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Marangoni Effects on Forced Convection of Power Law Fluids in Thin Film over an Unsteady Horizontal Stretching Surface with Heat Source

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Abstract

Objectives: In this paper, the effect of surface tension gradient on the transport phenomenon and temperature differences within a power-law liquid film on an unsteady horizontal stretching sheet with heat source is been investigated. **Method:** The flow of a fluid film and the heat transfer from the subsequent stretching surface is been found by applying similarity transformation. The numerical computation of the problem is done with aid of Maple. The system of governing non-linear partial differential equation is converted to a set of nonlinear ordinary differential equation using local similarity transformations, then solved numerically applying the Runge Kutta Felhberg-45 method. The effect of power law index n, Prandtl number Pr, the unsteadiness parameter S, Space dependent heat source parameter α_1, temperature dependent heat source parameter β_1, and film thickness parameter $\beta(=\eta)$ on heat transfer and flow are studied and graphically presented. And it was found that the surface tension gradient and nonuniform heating source have tremendous impact on controlling the rate of convective heat transfer near the boundary layer region.

Keywords: Maple, Power Law Fluids, Range Kutta Felhberg-45,Stretching Sheet, Thin Film

1. Introduction

The study of heat transfers and flow structure due to a stretching boundary is gaining importance due to its many engineering and industrial applications, such as extrusion of plastic sheets, drawing of thin sheets, production of papers, glass blowing, metal spinning, etc. In this process, the final product with desired characteristics are obtained by pulling the sheet through viscous liquid with controlled cooling the pioneering works in this field are due to Sakiadis[1] and Crane[2], Crane studied a stretching flow past a then presented a closed form of solution to it. Since from then many investigators have formulated various aspects of this stretching sheet problem in Newtonian/ non-Newtonian boundary layer flow and good amount of references can be found in the paper by Gupta and Gupta[3],for a non-Newtonian fluid by Rajagopal and Gupta[4], for a viscoelastic fluid by Dandapatand Gupta[5], Chen and Char[6], Andersson et al.[6,7], Siddheshwar and Mahabaleshwar[8], Abel et al.[9–11] and Liao[12]. The effects of heat transfer are very important in view of several physical and engineering applications. Vajravelu and Rollins[13], Zheng and Lin[14] investigated the marangoni convection of power-law fluids driven by power-law temperature gradient, Vajravelu and Nayfen[15] studied the influence of uniform heat source/sink (temperature dependent) on the stretching surface and heat transfer. Abo-Eldahad and El-Aziz[16] considered the effect of non-uniform heat source/sink with suction/injection. Abel et al.[9–11]further continued the work of Abo-Eldahad and El-Aziz[16]to

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fluids which are viscoelastic and for a power law fluid. Motion induced within the fluid due to variation of surface tension resulting from a non-uniform temperature distribution is an interesting fluid mechanical problem. Such a flow arising from the variation of surface tension is called Marangoni convection or thermocapillarily flow. Dandapatet al.[17] studied the effect of thermocapillarily in a fluid film on the unsteady stretching sheet. This problem is further extended by Dandapat et al.[18] to study the effect of different fluid properties. Arash Karimipour[19] investigated the effect of indentations on the parameters of heat transfer and fluid flow of an a no fluid in a 2D micro channel and proposed that solid volume fraction and Reynolds number significantly affected the heat transfer rate. Chen and Char[20] proposed the influence of Marangoni convection on the heat transfer and flow pattern within the power-law fluid on an unsteady stretching sheet. No or and Hashim[21]studied the effect of magnetic field and thermocapillarily on a thin film in an unsteady stretching surface.

The aim of the present paper is to investigate the Marangoni effect on forced convection of power-law fluid in a thin film on an unsteady horizontal stretching surface with a heat source.

2. Mathematical Formulation

The fluid flow modelled as two dimensional, unsteady, incompressible viscous laminar flow on a thin horizontal elastic stretching sheet, emerging at the origin of a mutually perpendicular Cartesian coordinate system as shown in the Figure 1. The set of governing equations of conservation of mass, momentum and energy are given by

$$\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=0 \tag{1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{K}{\rho}\frac{\partial}{\partial y}\left(\left|\frac{\partial u}{\partial y}\right|^{n-1}\frac{\partial u}{\partial y}\right) \tag{2}$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\frac{q^m}{\rho C_p} \tag{3}$$

subjected to the boundary conditions

$$K\left|\frac{\partial u}{\partial v}\right|^{n-1}\frac{\partial u}{\partial v}=\frac{\partial\sigma}{\partial T}\frac{\partial T}{\partial x}\text{ at } y=l \tag{4a}$$

$$v=u\frac{\partial l}{\partial x}+\frac{\partial l}{\partial t}\text{ at } y=l \tag{4b}$$

$$\frac{\partial T}{\partial y}=0\text{ at } y=l \tag{4c}$$

$$u=u_s,\ v=0,\ T=T_s\text{ at } y=0 \tag{4d}$$

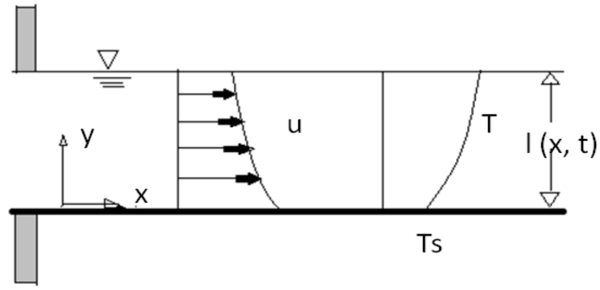


Figure 1. Representation of the physical configuration.

u and v represents velocity components in x and y directions respectively, ρ and t are temperature and density of the fluid, consistency coefficient is given by K ,where n is the power law index withC_p being the specific heat at constant pressure. q^m represents the non-uniform heat source given as

$$q^m=\frac{\rho k u_s(x)}{xK}\left[A'(T_s-T_0)f'+(T-T_0)B'\right] \tag{5}$$

WithA' and B' are coefficient of space and temperature dependent heat source respectively. The fluid is termed as shear thinning or pseudo plastic for 0<n<1, Newtonian fluid for n equal one with K= μ (absolute viscosity) and dilatants or shear thickening for n>1.

In equation (4a), the surface tension σ which is usually considered to vary linearly with temperature,

$$\sigma=\sigma_0[1-\gamma(T-T_0)] \tag{6}$$

and$\gamma=\frac{-1}{\sigma_0\left(\frac{\partial\sigma}{\partial T}\right)}$ is the temperature coefficient of surface tension, γ is a non-negative fluid property. Assuming the effect of interfacial shear by the surrounding air is considered to be negligible. However, surface-tension gradient $\frac{\partial\sigma}{\partial x}$ induced thermally along the interface is,

$$\frac{\partial\sigma}{\partial x}=\frac{\partial\sigma}{\partial T}\frac{\partial T}{\partial x} \tag{7}$$

The stretching sheet velocity and temperature are given by

$$u_s=\frac{bx}{(1-at)} \tag{8}$$

and $T_s=T_0-T_{ref}\left[\frac{b^{2-n}x^2}{2(K/\rho)}\right](1-at)^{-3/2}$ (9)

respectively. In the equations (8) and (9) a and b are non- negative constants, T_0 is the temperature at the slit and T_{ref} a constant reference temperature for all $t<\frac{1}{a}$ Using the standard definition of the stream function such as $u = \frac{\partial \psi}{\partial y}$ and $v = -\frac{\partial \psi}{\partial y}$, we define below the following similarity variables:

$$\xi = \left(\frac{K/\rho}{b^{2-n}}\right)^{(n-1)/2(n+1)} (1-at)^{3(n-1)/2(n+1)} (x)^{(1-n)/(n+1)}, \quad (10)$$

$$\eta = \left(\frac{b^{2-n}}{K/\rho}\right)^{1/(n+1)} (x)^{(1-n)/(n+1)} (1-at)^{(n-2)/(n+1)}\, y, \quad (11)$$

$$\psi = \left(\frac{K/\rho}{b^{2-n}}\right)^{1/(n+1)} (x)^{2n/(n+1)} (1-at)^{(1-2n)/(n+1)}\, f(\xi,\eta), \quad (12)$$

$$T = T_0 - T_{ref}\left[\frac{b^{2-n}x^2}{2(K/\rho)}\right](1-at)^{-3/2}\theta(\xi,\eta), \quad (13)$$

Using equations (10) – (13), the equations (2), (3) and boundary conditions (4) can be rewritten as

$$(|f''|^{n-1}f'')' + \frac{2n}{n+1}ff'' - (f')^2 - S\left(f' + \frac{2-n}{n+1}\eta f''\right) = \frac{1-n}{1+n}\xi\left[\left(\frac{3}{2}S + f'\right)\frac{\partial f'}{\partial \xi} - f''\frac{\partial f}{\partial \xi}\right] \quad (14)$$

$$\frac{1}{Pr}\xi^2\theta'' + \frac{2n}{n+1}f\theta' - 2f'\theta - S\left(\frac{3}{2}\theta + \frac{2-n}{n+1}\eta\theta'\right) + [\alpha_1 f' + \beta_1\theta] = \frac{1-n}{1+n}\xi\left[\left(\frac{3}{2}S + f'\right)\frac{\partial \theta}{\partial \xi} - \theta'\frac{\partial f}{\partial \xi}\right] \quad (15)$$

$$f(\xi,\beta) + \frac{1-n}{2n}\xi\frac{\partial f}{\partial \xi}(\xi,\beta) = M\xi\left[\theta(\xi,\beta) + \frac{1-n}{2(1+n)}\xi\frac{\partial \theta}{\partial \xi}(\xi,\beta)\right], \quad (16a)$$

$$f(\xi,\beta) + \frac{1-n}{2n}\xi\frac{\partial f}{\partial \xi}(\xi,\beta) = \left(\frac{2-n}{2n}\right)S\beta, \quad (16b)$$

$$\theta'(\xi,\beta) = 0, \quad (16c)$$

$$f'(\xi,0) = 1, \quad f(\xi,0) + \frac{1-n}{2n}\xi\frac{\partial f}{\partial \xi}(\xi,0) = 0, \quad \theta(\xi,0) = 1. \quad (16d)$$

where, $S = a/b$ the unsteadiness parameter, $Pr = Kb^{n-1}/\rho\alpha$ is the Prandtl number (modified), $\alpha_1 = \frac{kA^*}{KC_n}$ is the space-dependent heat source parameter and $\beta_1 = \frac{kB^*}{KC_n}$ is the temperature dependent heat source parameter, β the value of η in free surface, $M = \frac{\gamma\sigma_0 T_{ref}}{K(b^{3n-2}K/\rho)^{1/2}}$ is the Marangoni number and prime indicates the partial derivatives with respect to η. Note the dimensionless quantity β the film thick nessis unknown and must be determined to be a part of the boundary value. The actual thickness $l(x,t)$ of the film can be found from eqn.11 noting that $\eta = \beta$ as $y = l$, to be

$$l(x,t) = \beta\left(\frac{K/\rho}{b^{2-n}}\right)^{1/(n+1)} (x)^{(1-n)/(n+1)} (1-at)^{(n-2)/(n+1)}$$

It should be noted that for Newtonian fluid the actual film thickness depends on time only. The above system of non-linear partial differential equations (14) and (15) and subjected to the BC's (16a) to (16d), are converted to non-linear ODE's by local non-similarity methodPostelnicu[25] are then solved numerically by Range Kutta Felhberg-45 method.

3. Results and Discussion

We present in this paper, that the Marangoni effects on the heat transfer and flow within a power-law fluid over an unsteady stretching sheet by the influence of heat source is numerically investigated. The computation is done with aid of maple, asy mbolic computation software. The effect of unsteadiness parameter S, thepower-law index n, Prandtl number Pr, α_1 and β_1 space dependent and temperature dependent heat source parameter and film thickness parameter $\beta(=\eta)$ on flow and heat transfer are shown graphically in Figures 2–11.

Figure 2, is the plot of horizontal velocity profile $f'(\eta)$ versus η for varying values of power law index n. From

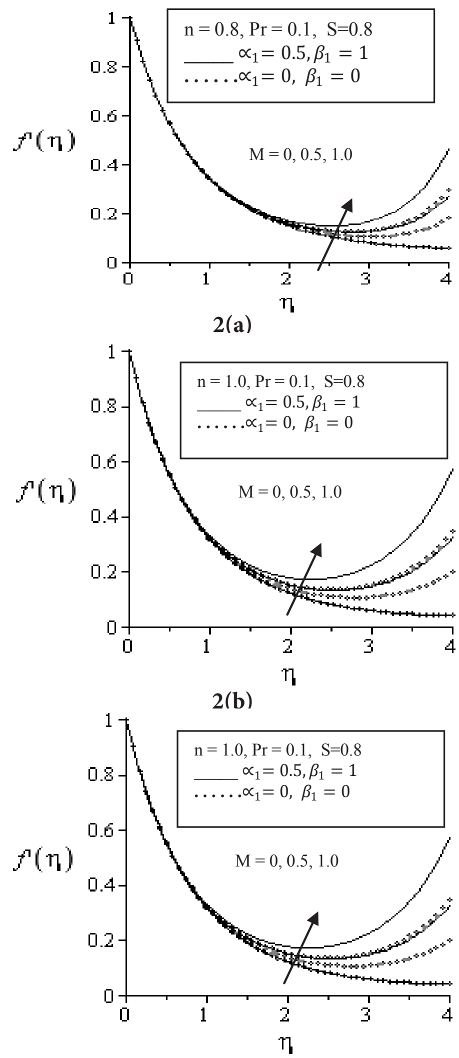


Figure 2. Plot of horizontal velocity f'(η) versus film thickness η for different values of Marangoni number M for power law index **a)** n=0.8, **b)** n=1, **c)** n=1.2.

the Figure 2 shows that the increase in n decreases the horizontal velocity. The in crease in the value of n implies the increase in the drag which decreases the velocity. From the Figure 2 we observe that when the Marangoni effects are not taken into account (M=0) the horizontal velocity decreases but in the presence of Marangoni effect ($M \neq 0$) the horizontal velocity decreases initially up to the value of n around 2.5 and then increases to its free surface velocity. It is also observed that surface velocity of shear thickening fluid < surface velocity of Newtonian fluid < surface velocity of shear thinning i.e., shear thinning fluids are more amenable to flow than that of shear thickening. Also observed that when space dependent heat source $\propto_1$, and space dependent temperature source β_1, increases, the energy is realized into the system which increases the horizontal velocity. The similar result has been observed by Chen[22]in the absence of heat source. The plot of surface temperature θ versus η Figure 3 for different values of Marangoni number M. It is observed from the figure that increases in M decreases for all values of η. This is due to the increase in film thickness because of the rmo capillary effects. It is also observed that with the increase in the values of power-law index, $\propto_1$ and β_1 increases, the temperature from T_{∞} to the actual temperature T. It is clear from the plot that increases in η decreases the θ. The increase in the value of η increases the boundary layer thickness and thus broadens the temperature distribution. Figures 4 and 5 are the plot of horizontal velocity and temperature respectively versus η for different values of Prandtl number. It can be observed that the increase in the values of Pr reduces the horizontal velocity profile and temperature. This observation is true in all the three kinds of fluids. It is evident from Figure 5 that the large values of Pr, results in decreasing the thickness of boundary layer. Figures 6andFigures 7shows the effect of unsteadiness parameters S with η for all three types of fluids. It is observed that increase in the value of S, horizontal velocity decreases initially and then increases with η after reaching the minimum value, where as in the case temperature decreases monotonically decreases for all values of η. Figure 8 is the plot of θ versus η for different values of space dependent heat source $\propto_1$ and temperature dependent heat source β_1 only positive values of $\propto_1$ and β_1 are considered in plotting the graph which corresponds to internal heat generation. From the plots it is clear that increase in the values of $\propto_1$ and β_1 increases the temperature θ. The increase in the value of $\propto_1$ and β_1 increases the energy released and leads to

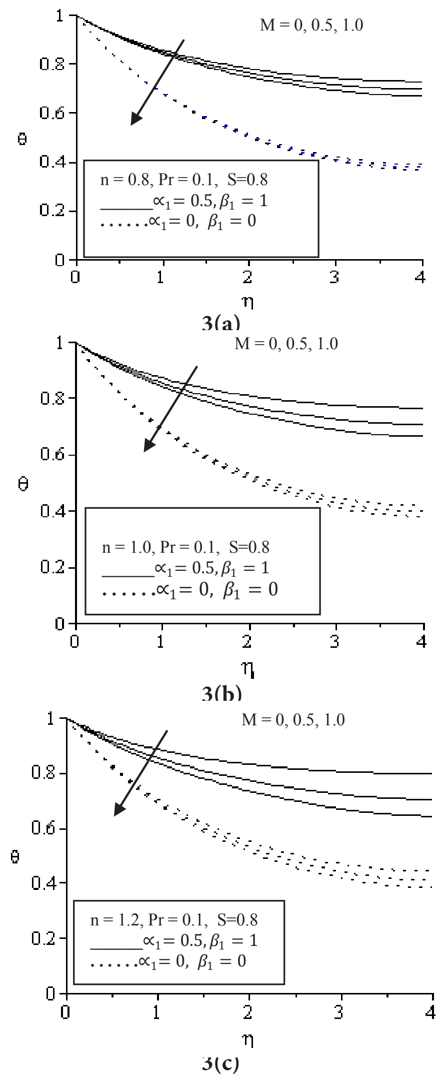
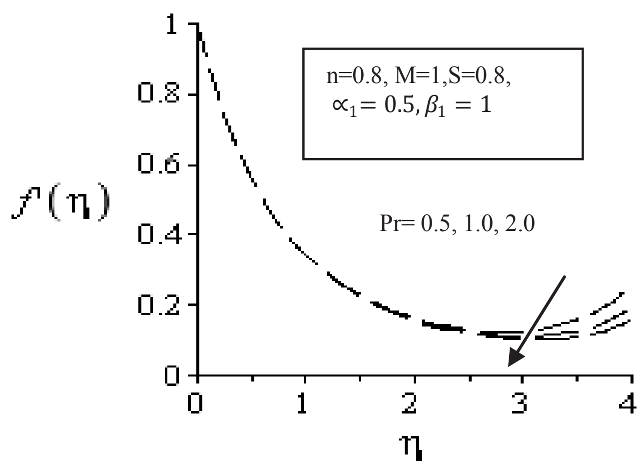
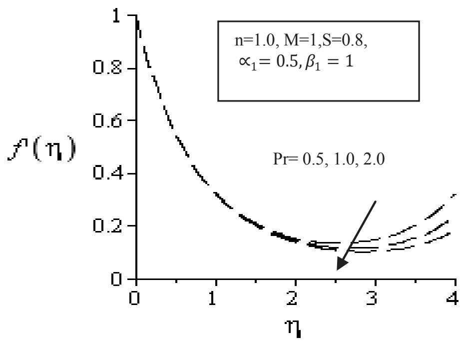


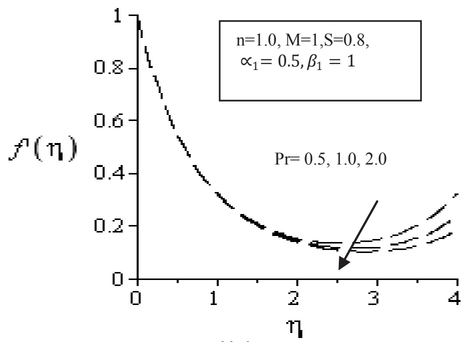
Figure 3. Plot of temperature θ versus η for different values of M for **a)** n=0.8, **b)** n=1, **c)** n=1.2



4(a)

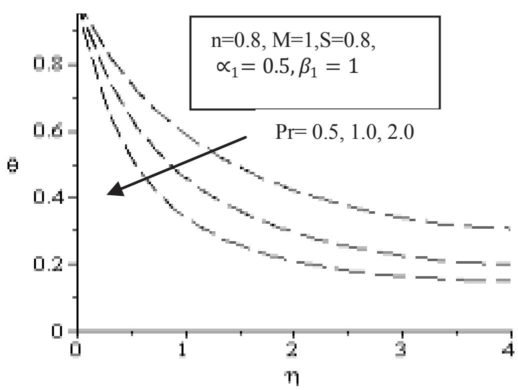


4(b)

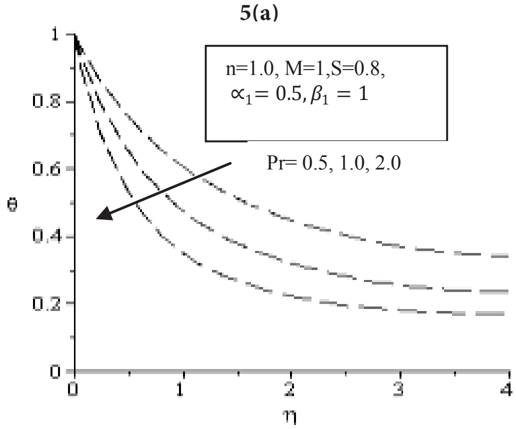


4(c)

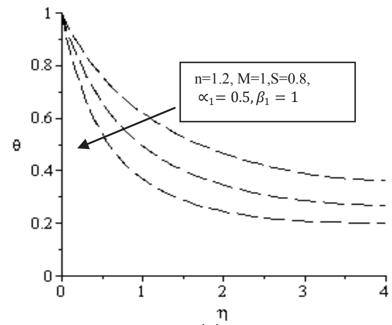
Figure 4. Plot of horizontal velocity f'(η) versus film thickness η for different values of Pr for **a)** n=0.8, **b)** n=1, **c)** n=1.2.



5(a)

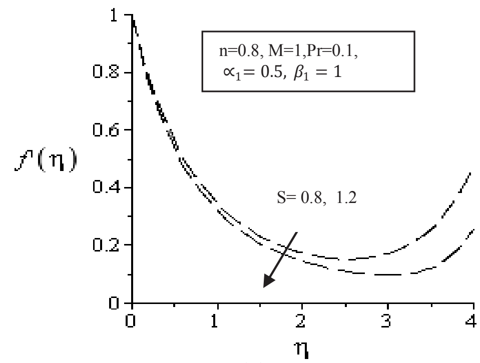


5(b)

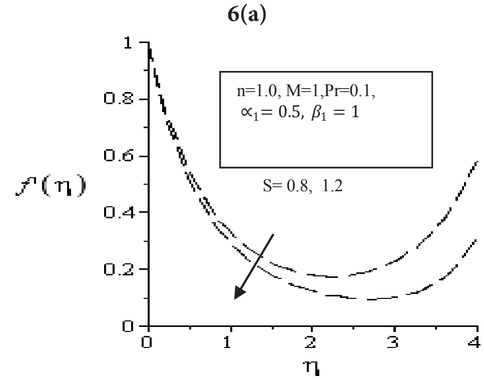


5(c)

Figure 5. Plot of temperature θ versus η for different values of Pr for **a)** n=0.8, **b)** n=1, **c)** n=1.2.



6(a)



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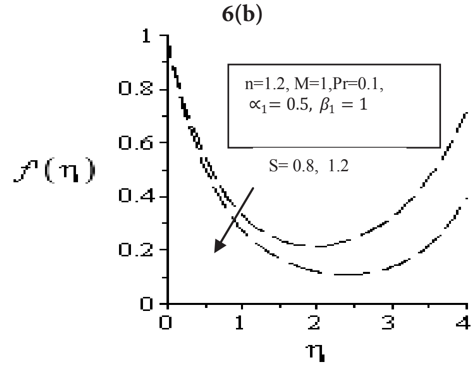
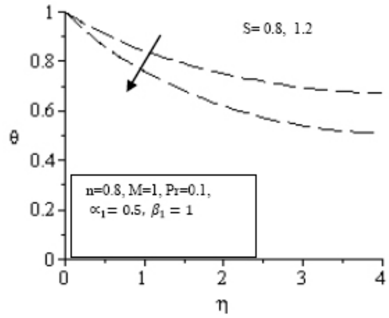
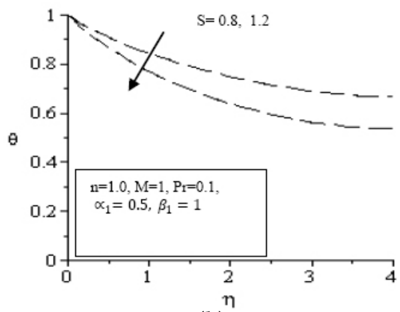


6(c)

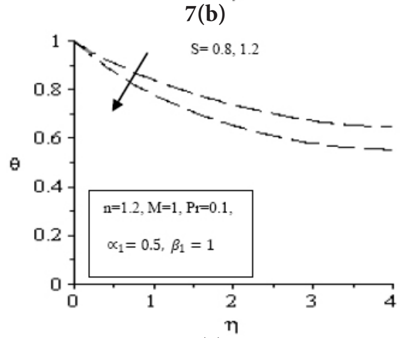
Figure 6. Plot of horizontal velocity f'(η) versus film thickness η for different values of S for **a)** n=0.8, **b)** n=1, **c)** n=1.2.



7(a)

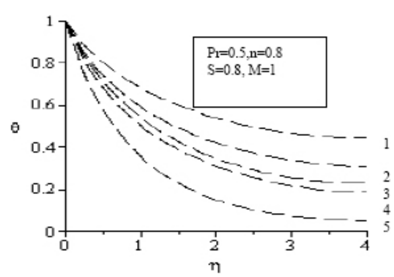


7(b)

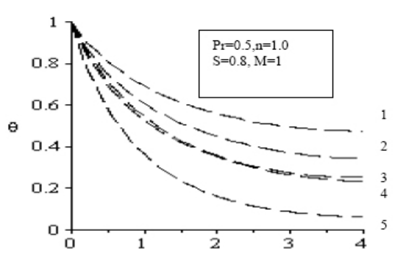


7(c)

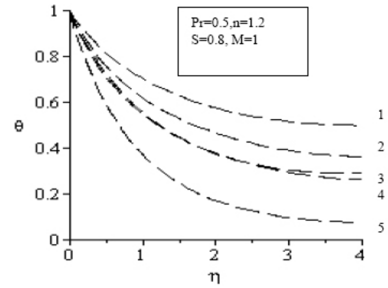
Figure 7. Plot of temperature θ versus η for different values of S for **a)** n=0.8, **b)** n=1, **c)** n=1.2.



8(a)

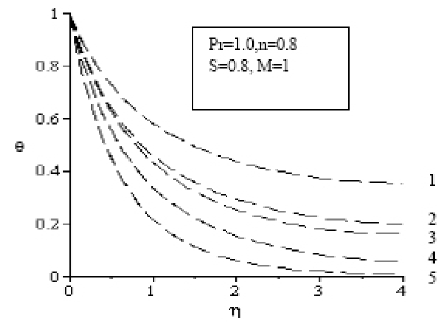


8(b)

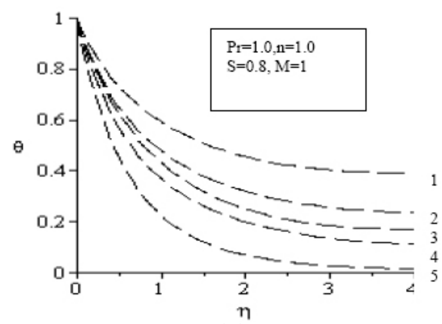


8(c)

Figure 8. Plot of temperature θ versus η for different values of n and different values of and for Pr=0.5 1) 3), 4) 5)



9(a)



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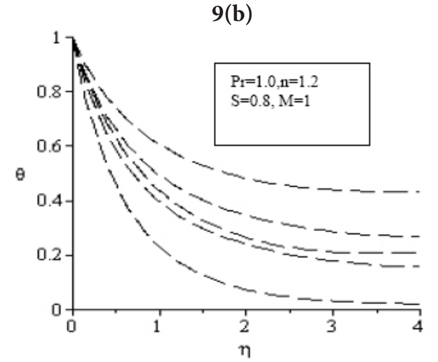
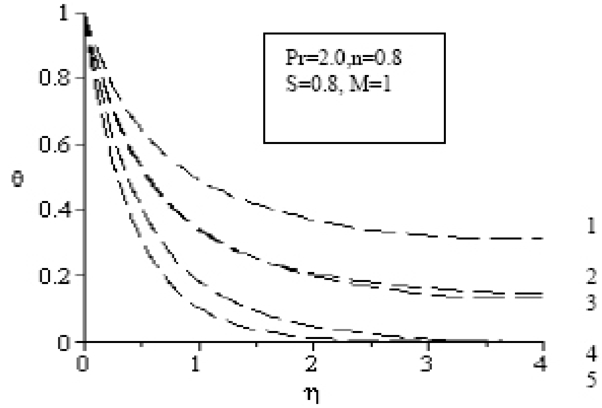


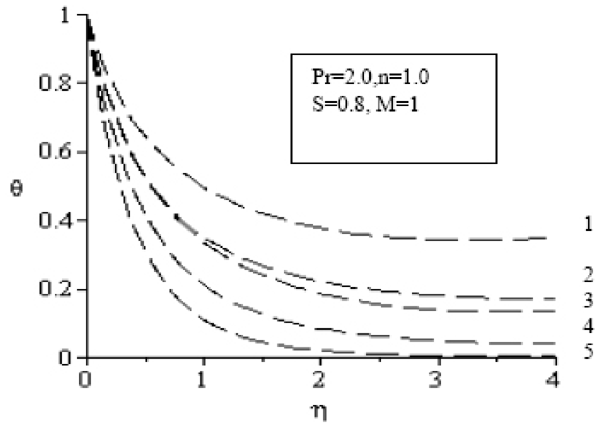
9(c)

Figure 9. Plot of temperature θ versus η for different values of n and different values of and for Pr=1.0 1) 3), 4) 5)

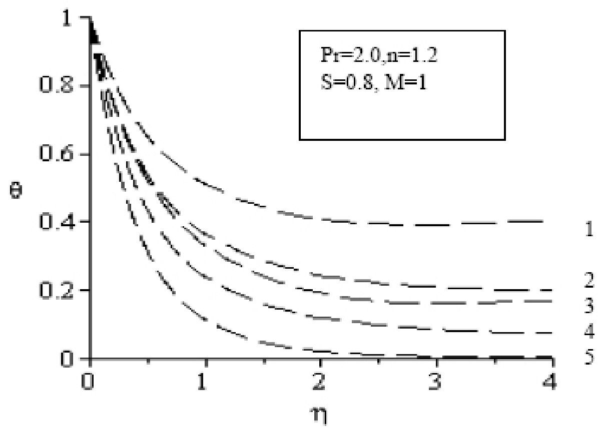
a larger thermal diffusion layer that may increase thermal boundary layer thickness which causes the increase in temperature. On comparing the Figures 8, 9 and 10 we observe that on increasing the Prandtl number Pr decreases the temperature θ, hence temperature distribution is found to be in a narrow region within the liquid film. Figure 11 shows the effect of film thickness β with unsteadiness parameter S for n < 1 and n > 1. It is observed that increase in Marangoni number increases β for all n less than one and n greater than one.



10(a)

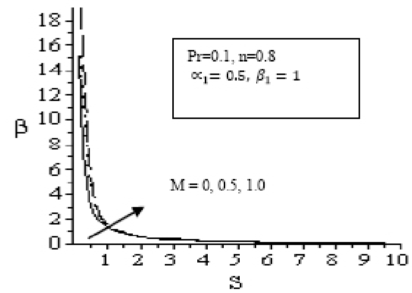


10(b)

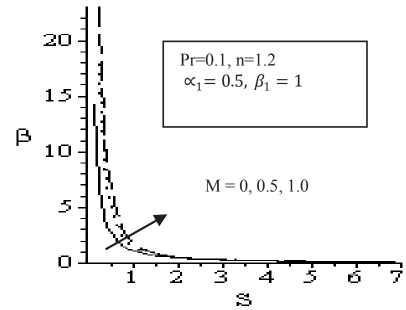


10(c)

Figure 10. Plot of temperature θ versus η for different values of n and different values of and for Pr=2.0 1) 3), 4) 5)



11(a)



11(b)

Figure 11. Plot of Film thickness β versus unsteadiness parameter S for **a)** n = 0.8, **b)** n=1.2 with Pr = 0.1 andξ =1.

4. Conclusions

These are some of the few conclusions drawn from the present study:

- The Marangoni number alters the horizontal velocity and temperature. The in crease in the value of M the flow temperature consistently cools down.
- The increase in the power law index n is to reduce the horizontal velocity and there by decreases the boundary layer thickness.
- (Boundary layer thick ness)$_{\text{shear thinning}}$ < (Boundary layer thick ness) $_{\text{New ton ian}}$ < (Boundary layer thick ness) $_{\text{shear thickening.}}$
- Increasing the value of Pr decreases the thermal boundary layer thickness.
- Increase in the space dependent and temperature dependent heat source is to increase the horizontal velocity and temperature.

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