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A Computationally More Efficient Distance based VaR Methodology for Real Time Market Risk Measurement

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Abstract

Background/Objectives: The main objective of this paper is to compute VaR (Value at risk) which requires minimal resources and the computing is done in real-time with utmost accuracy. **Method/Statistical Analysis:** The paper presents a methodology which helps in computing VaR in real time and with most accuracy. Very less computational resources are required from computing VaR. The VaR computing methodology proposed in this paper converges as the returns on the portfolio ranges increases. **Findings:** It has been presented in the paper that the number of valuations required for computing the VaR is dependent on the number of instruments added to the portfolio and is independent of the number of instruments already existing at the time computing VaR. The method proposed in this paper can be used for computing VaR in real time.

Keywords: Market Risk, Portfolio Instruments, Risk Assessment, Real-Time Market Risk Measurement, VaR

1. Introduction

Many studies have been conducted in the past for managing risk of an organisation considering cash flows, working capital, equity etc. Mojtaba Rezaei[1] considered risk related to cash flows and whereas Maryam Nakhaei[2], considered risk related to capital structure. Managing market risk of various portfolios has become very essential for managing wealth of an organization or an individual.

Managing market risk has the following objectives

- Compute the exposures against counterparties at various aggregation levels.
- Compute the regulatory capital charge for each instrument based on market-to-market (MTM) value and risk.
- Allocate scarce resources like capital, risk limits, accounting capital to various facilities.
- Introduce the firm's financial reliability and risk-management technology to regulators, pledged counterparties, rating agencies, auditors, the financial press, and others whose knowledge improves regulatory conduct and the firm's terms of instrument and compliance.
- Enhance the performance of facilities, by improving the risk reward ratio.
- Protect the firm from bankruptcy costs.

Market risk is measured in terms of VaR which is computed as the maximum loss that a portfolio can suffer at a presumed confidence over a given time horizon. The risk appetite of the firm can be defined by introducing VaR limits to allocate capital to different business areas at various facilities. Financial institutions use the historical simulation approach for computing market risk VaR as it is the most straight forward method that has no assumptions regarding the distribution of portfolio returns either

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implicitly or explicitly. This limitation makes it unsuitable for real time checking against VaR limits. The methodology proposed in this work involves usage of stored closing prices of the instruments within in the portfolio for N past data points. The portfolio is revalued only once using the current market data of the risk factors. This is taken as $(N+1)^{th}$ data point. The prices at the $(N+1)^{th}$ data point in conjunction with the stored historical prices of the instruments is used in the proposed VaR calculation algorithm. The proposed algorithm can be used in real time as the computational complexity of the proposed algorithm is always much less than the historical simulation approach.

The widely accepted measure for calculating market risk during the 1990's is VaR. In 1922 New York Stock Exchange enforced capital requirements on member firms which required calculation of losses that the portfolio can have for a set time horizon. A quantitative example based on the "spread between probable losses and gains proposed Leavens[3] is considered as the first VaR measure ever published. Markowitz[4] published VaR measures based on the covariance between risk factors for market risk measurement. Tobin[5] calculated VaR measures based on Liquidity preference theory. The theory explains the distribution of wealth among cash and other alternative monetary assets. The cash component doesn't yield any interest and is used to absorb the losses that occur on the other monetary assets. William Sharpe[6] described his VaR measure using relatively few parameters without losing much information making it a low cost analysis. The measure is used in deriving the Sharpe's[7] Capital Asset Pricing Model (CAPM) that establishes the risk and return relationship.

The more volatile markets in the 1980's resulting due to multiplying sources of market risk demanded development of more sophisticated VaR measures. During this period proprietary VaR measures were developed by financial institutions. The explosion of derivative instruments and disclosed losses in the early 1990's stimulated the arena of financial risk management. JP Morgan's Risk Metrics service to measure VaR was revealed to experts at financial organizations and businesses. Further the Basel Committee promoted the use of proprietary VaR models for calculating regulatory capital. A "VaR debate" emerged regarding the subjectivity of risk based on the issued identified by Markowitz. Studies on the Japanese and Singaporean data made by *Halton*[8], Tse[9] and Tse and Tung[10] revealed that volatility forecasts using the ARCH models are inferior compared to the exponentially weighted moving average (EWMA) model. The performance of RiskMetrics is analysed by Pafka and Kondor[11]. Their studies revealed that due to the presence of fat tails in financial data the risk is underestimated by assuming normally distributed returns. Fan et al.[12] did experiments using the EWMA and simple moving average (SMA) for calculating 95% VaR on two stock indices of Shenzhen and Shanghai. The studies exposed that the optimal decay factor for both the indices is less than value determined via RiskMetrics ($0.9 < \lambda < 1$). The fluctuations in the Chinese stock market and their memory lengths are better reflected by calculating the decay factor with EWMA method. Studies by So and Yu[13] on estimation of value at risk at various confidence levels using IGARCH (1, 1), RiskMetrics, GARCH (1, 1) and FIGARCH (1, d, 0) on 4 exchange rates and 12 stock indexes disclosed that the effect of volatility modes for estimating value at risk is less significant in the forex market in comparison to the equity market. Empirical results of efficiency presented in Galdi and Pereira[14] by calculating VaR using EWMA, GARCH and stochastic volatility (SV) models using 1500 observations for a sample proved that VaR computed by EWMA model has lower exceptions than by GARCH and SV models. Investigations of Patev et al.[15] for volatility forecasting on the thin emerging Bulgarian stock markets suggested that both EWMA with GED distribution and EWMA with t-distribution have good performance to model and forecast volatility of stock returns. Most research in the VaR literature emphasize on the computation of the VaR for financial assets like equities or bonds, usually dealing with modelling for negative returns. Recent studies on VaR include the books of Jorion[16] and Dowd[17], papers by Danielsson and de Vries[18], van den Goorbergh and Vlaar[19], Giot and Laurent[20] and Vlaar[21].

Among the methodologies discussed above historical simulation shows better unconditional coverage compared to sophisticated methods like GARCH. The regulatory back tests favour unconditional coverage performance measures of VaR estimates, providing no incentives to adopt different VaR methodology for better conditional coverage. Hence most of the banks implement the historical simulation methodology for VaR calculation. In this work we come up with a new methodology to calculate VaR using historical simulation that requires less computational resources compared to the conventional historical simulation approach.

2. Conventional Historical Simulation

Historical simulation approach involves identifying the risk factors that affect the instruments within the portfolio and generating the scenarios of the risk factors for the data point ahead depending on historical data using the formula

$$Rf(t+1) = Rf(t) * Rf(i+1)/Rf(i) \quad (1)$$

Where:

Rf(t+1) is the risk factor valueat next data point.
Rf(t) is the risk factor value on the data point of calculation.
Rf(i) and Rf(i+1) are the risk factor values on successive data points (i = 1 to N).

The value of each instrument with the set of possible scenarios is determined and the prices calculated using the scenarios are aggregated to get N different portfolio values. The difference between the current value and the N different values of the portfolio is calculated to get N different portfolio returns. These returns are sorted and the $\{floor[(1-\alpha)^*N]\}^{th}$ term is reported as the α% one day VaR.

3. Proposed Algorithm

The proposed algorithm is divided into seven steps as below.

1. Calculation of volatility of returns.
2. Calculation of Lower and Upper bounds of the future data point using the current data point.
3. Calculation of the fractional distance from the upper bound to the actual value.
4. Calculation of the differences in consecutive fractional distances.
5. Generation of possible fractional distances using the current fractional distance and the differences calculated in above step.
6. Calculation of the losses using the bounds on returns and possible fractional distances.
7. Sort the losses and get the loss at required percentile.

Notations

S_n - Closing value of portfolio on data point "n"
R_n - Portfolio Return on data point "n"
σ - Volatility of portfolio returns
μ - Mean of portfolio returns
d_n - Fractional distance from Lower boundary on data point "n"
Δd_n - Difference in fractional distances on data point "n"
LB_n - Lower boundary on data point "n"
UB_n - Upper boundary on data point "n"
L_i - i^{th} expected loss
N - Number of past data points

Example

σ = 30.8288619
k = 5
k σ = 147.4507425

Table 1 shows the calculations related to parameters used in the mathematical expressions and Table 2 shows the calculations related to VaR

Table 1. Calculation of Required Parameters

S_n	R_n	$LB_n = S_{n-1} - k\sigma$	$UB_n = S_{n-1} + k\sigma$	$d_n=(UB_n - S_n)/2k\sigma$	Δd_n
1946.05	33.80	1758.10569	2066.39431	0.390362479	
1955.00	08.95	1791.90569	2100.19431	0.470968763	0.080606284
1926.70	-28.30	1800.85569	2109.14431	0.591797096	0.120828333
1916.75	-09.95	1772.55569	2080.84431	0.532274951	-0.059522145
1968.55	51.80	1762.60569	2070.89431	0.331975633	-0.200299317
1971.90	3.35	1814.40569	2122.69431	0.489133559	0.157157926
1945.60	-26.3	1817.75569	2126.04431	0.585309669	0.09617611
1963.60	18	1791.45569	2099.74431	0.441613154	-0.143696514
1982.15	18.55	1809.45569	2117.74431	0.439829112	-0.001784043
1944.45	-37.7	1828.00569	2136.29431	0.622288004	0.182458892
1900.65	-43.8	1790.30569	2098.59431	0.642074658	0.019786653

Table 2. VaR Calculation

Generated Distances $d = 0.642074658 + \Delta d_n$	$L_i d^* (-k\sigma) + (1-d)^* k\sigma$	Sorted Losses
0.722681	−65.6689	−95.7054
0.762903	−77.5305	−88.2441
0.582553	−24.3449	−77.5305
0.441775	17.1705	−70.2605
0.799233	−88.2441	−65.6689
0.738251	−70.2605	−47.7331
0.498378	0.47829	−41.3719
0.640291	−41.3719	−24.3449
0.824534	−95.7054	0.478288
0.661861	−47.7331	17.17054

90^{th} percentile VaR = −88.2441

4. Comparison between Historical and Proposed Approach

The calculations that are done as a part of VaR calculations can be divided into valuations and computations. Valuations involve instrument pricing that require lot of computational power. Computations involve simple arithmetic like adding instrument prices to get portfolio values etc. Therefore our objective should be to reduce the number of valuations.

Consider a portfolio of "I" instruments. The proposed algorithm is compared with the historical simulation approach considering the four cases as described below.

1. The instruments within the portfolio doesn't not change compared with previous data point.
2. I_{new} new instruments are added to the portfolio compared to previous data point.
3. I_{del} instruments are expired and deleted from the portfolio compared to previous data point.
4. I_{new} new instruments are added and I_{del} instruments are expired and deleted from the portfolio compared to previous data point.

The comparisons made between the Historical Simulation and proposed algorithm is shown in the Table 3.

In each of the above mentioned cases the price of each instrument for N data points is to be stored in the database for implementing the proposed algorithm. However in case of historical simulation there is no such requirement. The expired instruments are deleted from the portfolio before starting the valuation process. In case of the proposed algorithm, the new instruments are valued assuming that the instrument is traded on that particular data point with the corresponding market data of risk factors.

These prices are stored in the database against the corresponding data point for using them at a future data point. When an instrument expires, its historical prices stored in the database are deleted. From Table 3, it can be inferred that the number of valuations required for historical simulation approach is always much greater than that required for the proposed algorithm. Therefore the proposed algorithm is much faster and requires less computational resources than the historical simulation approach and can be used in real time. The storage space required to store the data of a portfolio

Table 3. Number of valuations

Case	Valuations using Historical Simulation	Valuations Using Proposed Algorithm
1	N^*I	I
2	$N^*(I+ I_{new})$	$I + N^* I_{new}$
3	$N^*(I- I_{del})$	$I- I_{del}$
4	$N^*(I+ I_{new} - I_{del})$	$I+ N^*I_{new} - I_{del}$

5. Evaluation of the Proposed Algorithm

To evaluate the model VaR is calculated for 100 data points using both historical simulation and proposed approach for S&P CNXNIFTY index. The accuracy of the proposed algorithm is validated by applying Kupiec[22] test and Mixed Kupiectest. Kupiec's test measures whether the number of exceptions where the actual loss exceeded the measured VaR is in-line with the confidence level. Kupiec's test also called as POF-test (proportion of failures). POF-test requires information regarding, the number of exceptions (e), number of observations (X) and the confidence level (c) for its implementation. The test static is given by equation (2).

$$LR_{POF} = -2\ln\{[(1-p)^{X-e}p^{e}]/[(1-(e/X))^{X-e}(e/X)^{e}]\} \qquad (2)$$

Where $p = 1 - c$

LR_{POF} should be asymptotically χ^2 distributed with one degree of freedom. When the test static is less than the critical value the model passes POF test. The computations achieved using Kupiec's Test is shown in the Table 4.

Table 4. Kupiec's Test for 99% one day VaR

Year	Number of Exceptions Historical Simulation (99%)	Number of Exceptions Proposed Method (99%)	Historical Simulation LR_{POF}(99%)	Proposed Method LR_{POF} (99%)
2005	7	6	5.424052	3.498777
2006	8	4	7.733551	0.769138
2007	7	4	5.533804	0.781362
2008	7	2	5.645647	0.092812
2009	1	1	1.092701	1.092701
2010	5	3	1.936586	0.090944
2011	6	2	3.670885	0.092812
2012	1	1	1.176491	1.176491
2013	12	12	19.09467	19.09467

From Table 4 it can be observed that the test static exceeds the critical value of 6.635 during the year 2013 for both historical simulation and proposed method for VaR calculation. In all the other years the test static is less than the critical value and the proposed model passes POF test. Also the number of exceptions in the proposed approach is less than the historical simulation approach. Haas[23], proposed the mixed kupeic's test that measures both the independence and coverage. The test static for independence is given by equation (3).

$$LR_{ind} = \sum_{i=1}^{n} [-2\ln((p^*(1-p)^{vi-1})/((1/v_i)^*(1-1/v_i)^{vi-1})))] -2\ln((p^*(1-p)^{v-1})/((1/v)^*(1-1/v)^{v-1}))) \quad (3)$$

Where

v_i the time between exceptions i and i−1
v is the time to first exception
n is the number of exceptions

The computations resulted through application of Kupec's test that measures the independence and coverage is shown in the Table 5.

The LR_{ind} -statistic is χ^2 distributed with n degrees of freedom and the LRMix-statistic is χ^2 distributed with n + 1 degrees of freedom. When the test if the test static is less than the critical value the model passes mixed kupiec test. From Table 3, it can be inferred that the proposed model breached the critical value of the test for independence and mixed kupiec test only for the year 2013. In all the other years the test static is less than the critical value and the proposed model passes the test.

Table 5. Mixed Kupiec Test for Proposed Model for 99% one day VaR

Year	Number of Exceptions Proposed Method (99%)	Proposed Method LR_{ind} (99%)	Critical value For LR_{ind} (99%)	Proposed Method $LR_{Mix} = LR_{POF} + LR_{ind}$ (99%)	Critical value For $LR_{Mix} = LR_{POF} + LR_{ind}$ (99%)
2005	6	2.311869	16.812	5.810646	18.475
2006	4	7.798117	13.277	8.567255	15.086
2007	4	3.161443	13.277	3.942805	15.086
2008	2	−1.42182	9.210	−1.32901	11.345
2009	1	−0.132848	6.635	0.959853	9.210
2010	3	2.674885	11.345	2.765829	13.277
2011	2	−1.05561	9.210	−0.9628	11.345
2012	1	−0.23815	6.635	0.938341	9.210
2013	12	36.30045	26.217	55.39512	27.688

6. Mining Rule for 1-day VaR

The 99^{th} percentile one day VaR calculated using the proposed method is converted as a percentage of the current closing value of the Nifty index as given in equation (6.1).

$$\%VaR(t) = \{VaR(t)/S(t)\}^*100 \quad (6.1)$$

Where

VaR(t) is the VaR calculated using the proposed method
S(t) is the Closing value of Nifty index

The one day, percentages of actual returns is calculated using equation (6.2) over the same period. The 99^{th} percentile, one day, percentage of actual returns when sorted in ascending order is equal to 4.384%.

$$\%R(t) = \{ R(t)/S(t_1)\}*100 \quad (6.2)$$

Where

$R(t) = S(t_2) - S(t_1)$
$S(t_1)$ is the closing value of Nifty index on t_1
$S(t_2)$ is the closing value of Nifty Index on t_2
$t_1 - t_2 = 1$ for one day VaR and 10 for ten day VaR

Computing the 99^{th} percentile loss percentage from the actual daily returns of Nifty Index

1. Compute the daily returns of the Nifty index which is the difference between values of the Nifty Index for recent successive trading days as specified by window size, i.e 100 trading days.
2. Convert the returns as a percentage of closing value of index using the equation (6.2) for each time point in the specified window size (100 trading days).
3. Sort the values computed in step 2 in ascending order.
4. Take the 99^{th} percentile loss percentage (L_{99}) of sorted values which corresponds to floor 2^{nd} element from the top in the sorted list of step 3.

Computing the average 99^{th} percentile loss percentage

1. Compute the 99^{th} percentile VaR
2. Convert the VaR obtained as a percentage of closing value of index using the equation (6.1).
3. Take the mean of the values calculated in step 2 for the specified time period.
4. The mean calculated in step 3 represents the average 99^{th} percentile loss percentage (L_{99}) calculated using the proposed method.

Mining Rules

One day and two day over 250 days moving avarage of percentage of VaR are shown in the Figuers 1 and 2. From Figures 1 and 2. it can be observed that during stock market crash the 250 day moving average of 99th percentile oneday VaR expressed as a percentage of closing value of the Nifty index continuously increased in magnitude. Therefore we can say that during the period of stock market crash the VaR shows a trend. Therefore the data corresponding to one crash period can be used to determine the parameters for the following crash period. The period 02-May-00 to 30-Apr-01 represents the stock market crash due to the dotcom bubble and the period 10-Oct-07 to 06-Apr-09 represent the stock market crash due to the subprime crisis.

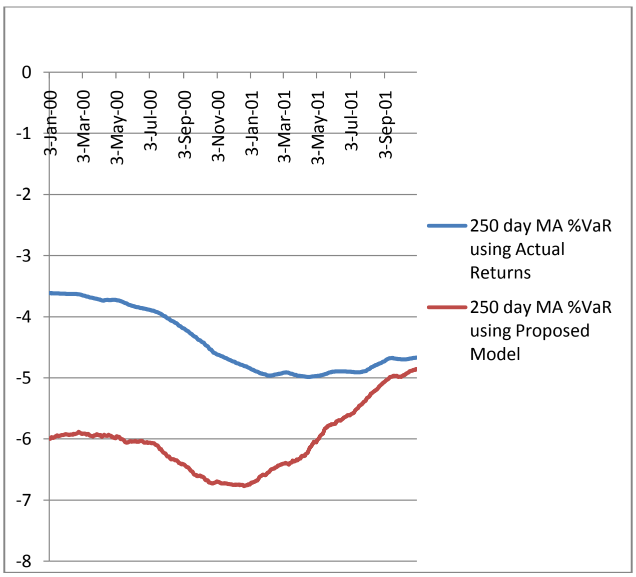

Figure 1. 250 day Moving Average of percentage VaR for the period 03-Jan-00 to 31-Oct-01.

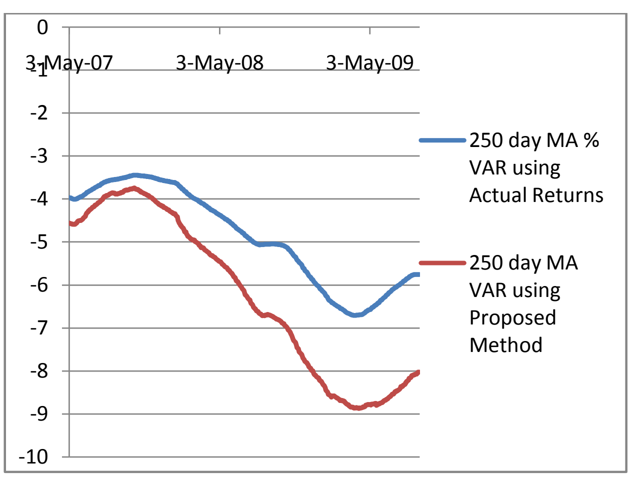

Figure 2. 250 day Moving Average of percentage VaR for the period 03-May-07 to 31-Aug-09.

H_0: The average 99th percentile one day percentage VaR during the period of stock market crash calculated using the proposed method represents the 99th percentile or above VaR during the following stock market crash.

In order to check the above rule the average 99th percentile one day percentage VaR during the period 02-May-00 to 30-Apr-01 is calculated an is validated against the data of the period 10-Oct-07 to 06-Apr-09. The null hypothesis is accepted if the LR_{POF} for the period 10-Oct-07 to 06-Apr-09 is below the critical value. The average 99th percentile one day percentage VaR during the period 02-May-00 to 30-Apr-01 calculated using the proposed method is equal to -6.0534% and using the actual data is equal to -4.7929%. The LR_{POF} calculated for both the cases is equal to 0.46 and 17.29 respectively. The test static (LR_{POF}) exceeds the critical value of 6.635 when the actual data is used to calculate the 99th percentile one day VaR. However when the average 99th percentile one day percentage VaR is calculated using the proposed method the test static (LR_{POF}) is less than the critical value. Hence the null hypothesis is accepted when the average 99th percentile one day percentage VaR is calculated using the proposed method.

7. Conclusions

The number of valuations required is dependent on the number of new instruments added to the portfolio and is independent of the number of instruments already existing in the portfolio. The proposed model uses less computational resources and can be used in real time measurement of market risk VaR. However there is a cost to store the historical prices of the instruments within the portfolio which are used in the VaR calculation algorithm.

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