

Generation of Random NLFM Signals for Radars and Sonars and their Ambiguity Studies

B. L. Prakash^{1*}, G. Sajitha¹ and K. Raja Rajeswari²

¹Department of Electronics and Communication Engineering, K.L. University, Guntur - 522502, Andhra Pradesh, India; blprakash@gmail.com, sajitha875@gmail.com

²Viswanadha Institute of Technology and Management, Visakhapatnam 531173, Andhra Pradesh, India; krr@yahoo.com

Abstract

Background/Objectives: In modern radars waveform design plays a vital role. In the design of radars the most significant parameter is the range resolution. With the help of the signal processing tools like auto correlation and ambiguity function various waveform designs can be achieved. **Methods/Statistical Analysis:** In the proposed paper the discrete frequency coding method of random NLFM (Non Linear Frequency Modulation) signal is described which results in a random like frequency evolution. Random NLFM enables to achieve the thumbtack ambiguity function while minimizing the crosstalk between frequencies. **Findings:** Random NLFM signals for different lengths are generated which are used in radars and sonars and also their ambiguity plots are obtained. The PSLR (Peak Side Lobe Ratio) and ISLR (Integrated Side Lobe Ratio) for these signals are evaluated and a comparison is made among these signals to identify best NLFM waveform. **Application/Improvements:** Non Linear Frequency Modulation waveforms using algebraic random number generators that are quite robust to electronic counter measures and have long duration for long range applications. In future applications of NLFM especially in RADAR/SONAR applications, this idea may result into new methodology.

Keywords: Ambiguity Function, Discrete Frequency Coding, ISLR, NLFM, PSLR, Pulse Compression, Radar

1. Introduction

In modern radars pulse compression is used to improve the signal to noise ratio and resolution in the range¹⁻⁴. To obtain significant time-bandwidth product BT the transmitted pulse is modulated. The transmitted pulse is actually compressed in time domain. Pulse compression brings the advantage of high energy of a long pulse with the high resolution of a short pulse together. Popular pulse compression techniques are frequency coded as well as phase coded signals²⁻⁶. Linear Frequency Modulated (LFM) signals attain good ambiguity function but has the drawback of high grating lobes. Discrete frequency enables to achieve the thumbtack ambiguity function with random like frequency evolution³⁻⁹.

2. Non-Linear Frequency Modulation (NLFM) Waveform

Pulse compression obtained with LFM suffers from side lobes. These side lobes can be decreased by using different side lobe suppression techniques. The shaping is obtained by varying the pulse amplitude along the time axis. In LFM, frequency varies in direct proportion to time very much similar to amplitude change with frequency. Shaping the spectrum by amplitude weighting an LFM pulse has a serious problem.

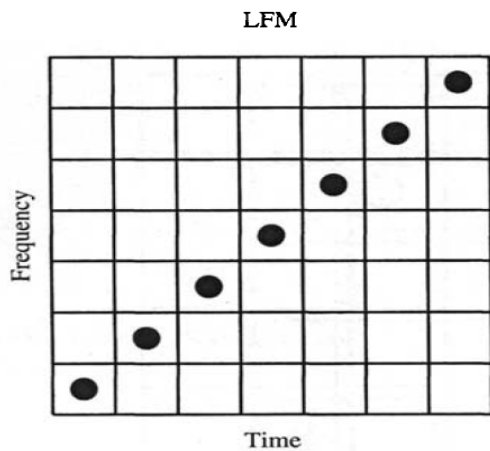
The spectral energy could be reduced at the edges giving a window shaped spectrum by reducing the signal amplitude at the pulse edges with constant pulse amplitude so as to spend less time in each spectral interval near

*Author for correspondence

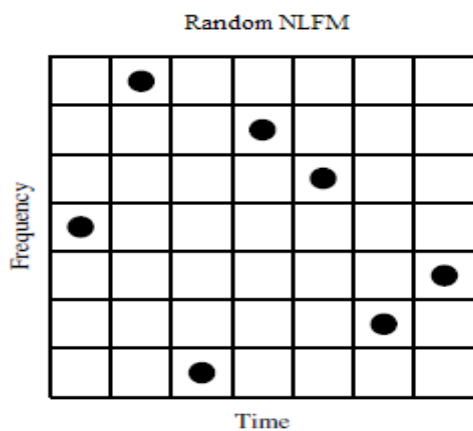
the band edges or both⁸⁻¹³. This approach in which variable sweep rates are used is called Non Linear Frequency Modulation¹²⁻¹⁸.

3. Discrete Frequency Coding

Discrete frequency coding is similar to LFM. LFM generates high grating sidelobes and amplitude weighting is necessary to reform the spectrum. In discrete LFM the hopping orders of frequencies are varied in a linear fashion where as in the case of discrete NLFM the frequency is hopped in a random fashion as indicated in Figure 1.



(a) Quantized linear FM



(b) Discrete random frequency coding

Figure 1. Binary matrix representation.

Figure 1 represents a pair of hopping orders that are possible out of $M!$ feasible orders that will meet the limitation of just one dot per column and per row.

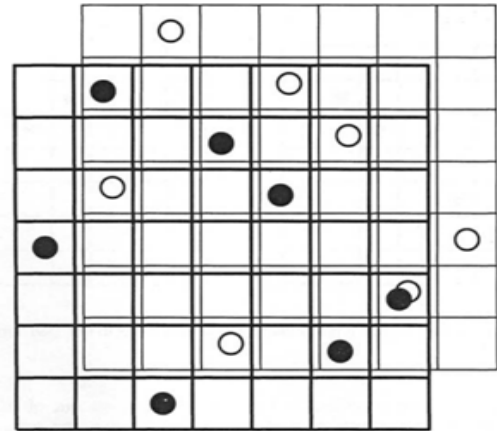


Figure 2. Example of one coincidence occurring at $\tau/t_b=1$ and $v / \Delta f = 1$.

It is noticed that in the case of LFM the delay and Doppler movements of equivalent number of units [$\tau = m t_b, v = m f, m = 0, \pm 1, \dots, \pm(M - 1)$] will generate an overlap of dots, and the quantity of matching dots will be $N = M - |m|$. It generates a diagonal ridge across the line $v = f \tau/t_b$ in the ambiguity function.

3.1 Generation of Discrete Frequency (Random NLFM) Signals

In the generation of random NLFM signals a relatively long pulse of τ' length is divided directly into N subpulses with each pulse of width τ_1 ($\tau' = N\tau_1$). A group of 'N' sub pulses is called burst signal. The frequency is enhanced by Δf derived from one of the sub-pulses to another sub-pulse within each burst. The bandwidth of the whole burst is given by $N\Delta f$. Much more specifically,

$$\tau_1 = \tau' / N$$

where, $N =$ group of pulses

The frequency for the i^{th} sub pulse is given as

$$f_i = f_0 + i\Delta f; \quad i = 1, N$$

$$f_0 \gg \Delta f$$

$$\Delta f \tau' = N^2$$

Depending on a predetermined concept or logic the frequencies for the subpulses are usually selected randomly. The rows and columns are given as $i = 1, 2, \dots, N$ and $j = 0, 1, 2, \dots, (N - 1)$ respectively. Here the rows represent sub-pulses and the columns represent frequency. The dot signifies the frequency issued for the particular subpulse. The frequency assignments shown in the Figure

2 are usually chosen randomly. There will be a total of $N!$ possible ways to assign the dots (achievable codes) for a matrix of size $N \times N$.

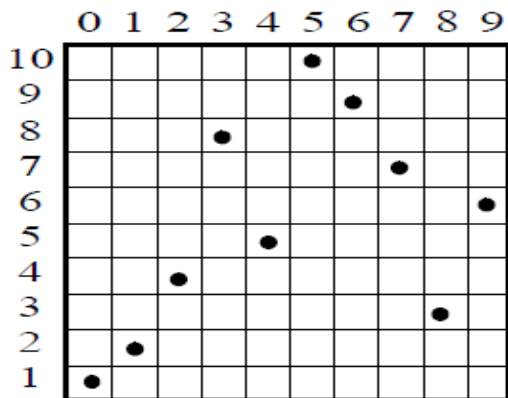


Figure 3. Discrete frequency code of length $N_D = 10$.

A series of arrangement of dots for which the resultant ambiguity function reaches an ideal or possibly a thumbtack response is termed as discrete frequency code. Discrete frequency coding (random NLFM) can acquire near thumbtack response with following logic: there must be only one frequency per time slot (row) along with frequency slot (column). The achievable number of discrete frequency codes for an $N \times N$ matrix is usually lesser than $N!$. As N becomes larger the code density $N_D/N!$ is considerably less.

Discrete frequency codes can be generated by several analytical methods. Two different methods are described here. In the first method: Let the number of sub-pulses be chosen as

$$N = q - 1$$

The primitive root of q is specified as γ .

$\gamma, \gamma^2, \gamma^3, \dots, \gamma^{q-1}$ modulo q produce each and every integer from 1 to $q - 1$.

For an $N \times N$ matrix the rows and columns for the first method are given by

$$i = 0, 1, 2, \dots, (q-2)$$

$$j = 1, 2, 3, \dots, (q-1)$$

A dot is placed if

$$i = (\gamma)^j \text{ mod } q$$

A new code is generated by deleting the first row and column of the matrix obtained using the first method.

Using this method of coding a discrete frequency code (random NLFM) of size $N = q - 2$ is produced.

For a discrete frequency signal the normalized complex envelope is given as

$$r(t) = \frac{1}{\sqrt{N\tau_1}} \sum_{l=0}^{N-1} r_l(t - l\tau_1) \tag{2}$$

$$\text{where, } r_l(t) = \begin{cases} \exp(j2\pi f_l t) & 0 \leq t \leq \tau_1 \\ 0 & \text{elsewhere} \end{cases}$$

The output of the matched filter for discrete frequency random NLFM signal is calculated as

$$\chi(\tau, f_D) = \frac{1}{N} \sum_{l=0}^{N-1} e^{j2\pi f_D \tau} \left[\Phi_l(\tau, f_D) + \sum_{\substack{q=0 \\ q \neq l}}^{N-1} \Phi_q(\tau - (l-q)\tau_1, f_D) \right] \tag{3}$$

$$\Phi(\tau, f_D) = \left(\tau_1 - \frac{|\tau|}{\tau_1} \right) \frac{\sin \alpha}{\alpha} \exp(-j\beta - j2\pi f_q \tau) \quad |\tau| \leq \tau_1 \tag{4}$$

$$\text{where, } \alpha = \pi(f_l - f_q - f_D) \tau_1 - |\tau|$$

$$\beta = \pi(f_l - f_q - f_D) \tau_1 + |\tau|$$

4. Ambiguity Function of Random NLFM Signals

An ambiguity function is a two-dimensional function of time delay and Doppler frequency $\chi(\tau, \nu)$ showing the distortion of a returned pulse due to the receiver matched filter.

The ambiguity function (AF) can be defined as

$$|\chi(\tau, \nu)| = \left| \int_{-\infty}^{\infty} u(t) u^*(t + \tau) e^{j2\pi \nu t} dt \right|^2 \tag{5}$$

where, u is the complex envelope of the signal.

A target moving toward the radar is indicated by positive ν . The ambiguity function of random NLFM signals shows the near thumbtack response. Except some side lobes near to the origin the remaining side lobes have amplitude of $1/N$ and that are close to the origin have an amplitude $2/N$, which is typical nature of discrete frequency code. For a discrete frequency random NLFM the compression rate is given as N . The most important method for learning and evaluating radar signals is the ambiguity function. Here in this paper ambiguity dia-

grams for random NLFM pulses of different lengths are plotted and also their PSLR and ISLR (which are defined below) are compared.

5. Peak Side Lobe Ratio (PSLR)

It is defined as the ratio of the peak value of side lobe to the maximum value of main lobe. PSLR represents the ability of the radar to identify a weak target from a nearby strong one.

The peak side lobe ratio can be calculated using the formula

$$PSLR = 20 \log \left[\frac{\max(\text{sidelobe peak})}{(\text{Mainlobe peak})} \right]$$

6. Integrated Side Lobe Ratio (ISLR)

The ISLR is defined as the ratio of total power (energy) in the side lobes to the power in the main lobes. ISLR characterizes the ability to detect weak targets in the neighbourhood of bright targets.

The integrated side lobe ratio can be calculated using the formula

$$ISLR = 10 \log \left[\frac{\text{Total energy in the sidelobe}}{\text{Energy in the mainlobe}} \right]$$

7. Results

7.1 Ambiguity Plot for Random NLFM Pulse of Length p=53

The following are the amplitude, phase, frequency response and zero doppler cut for random NLFM signal of length p=53.

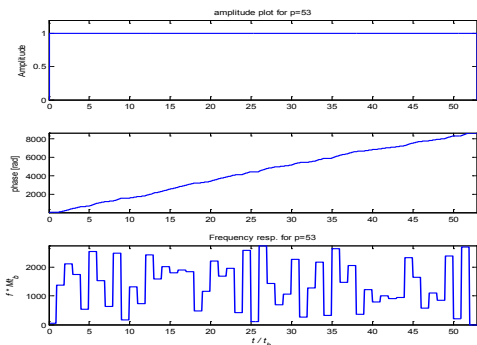


Figure 4. Amplitude, phase and frequency response for random NLFM signal of length p=53.

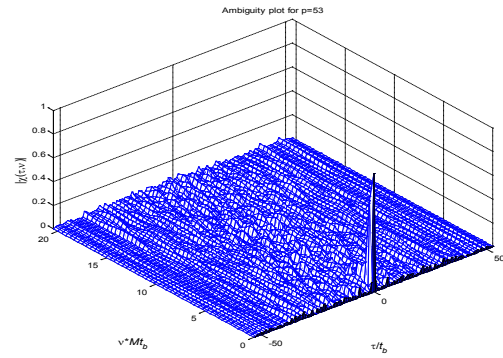


Figure 5. Ambiguity plot for random NLFM signal of length p=53.

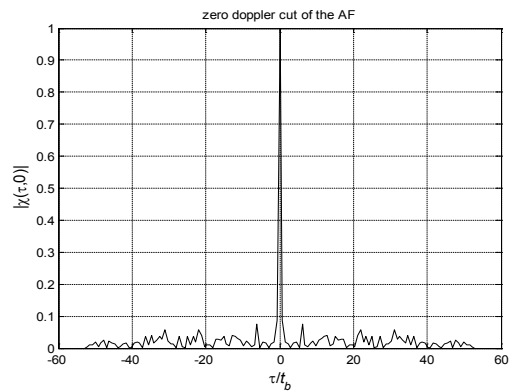


Figure 6. Zero doppler cut of the AF of the length p=53.

7.2 Ambiguity Plot for Random NLFM Pulse of Length p=71

The following are the amplitude, phase, frequency response and zero doppler cut for random NLFM signal of length p=71.

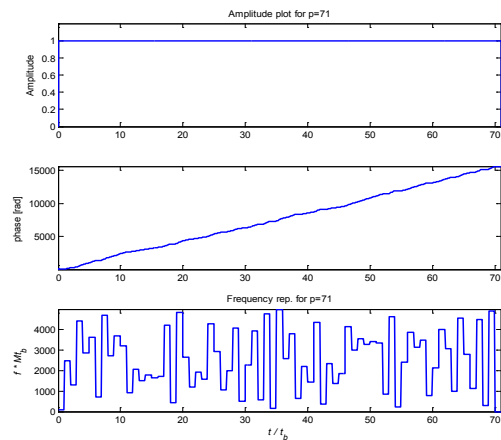


Figure 7. Amplitude, phase and frequency response for random NLFM signal of length p=71.

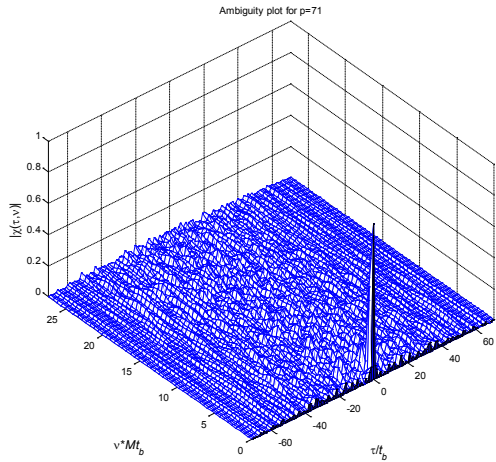


Figure 8. Ambiguity plot for random NLFM signal of length $p=71$.

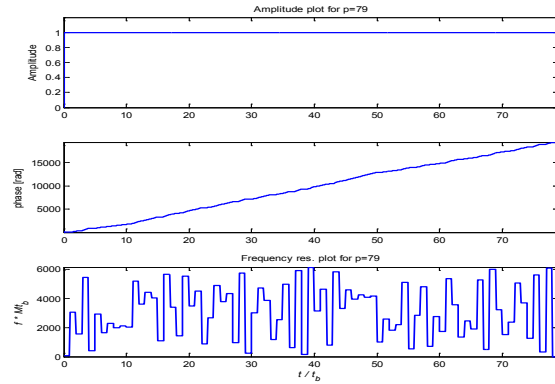


Figure 10. Amplitude, phase and frequency response for random NLFM signal of length $p=79$.

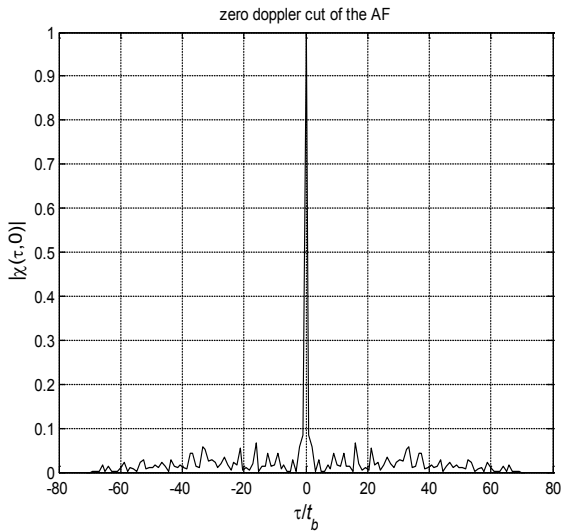


Figure 9. Zero doppler cut of the AF of the length $p=71$.

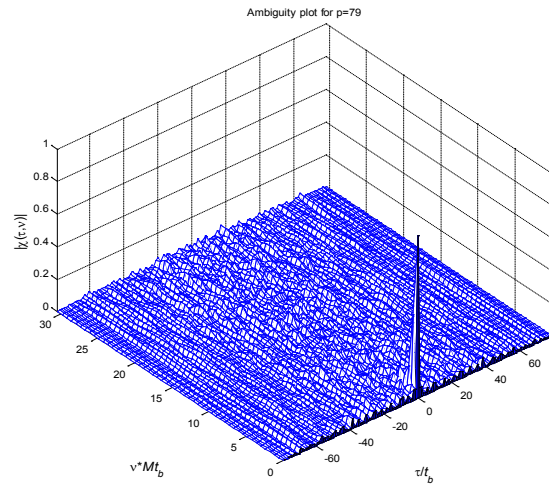


Figure 11. Ambiguity plot for random NLFM signal of length $p=79$.

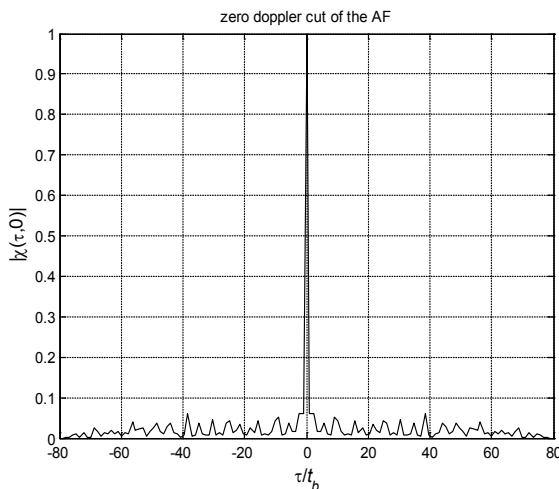


Figure 12. Zero doppler cut of the AF of the length $p=79$.

7.3 Ambiguity Plot for Random NLFM Pulse of Length $p=79$

The following are the amplitude, phase, frequency response and zero doppler cut for random NLFM signal of length $p=79$.

8. Comparison of PSLR and ISLR of Random NLFM Signals of Different Lengths

The prime numbers up to length 47 of random NLFM signals gave poor results hence in this paper we considered the prime numbers from the length 53 onwards. Above 53 the lengths for which the PSLR and ISLR values are not good are omitted. And a comparison is made only among those lengths which have produced good results.

In this paper the random NLFM signals for prime numbers of lengths $P = 53, 71, 79, 83$ and 103 are compared (Table 1). The PSLR and ISLR computed for lower lengths are considerably less and hence are ignored. The following table represents the comparison of the PSLR and ISLR of random NLFM signals for the following prime numbers. The ambiguity diagrams are plotted only for $p= 53, 71$ and 79 .

Table 1. Comparison of the PSLR and ISLR of random NLFM signals

S.No	Prime Number	PSLR in dB	ISLR in dB
1	$P=53$	-17.0836	-4.0443
2	$P=71$	-17.5252	-4.3159
3	$P=79$	-17.9525	-4.9689
4	$P=83$	-18.3816	-5.0633
5	$P=103$	-18.7349	-5.3180

9. Conclusion

In this paper the novel idea of random Non Linear Frequency Modulation is introduced. The ambiguity diagrams are developed for the lengths of $p = 53, 71$ and 79 lengths respectively. The Doppler resolution plots are also obtained. The PSLR and ISLR values are obtained for five different lengths and a comparison is made among them. The ambiguity plots show very narrow main lobe and the envelope of it along Doppler axis vary randomly in random NLFM. As the length is increasing the waveform exhibits better PSLR and ISLR. From the ambiguity

plots it can be observed that as the length of the pulse increases, the range side lobes are drastically reduced. In the communications random numbers play vital role in many applications. The NLFM generation using random numbers is entirely different concept compared with other methods of NLFM generation. on Linear Frequency Modulation waveforms using algebraic random number generators that are quite robust to electronic counter measures and have long duration for long range applications. In future applications of NLFM especially in RADAR/SONAR applications, this idea may result into new methodology.

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