].

Definition 2.1[6,8]

Let V be a finite non empty set. Let E be the collection of all two element subsets of V. A fuzzy Graph G= (σ,μ) is a set with 2 functions σ : V -> [0,1] and μ:E->[0,1] such

that $\mu(x,y) \leq \sigma(x) \wedge \sigma(y)$ for all $x, y \in V$.

The order p and size q of a fuzzy graph G= (σ,μ) are defined to be $p = \sum_{x \in V} \sigma(x)$ and $q = \sum_{xy \in E} \mu(x,y)$

Definition 2.2[6]

Degree of an edge uv of G is defined by deg e = degu + degv

Definition 2.3[6]

Let G = (σ,μ) be a fuzzy graph on V. Let u,v $\in$ V, we say that u dominates v in G if $\mu(u, v) \sigma(x) \wedge \sigma(y)$. A subset S of V is called a dominating set in G if for every u $\in$ V-S, $\exists$ an u $\in$ S, such that u dominates v.

Definition 2.4[6]

A fuzzy dominating set S of a fuzzy graph G is called minimal fuzzy dominating set of G if for every node v$\in$S, S-{v} is not a dominating set

Definition 2.5[6]

The fuzzy domination number ? (G) is the minimum cardinality taken over all minimal fuzzy dominating set of G.

Definition 2.6[8]

Let G = (σ, μ) be a fuzzy graph. And let D be a subset of V. A vertex v $\in$ V-D is said to be fuzzy k-dominated if it is dominated by at least k vertices in D, (i.e.,) IN (v) $\cap$ D | $\geq$ k. In a fuzzy graph G every vertex in V-D is fuzzy k-dominated, then D is called a fuzzy k-dominating set.

Definition 2.7[8]

The minimum cardinality of a fuzzy k-dominating set is called the fuzzy k-domination number $\gamma_k (G)$

Definition 2.8[9]

A dominating set D of a fuzzy graph G is said to be a perfect dominating set if for each vertex v not in D, v is adjacent to exactly one vertex of D.

A perfect dominating set D of a fuzzy graph G is said to be a minimal perfect dominating set if for each vertex v in D, D-{v} is not a dominating set. A perfect dominating set with smallest cardinality is called a minimum perfect dominating set. It is denoted by γ_{pf} set of G. The cardinality of a minimum perfect dominating set is called the perfect domination number of the fuzzy graph G. It is denoted by $\gamma_{pf} (G)$.

Next in this section we define and discuss an example for perfect k domination number of a fuzzy graph.

3. Section

Definition 3.1

Let K be a positive integer. A vertex subset D of a fuzzy graph G = (μ, σ) is said to be a perfect k-vertex dominating set of G, if every vertex v of G not in D is adjacent to exactly k-vertices of D. The minimum cardinality of a perfect k-vertex dominating set of G is called the perfect k-domination number of the fuzzy graph G. It is denoted by $\gamma_{pf} (G)$.

Example
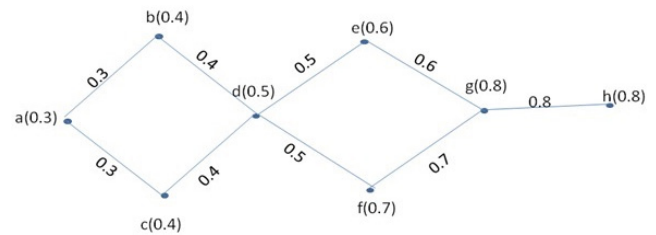
Figure 1. Fuzzy graph.

In the above example the perfect vertex dominating set is given by { b,c,g} and the perfect domination number is given by $\gamma_{pf} (G) = 1.6$

A perfect 2-dominating set is given by D = {a, d, g, h} and the perfect two domination number is given by $\gamma_{pf} (G) = 2.4$.

Observation 3.2

A perfect k-vertex dominating set of a fuzzy graph G is a k-dominating set, and hence $\gamma_k (G) \leq \gamma_{kpf} (G)$ for every graph G and positive integer k.

Proposition 3.3[8]

If G is a fuzzy graph with $\Delta (G) \geq k \geq 2$, then $\gamma_k (G) \leq > \gamma (G)+k-2$.

Theorem 3.4

If G is a fuzzy graph with $\Delta (G) \geq k \geq 2$, then $\gamma_{kpf} (G) \geq \gamma (G)+k-2$

Proof

Since a perfect k-dominating se is a k-dominating set we

have $\gamma_k(G) \le \gamma_{kpf}(G)$ (Observation 3.2)

But $\gamma_k(G) \ge \gamma(G)+k-2$ by (Proposition 3.3).

Hence we have $\gamma_{kpf}(G) \ge \gamma_k(G) \ge \gamma(G)+k-2$.

Proposition 3.5[8]

For a fuzzy graph G, $y_k(G) \ge \dfrac{kn}{(\Delta(G)+k)}$.

Theorem 3.6

For a fuzzy graph G $y_{kpf}(G) \ge \dfrac{kn}{(\Delta(G)+k)}$.

Proof

Since a perfect k-dominating se is a k-dominating set we have $\gamma_k(G) \le \gamma_{kpf}(G)$ (Observation 3.2)

But $y_k(G) \ge \dfrac{kn}{(\Delta(G)+k)}$ by (Proposition 3.3).

Hence we have $y_{kpf}(G) \ge y_k(G) \dfrac{kn}{(\Delta(G)+k)}$

4. Section

In this section we define the perfect edge domination number and perfect K-edge domination number of a fuzzy graph G and we discuss some results relating to these new parameters.

Definition 4.1

Let G = (σ,μ) be a fuzzy graph on V. A subset X of $V \times V$ is called an edge dominating set of G if every edge not in X is incident to some edge in X.

Definition 4.2

A subset X of $V \times V$ is said to be a minimal edge dominating set if no proper subset of X is an edge dominating set of G.

Definition 4.3

The minimum cardinality of an edge dominating set is called as the edge domination number of G and is denoted by $\gamma'(G)$.

Next we define perfect edge domination number of a fuzzy graph and discuss few results relating to this new parameter.

Definition 4.4

An edge dominating set X of a fuzzy graph G is said to be a perfect edge dominating set if every edge of E-X is

adjacent to exactly one edge in X.

The perfect edge dominating set X of a fuzzy graph G is said to be a minimal perfect edge dominating set if for each edge uv $\in$ X, X - {u,v} is not a perfect edge dominating set.

Definition 4.5

The cardinality of a minimum perfect edge dominating set is called as perfect edge domination number and is denoted by $\gamma'_{pf}(G)$.

Definition 4.6

An edge dominating set X of a fuzzy graph G is said to be a perfect k-edge dominating set if every edge of E-X is adjacent to exactly k edges in X.

Definition 4.7

The cardinality of a minimum perfect k-edge dominating set is called as perfect k-edge domination number and is denoted by $\gamma'_{pf}(G)$.

5. Result

For any fuzzy graph G, if the perfect edge dominating set exists then we have $y'_{kpf}(G) \ge y'_{pf}(G) \ge \gamma'(G)$.

Example 1
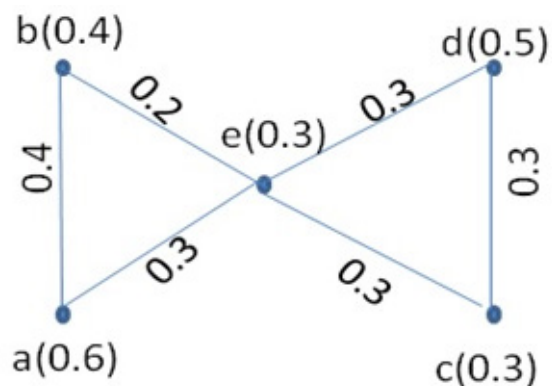

Figure 2.

In the above graph the edge dominating set is given by {be, cs}, $\gamma'(G) = 0.5$, but it is not a perfect edge dominating set.

The perfect edge dominating set is given by {ab, cd}, $\gamma'_{pf}(G) = 0.7$.

The perfect 2-edge dominating set is given by {ae,be,ed,ec}, $y'_{2_p}(G) = 1.1$

Example 2:

Figure 3.

In the above example the edge dominating set is {bd} which is also a perfect edge dominating set. Hence $\gamma'(G) = y'_{pf}(G)$

Theorem 5.1

A perfect edge dominating set S of a fuzzy graph G is a minimal perfect edge dominating set if for each edge uv ∈ S, one of the following conditions holds

- N(uv) ∩ S = Φ.
- There is an edge xy ∈ E - S such that N (xy) ∩ S = {uv}.

Proof:

Let S be a minimal perfect edge dominating set and uv ∈ S. Then clearly X' = S - {uv} is not a perfect edge dominating set and hence there exists an edge xy ∈ E - S such that xy is not adjacent to any edge of X'.

If uv = xy then we get (i) and
if uv ≠ x y then we get (ii).
The converse is obvious.

Theorem 5.2

Let X_1 and X_2 be two disjoint perfect edge dominating sets of a fuzzy graph G. Then we have $|X_1| = |X_2|$

proof

For every edge uv in X_1 there is a unique edge $u'v'$ in X_2 which is adjacent to uv. Also for ∧ every edge xy in X_2 there is an unique edge $x'y'$ in X_1 which is adjacent to xy.

corollary

If in a fuzzy graph G, there are perfect edge dominating sets X_1 and X_2 such that $|X_1| = |X_2|$, then $|X_1| \cap |X_2| = \Phi$.

Theorem 5.3

Every complete fuzzy graph G has a perfect 4-edge dominating set and the perfect 4-edge domination number is given by
$$y'_{4f}(G) = [(n-1)\wedge_{(\forall v \in G)}[\{\sigma(v_i)\} + (n-)\wedge_{(\forall_v j \in G - v_i)}\{\sigma(v_j)\}]\, 1 \le i \le j \le n, i \ne j.$$
where n is the number of vertices in G.

Proof

In a fuzzy graph with n vertices, each vertex is adjacent to remaining (n-1) vertices. If we consider an edge uv incident to 2 vertices, then the edge uv will be adjacent to exactly 2(n-2) edges. Let the perfect 4-edge dominating set S consist of the 2(n-2)+1 edges. The remaining edges are given by {(n-2)(n-3)}/2. Here we have the remaining (n-2) vertices will be adjacent to exactly 2 edges in [2(n-2)+1] edges. Since each edge is incident to two vertices, the remaining {(n-2)(n-3)}/2 edges will be exactly adjacent to 4 edges in E.

Since G is a complete fuzzy graph every edge of G is an effective edge.

Also γ (G) = ∧{σ (v): for all v ∈ V}.

Hence $y'_{4f}(G) = [(n-1)\wedge_{(\forall v \in G)}[\{\sigma(v_i)\} + (n-)\wedge_{(\forall_v j \in G - v_i)}\{\sigma(v_j)\}]$, $1 \le i \le j \le n, i \ne j$.

Example

Figure 4. Complete fuzzy graph.

Consider the complete fuzzy graph k_6 given below. Here n = 6, each vertex is adjacent to n-1 (i.e.,) 5 edges. Let S be

the set {ed, ef, ea, eb, ec, dc, db, da, df} which consists of 2(n-1)+1 edges. Here S represents a perfect -4 dominating set, and the perfect -4 domination number is given by y'_{4f} (G) = 5 (0.1) + 4(0.2) = 0.5 + 0.8 = 1.3.

If we consider any m X n matrix A where all the entries a_{ij}'s lies between (0,1) and select a minimum set S of values in A in such a way that no two elements of S are in the same row or same column. If we construct a bipartite fuzzy graph G by corresponding to every row a vertex in A and every column a vertex in B and by connecting the vertices, then the set S represents a perfect edge dominating set of the fuzzy bipartite graph G, and its cardinality gives the perfect edge domination number of G.

6. Conclusion

In this paper we defined two new domination parameters namely perfect k-domination in fuzzy graphs and perfect edge domination in fuzzy graphs and we proved results relating to these domination parameters. Perfect vertex domination and perfect k-domination in fuzzy graphs can be applied to the graphs arising from the interconnection networks of parallel computers. A system of parallel computers with processors and interconnection network can be modelled by the fuzzy graph G = (σ, μ), where the vertex membership σ represents the speed of the processors and μ : V X V-> [0,1], for all u,v ∈ V we have μ (u, v) ≤ σ (u) ∧ σ (v) where each processor is associated with a vertex of G and the direct communication link between two processor is indicated by the existence of an edge between the associated vertices. Also say if we have a limited resources such as disk, I/O units or software modules and we want to place a minimum number of these units at each processor with at most one per processor so that every processor is adjacent to at least one resource unit then we get a perfect k-dominating set in fuzzy graph.

7. References

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