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Stability Analysis of SIDR Model for Worm Propagation in Wireless Sensor Network

Arun Pratap Srivastava[1*], Shashank Awasthi[1], Rudra Pratap Ojha[2], Pramod Kumar Srivastava[2] and Saurabh Katiyar[2]

[1]G.L. Bajaj Institute of Technology and Management, Greater Noida - 201306, Uttar Pradesh, India; arun019@yahoo.com, shashankglbitm@gmail.com
[2]Galgotias College of Engineering and Technology, Greater Noida, Uttar Pradesh – 201306, India; rpojha@gmail.com, pramodpooja59@gmail.com, saurabh0277@gmail.com

Abstract

Background/Objectives: Security is essential concerns in wireless sensor network. To find the stability points when worms appear in the wireless sensor network. **Methods/Statistical Analysis**: By using ODE formulate the SIDR model by introducing the concept of dead nodes for wireless sensor network. Find the existence of positive equilibrium and perform the stability test with the help of Jacobian matrix. Some theorems are proposed for the analysis of model. **Findings:** The model explains that the inactive nodes are the nodes which die due to battery consumption and cannot be recharged because of remotely located in harsh region. Inactive nodes are not capable to transmit data from one sensor node to another sensor node. The model describes the nonlinear dynamics of Susceptible, Infectious, Dead and Recovered class of nodes. The entire dynamics of the transmission of worms can be analyzed by this mathematical model, propagating feat by worms in WSN can be determine with the help of threshold value of Ro. This model validates through extensive results by using MATLAB. **Application/Improvements:** Proposed model is useful to reduce the battery overhead, enhance the lifetime of wireless sensor network.

Keywords: Epidemic Model, Equilibrium, Stability, Wireless Sensor Network, Worms Propagation

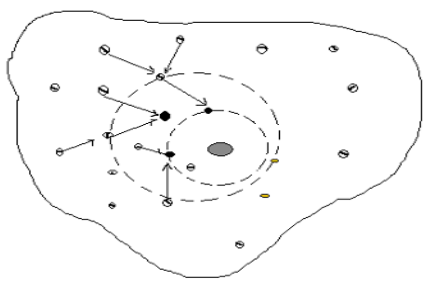
1. Introduction

WSN is a collection of small devices called sensor nodes. These nodes are deployed at remote places. The sensor nodes are able to transmit information from one place to another place via neighbor nodes. Sensor nodes are smart, self-organized, using radio communication and equipped with battery, microcontroller and sensors. WSN have so many advantages but also have some limitations like limited processing capacity, memory, power consumption and limited sensing capability[1] WSN have various applications like health care, military, defense, biodiversity, and disaster relief services etc[2]. In WSN, the information is transmitted from one senor node (source) to another node (sink). Sensor nodes are work in two modes, active mode and sleep mode. In sleep mode the node is not capable to send or receive the data. On the other hand, active node can transmit the data. Since WSN is widely use in mission critical applications. Therefore, security is vital for WSN. The attackers can attack on sensor node without physical contact in WSN. Such attack can create various vulnerabilities like buffer overflow, denial of services and worm attack[3]. Worms are self-propagating and can recruit the malicious code. Such code can further be resent to neighbor node repeatedly. Such kind of attacks is very dangerous for WSN. Due to this self-propagating nature of worm, there is a need to analyze the nonlinear dynamics of worm propagation.

In WSNs, one sensor node communicates to another sensor node through neighbor node. During communication, each sensor node consumes their individual energy provided through batteries. These batteries are not

*Author for correspondence

recharge because of locations. After some time, some sensor nodes are become inactive due to exhaust of energy. As we know that dead nodes are not capable to propagate the data. The inactive nodes that carry the worms are not capable to propagate the worms also. Therefore, total number of active nodes changes with time in WSN. Whereas the active number of nodes are a key factor in worm propagation process modeling.



Active node
Inactive node

Figure 1. Topological structure of WSN.

Some researchers have proposed mathematical model of worm propagation in WSNs by considering the dead nodes[5,6]. This model does not consider the communication radius and node density in WSNs. However, communication radius and node density is a key factor of worm propagation in WSN by considering communication radius and node density of sensor nodes in WSN, we proposed a more accurate model to discuss the transmission of worms in WSNs. The proposed model describes Susceptible (S), Infectious (I), dead (D), and Recovered (R) class of nodes. The entire dynamic of worm propagation can be analyzed by this model. The model also explains how communication radius and node density is an important factor in worm propagation analysis. Our model describes communication radius, distributed density of nodes and energy consumption of nodes. We also analyzed the stability of our model by finding the equilibrium and basic reproduction number of the model.

2. Related Work

There are so many models have been proposed to explore the spreading and controlling dynamics in WSN. Through introduction of removed state of a host, a new model was proposed to overcome the deficiencies of SIS model, called SIR model[9]. In SIR model[9], a host in stage of one among the three state of susceptible, infectious and removed state.

Recently the research on worm propagation in WSN is based on the mathematical models. There are so many models have been proposed to control and analyze the dynamics of worm propagation in WSN. One of the classical model was introduced by Kephart & White in 1991 called SIS model for worm propagation in internet. After that SIR (Susceptible, Infected, and Recovered) was also proposed by Zou-et al in 2005. These models effectively described the worm propagation process on the internet with the help of differential equation. The SIS model used for worm propagation analysis on internet[9].

In SIR model[10] it was assumed that the host node be working forever but this concept fails in WSNs due to battery power limitation of sensor nodes. To reduce the limitations of SIR model, an improved model was proposed by Wang and Li in 2008, called iSIRS model. This model has four states susceptible state, infectious node state, recovered node state, and dead node state in WSNs. At any moment t, the number of nodes in S, I, R and D is referred as $S(t), I(t), R(t)$ and $D(t)$, respectively. However, the iSIRS model does not discuss about the stability of model and its dynamic behavior. Therefore, to reduce shortcomings of iSIRS model, a new model proposes to investigate the performance and controlling of the worm propagation in WSNs.

3. Model Formulation

Different subclass of sensor nodes at any time t, are Susceptible $S(t)$, Infectious $I(t)$, Dead $D(t)$ and Recovered $R(t)$ of total size $N(t)$ i.e.,

$$N(t) = S(t) + I(t) + R(t) + D(t) \tag{3.1}$$

for any time $t \geq 0$.

In Figure 2, we describe the dynamical transfer of sub class. The SIRD model is given by:

$$\left.\begin{aligned}
\dot{S} &= b - \frac{r^2}{R^2}\alpha\, kSI - \lambda S + \delta R, \\
\dot{I} &= \frac{r^2}{R^2}\alpha\, kSI - (\rho + \mu) I, \\
\dot{R} &= \rho I - (\delta + \eta) R \\
\dot{D} &= \lambda S + \mu I + \eta R
\end{aligned}\right\} \tag{3.2}$$

where, b is the constant recruitment to susceptible, α is coefficient of transmission of worms, ρ is the recovery rate, δ is the rate at which a recovered node become susceptible, λ, μ, η is the rate at which all node susceptible, infectious and recovered become dead node due to worm attack respectively and k is the mean number of worms per unit time in an infectious state.

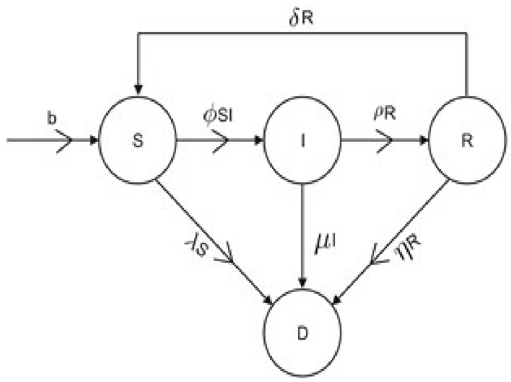


Figure 2. Transition diagram for the flow of worms in wireless sensor network.

For convenience, let $\phi = \frac{r^2}{R^2}\alpha k$,then the system of equation (3.2) can be written as:

$$\left.\begin{aligned} \dot{S} &= b - \phi SI - \lambda S + \delta R, \\ \dot{I} &= \phi SI - (\rho + \mu) I, \\ \dot{R} &= \rho I - (\delta + \eta) R \\ \dot{D} &= \lambda S + \mu I + \eta R \end{aligned}\right\} \qquad (3.3)$$

Clearly the above first four equation of (3.3) are independent of D so we will discuss the reduced system in the domain $\Gamma = \left\{(S, I, R,) \in \Re_+^3\right\}$ .It can be verified that Γ is positively invariant for all t greater than or equal to zero.

4. Local Stability and Existence of Positive Equilibrium

For equilibrium points, we have $\dot{S} = 0;\ \dot{I} = 0;\ \dot{R} = 0;$ and after a straight forward calculation ,we set equilibrium point as: $P_0 = (S_0, I_0, R_0) = (\frac{b}{\lambda}, 0, 0)$ for worm free state and $P^* = (S^*, I^*, R^*)$ for endemic state, with,

$$S^* = \frac{b}{\lambda R_0},\quad I^* = \frac{(\eta + \delta) b}{\{\mu(\eta + \delta) + \eta\rho\}}\left(\frac{R_0 - 1}{R_0}\right)$$, and

$$R^* = \left(\frac{R_0 - 1}{R_0}\right)\frac{\rho(\eta + \delta) b}{\{\mu(\eta + \delta) + \eta\rho\}}$$

where R_0 is the basic reproduction number[12] given by $R_0 = \frac{\phi b}{\lambda(\mu + \rho)}$. It is clear that P^* exist and unique if and only if R_0 greater than one.

5. Worm Free Equilibrium and its Stability

Theorem 1. If basic reproduction number R_0 is less than unity then the system (3.1) is locally asymptotically stable at worm free equilibrium P_0.

Proof. The Jacobian matrix at worm free equilibrium point P_0 is

$$J(P_0) = \begin{pmatrix} -\lambda & -\phi S_0 & \delta \\ 0 & \phi S_0 - (\rho + \mu) & 0 \\ 0 & \rho & -(\eta + \delta) \end{pmatrix} \qquad (5.1)$$

Eigenvalues of (5.1) are:

$$\omega_1 = -\lambda, \omega_2 = \phi S_0 - (\rho + \mu), \omega_3 = -(\delta + \eta).$$

It is clear that $\omega_1 < 0,\ \omega_3 < 0,$ and $\omega_2 < 0$ if

$\phi S - (\rho + \mu) < 0 \Rightarrow R_0 < 1$ .Therefore the system is locally asymptotically stable at worm free equilibrium point P_0, which proves the theorem.

Theorem 2. The system (3.1) is globally asymptotically stable if R_0 is less than or equal to one at worm free equilibrium P_0.

Proof. Consider the Lyapunov function $L(t): \Re^3 \to \Re^+$ defined by defined by $L(t) = \omega I$. Its

$$\dot{L} = \omega \dot{I} = \omega(\phi S_0 - (\rho + \mu)) \le \frac{\omega}{(\rho + \mu)}\left[\frac{\phi S_0}{(\rho + \mu)} - 1\right] \le (R_0 - 1) I$$

derivative w.r.t. time t, we get

If $R_0 \le 1$ then $\dot{L} \le 0$ holds. Furthermore $\dot{L} \le 0$ iff $I = 0$ .Therefore, the largest invariant set in

$\left\{(S,I,R)\in\Gamma:\dot{L}\leq 0\right\}$ is the singleton set $\{P_0\}$. Hence the global stability of P_0 follows from LaSalle's invariance principle[11], when $R_0\leq 1$.

6. Stability of the Endemic Equilibrium

Theorem 3. The system (3.1) locally asymptotically stable when its all eigen values are less than zero at endemic equilibrium P^*.

Proof. The Jacobian matrix associated with the endemic equilibrium is

$$J(P^*)=\begin{pmatrix} -(\phi I^*+\lambda) & -\phi S^* & \delta \\ \phi I^* & \phi S^*-(\rho+\mu) & 0 \\ 0 & \rho & -(\eta+\delta) \end{pmatrix} \tag{6.1}$$

Eigenvalues of (6.1) are the roots of the characteristic equation

$$v^3+b_1v^2+b_2v+b_3=0 \tag{6.2}$$

where, $b_1=1+\eta+\delta+\lambda$, $b_2=\lambda(\eta+\delta)$, $b_3=\rho\eta+\delta\mu+\eta\mu$. All the coefficient of the equation (6.2) are positive therefore according to Routh-Hurwitz criteria it follows that all the roots of equation (6.2) have negative real parts. Therefore, the endemic equilibrium point P^* is locally asymptotically stable. This completes the proof.

7. Simulation and Result

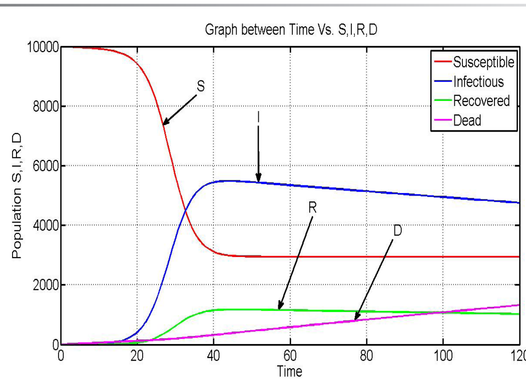


Figure 3. Dynamic demeanor of the system for different classes.

b=0.33; k =7; α =0.01; ρ =0.13; R = 600; λ =0.0006; μ =0.0008; r =15; η =0.006; δ =0.6

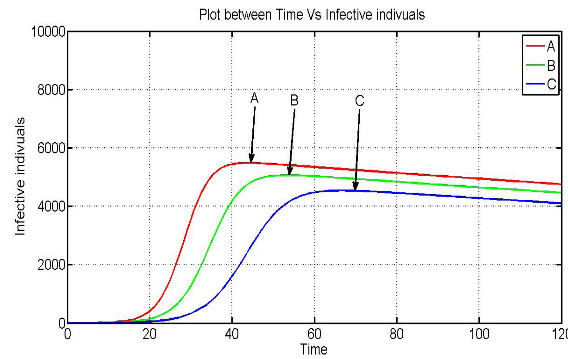


Figure 4. Dynamic demeanor of infectious class with time and variation in radius.

A) b=0.33; k =7; α =0.01; ρ =0.13; R = 600; λ =0.0006; μ =0.0008; r =15; η =0.006; δ =0.6;
B) b=0.33; k =7; α =0.01; ρ =0.13; R = 600; λ =0.0006; μ =0.0008; r =14; η =0.006; δ =0.6;
C) b=0.33; k =7; α =0.01; ρ =0.13; R = 600; λ =0.0006; μ =0.0008; r =13; η =0.006; δ =0.6

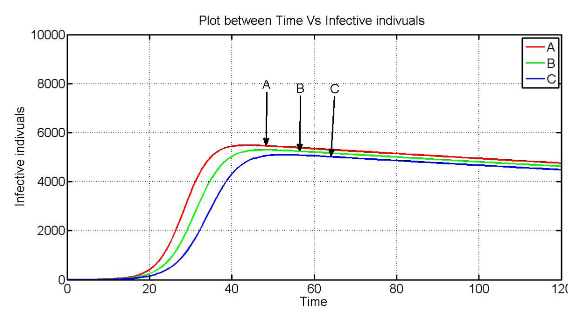


Figure 5. Dynamical demeanor of infectious class with time and variation in coverage area.

(A) b=0.33; k =7; α =0.01; ρ =0.13; R = 600; λ =0.0006; μ =0.0008; r =15; η =0.006; δ =0.6:
(B) b=0.33; k =7; α =0.01; ρ =0.13; R = 620; λ =0.0006; μ =0.0008; r =15; η =0.006; δ =0.6;
(C) b=0.33; k =7; α =0.01; ρ =0.13; R = 640; λ =0.0006; μ =0.0008; r =15; η =0.006; δ =0.6;

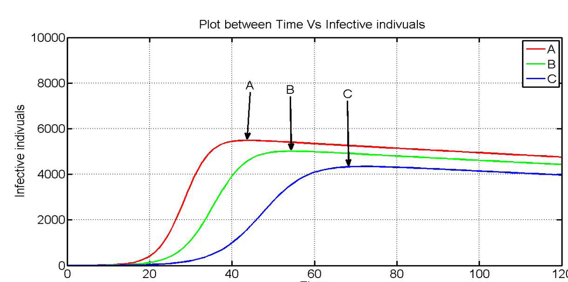


Figure 6. Dynamical demeanor of infectious class with respect to time and variation in k

(A)b=0.33; k =7; α =0.01; ρ =0.13; R = 600; λ =0.0006; μ =0.0008; r =15; η =0.006; δ =0.6;
(B) b=0.33; k =6; α =0.01; ρ =0.13; R = 600; λ =0.0006; μ =0.0008; r =15; η =0.006; δ =0.6;
(C) b=0.33; k =5; α =0.01; ρ =0.13; R = 600; λ =0.0006; μ =0.0008; r =15; η =0.006; δ =0.6;

8. Conclusion

In modern research, mathematical modeling is an important tool for analyzing and controlling the worm propagation in wireless sensor network. In this model, different behavior of network has studied. We derive an expression for basic reproduction number R_0 .Analytical result shows that if $R_0 \leq 1$ the worm free equilibrium P_0 is locally and globally asymptotically stable in worms can be eliminated from the wireless sensor network, when $R_0 \geq 1$ then worms will persist in system. For this to calculate the threshold value for communication radius on the basis we can fix the node to consume less energy as well as control the worm spreading. Here also studied that if coverage area of node is large infection rate is also large this is shown with the help of simulation. Node density effect is also studied and shown by simulation. This model also studied the energy consumption with variation of radius and node density. If coverage radius is large energy consumption will be large.

9. Acknowledgement

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