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Application of Cubature Kalman Filter for Bearings-only Target Tracking

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Abstract

Background/Objectives: The objective of this paper is to develop a novel estimation algorithm based target tracking simulator for underwater target tracking applications. **Methods/Statistical Analysis**: An own ship observes corrupted sonar bearings from a radiating target and finds out Target Motion Parameters (TMP) - viz., range, course, bearing and speed of the target. The issue is inherently nonlinear as the bearing measurement is non-linearly related to the target state. CKF is a new nonlinear filter for state estimation. The modeling of target state and measurement vectors is carried out. CKF is integrated into the model to result in evolution of simulator. Extensive performance evaluation of CKF with respect to bearings-only target tracking problem in Monte-Carlo simulation is carried out and the results are presented. **Findings:** CKF depends on spherical-radial cubature rule that makes it potential to numerically figure variable moment integrals encountered within the nonlinear filter. The underwater passive target tracking following Cubature Kalman filter is explored in this paper. **Application/Improvements:** The results obtained are satisfactory and UKF can be used in futuristic submarines in Indian Navy owing to its advantages as envisaged in this paper.

Keywords: Cubature Kalman (CKF) Filter, Estimation, Kalman Filter, Simulation, Sonar, Underwater Target Tracking

1. Introduction

In ocean atmosphere, 2-dimensional bearings-only target motion analysis is mostly used. An observer monitors corrupted sonar bearings from a radiating target in passive listening mode. It's assumed that the hull mounted sonar within the observer platform picks up the signal and generates bearing measurements of the target. For range observability, the observer usually performs S maneuver on line of sight as shown in Figure 1, as bearing measurements are only available to discover out the target motion parameters viz, range, course, bearing and speed of the target. In Bearings- Only Target (BOT), range measurements are inaccessible, the bearing measurements are non-linearly associated with target states, creating the total method inherently non-linear. For presenting the ideas in clarity, it's assumed that the target is moving at constant speed[1-4].

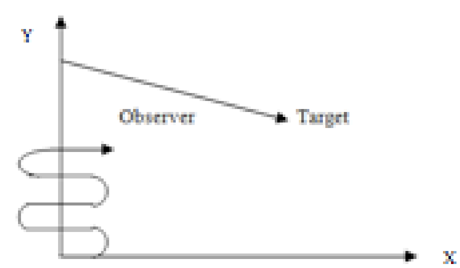

Figure 1. Observer in S-maneuver.

The authors are motivated by the paper written by Pie. H. Leong et.al[1]. Extensive mathematical modeling is carried out for Cubature Kalman Filter (CKF), Range-Parameterized CKF and Gaussian-Sum CKF for bearings-only target tracking[1]. Their effort is highly appreciated. However, the algorithms are evaluated using target position errors only. Target course error and target speed error are also required for passive target tracking applica-

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tions. As this research work is useful for surveillance in sea waters, some more information like errors in target course and speed are also required.

In intercept guidance algorithm and various other weapon guidance algorithms, the future position of target location is determined by present position as well as course and speed of the target. Target position error does not provide any information regarding the course and speed errors. Various tactical scenarios also need to be considered for performance evaluation of the bearings-only target tracking algorithms.

The observer carried out one maneuver and the length of each leg is 15 minutes approximately. In general, one observer maneuver is not sufficient. The first observer maneuver makes the process observable and the second maneuver is required to get better solution and third maneuver is required to obtain the required solution. Additionally, fourth observer maneuver may be required in case of highly noisy bearing measurements or if the range increases with time (range is opening). In[1], sampling interval chosen is 1 minute which leads to availability of fewer bearing measurements. In real time environment, integration time at the sonar is 1 second approximately for bearing measurement. In this paper, it is chosen as 1 second.

The scenarios chosen in[1] are far away from practical view point. In highly nonlinear scenario, the turning rate of observer is infinite which is practically not feasible. In real time environment, the maximum possible observer turning rate is 0.5°/s and 1°/s in case of submarine and ship respectively. Another important issue is initializing the state vector of the target[2–7]. Here, the observer course is assumed to be target course with 180 degree addition. In BOT, target course and target speed cannot be found out initially.

Non-linear filters estimate the state of a non-linear stochastic process from Gaussian corrupted bearing measurements. The standard technique usually applied for non-linear applications is the Extended Kalman Filter (EKF). But, in case of severe non-linearity's such as BOT, the EKF is unstable and it diverges. Improved non-linear filtering algorithms such as UKF, CKF, Particle Filter (PF) etc. are discussed in literature[8–12]. Various scenarios considering different observer and target geometries, speeds and different ATBs are simulated in Mat lab/PC environment for performance evaluation of UKF and CKF.

Section 2 describes mathematical modeling of the bearing measurements, target state equation, measurement equations. In section 3, initialization of target state vector and its covariance matrix of CKF algorithm is elaborated. Performance evaluation of these algorithms is discussed in section 4. Summary and conclusion are presented in section 5.

2. Mathematical Modeling

Let the target state vector be X_s (k) where

$$X_s(k)=\begin{bmatrix} \dot{x}(k) & \dot{y}(k) & R_x(k) & R_y(k) \end{bmatrix}^T \tag{1}$$

W where, x *(k)* and y *(k)* are target velocity components R_x *(k)*and R_y*(k)* are range components respectively. True north convention is followed for all angles to reduce mathematical complexity and for easy implementation. The bearing measurement, B_m is modeled as

$$B_m(k+1)=\tan^{-1}\left(\frac{R_x(k+1)}{R_y(k+1)}\right)+\zeta(k) \tag{2}$$

Here ξ *(k)* is error in the measurement and this error is assumed to be zero mean Gaussian with variance σ_b^2. The measurement and plant noise are assumed to be uncorrelated to each other. Equation (2) is a nonlinear equation and is lineralized by using Taylor series expansion for R_x and R_y. The measurement matrix is given by

$$H(k+1)=\begin{bmatrix} 0 & 0 & \hat{R}_Y(K+1/K)\big/R^2(k+1/k) & -\hat{R}_x(K+1/K)\big/R^2(k+1/k) \end{bmatrix} \tag{3}$$

where $\hat{R}_X$ and $\hat{R}_Y$ are estimated values of range components. Since the true values of range components are not known, the estimated values are used in eqn.(3). The target state dynamic equation is given by

$$X_s(k+1)=\varphi X_s(k)+b(k+1)+\Gamma w(k) \tag{4}$$

where ϕ and $b$are transition matrix and deterministic vector respectively. The transition matrix is given by

$$\varphi=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ t & 0 & 1 & 0 \\ 0 & t & 0 & 1 \end{bmatrix} \tag{5}$$

where, t is sample time

$$b(k+1) = \begin{bmatrix} 0 & 0 & -(x_o(k+1) - x_o(k)) & -(y_o(k+1) - y_o(k)) \end{bmatrix} \tag{6}$$

x_o and y_o are observer position components.

$$\Gamma = \begin{bmatrix} t & 0 \\ 0 & t \\ t^2/2 & 0 \\ 0 & t^2/2 \end{bmatrix} \tag{7}$$

$w(k)$, the plant noise, is assumed to be zero mean white Gaussian with

$$E[\Gamma(k)w(k)w^T(k)\Gamma^T(k) = Q\delta_{ij} \tag{8}$$

where

$$\delta_{ij} = \sigma_w{}^2 \quad \text{if } i = j \\ = 0 \text{ otherwise} \tag{9}$$

$$Q = \begin{bmatrix} t_s{}^2 & 0 & t_s{}^3/2 & 0 \\ 0 & t_s{}^2 & 0 & t_s{}^3/2 \\ t_s{}^3/2 & 0 & t_s{}^4/8 & 0 \\ 0 & t_s{}^3/2 & 0 & t_s{}^4/8 \end{bmatrix} \tag{10}$$

The measurement and plant noises are assumed to be uncorrelated to each other.

3. Cubature Kalman Filter Algorithm

1. The unit sigma points are calculated as

$$\xi^{(i)} = \begin{cases} \sqrt{n}e_i, \ i = 1,....,2n \\ -\sqrt{n}e_{i-n}, i=n+1,....,2n. \end{cases} \tag{11}$$

where e_i denotes a unit vector in the direction of the coordinate axis i.

2. Approximating the integral as

$$\int g(x)N(x \mid m,P)dx \approx \frac{1}{n}\sum_{i=1}^{2n} g(m + \sqrt{P}\xi^{(i)} \tag{12}$$

where $\sqrt{P}$ is a matrix square root defined by $P = \sqrt{P}\sqrt{P}^T$

It is easy to see that the approximation above is a special case of the unscented transform with parameters $\alpha = 1, \beta = 0$ and $k = 0$

With this parameter selection the mean weight is zero and the unscented transform is effectively a 2n-point approximation as well. Using third order spherical cubature integration rule, the CKF algorithm is as follows.

- Prediction:

1. The sigma points are calculated as

$$X_{k-1}^{(i)} = m_{k-1} + \sqrt{P_{k-1}}\xi^{(i)} \quad i = 1,.......2n \tag{13}$$

where the unit sigma points are defined as

$$\xi^{(i)} = \begin{cases} \sqrt{n}e_i, \ i = 1,....,2n \\ -\sqrt{n}e_{i-n}, i = n+1,....,2n. \end{cases} \tag{14}$$

2. The sigma points are propagated through the dynamic model

$$\hat{X}_k^{(i)} = f(X_{k-1}^{(i)}), \quad i = 1,....,2n \tag{15}$$

3. The predicted mean m_k^- and the predicted covariance P_k^- are calculated as

$$m_k^- = \frac{1}{2n}\sum_{i=1}^{2n}\hat{X}_k^{(i)}, \\ P_k^- = \frac{1}{2n}\sum_{i=1}^{2n}(\hat{X}_k^{(i)} - m_k^-)(\hat{X}_k^{(i)} - m_k^-)^T + Q_{k-1}. \tag{16}$$

- Updation

1. The sigma points are formed as

$$X_k^{-(i)} = m_k^{(i)} + \sqrt{P_k^-}\xi^{(i)}, \quad i = 1,.....,2n, \tag{17}$$

where, the unit sigma points are defined as in Equation

2. Sigma points are propagated through the measurements model as

$$\hat{y}_k^{(i)} = h(X_k^{-(i)}), \quad i = 1...2n \tag{18}$$

3. The predicted mean μ_k, the predicted covariance of the measurement C_k, and the cross-covariance of the state and the measurement S_k are calculated as

$$\mu_k = \frac{1}{2n}\sum_{i=1}^{2n}\hat{y}_k^{(i)},$$
$$S_k = \frac{1}{2n}\sum_{i=1}^{2n}(\hat{y}_k^{(i)} - \mu_k)(\hat{y}_k^{(i)} - \mu_k)^T + R_k, \quad (19)$$
$$C_k = \frac{1}{2n}\sum_{i=1}^{2n}(X_k^{-(i)} - m_k^-)(\hat{y}_k^{(i)} - \mu_k)^T.$$

4. The filter gain and K_k the filtered state mean m_k and covariance P_k are calculated as

$$K_k = C_k S_k^{-1},$$
$$m_k = m_k^- + K_k[y_k - \mu_k], \quad (20)$$
$$P_k = P_k^- - K_k S_k K_k^T.$$

4. Performance Evaluation of the Algorithm

As the aim of this paper is to analyse the performance of CKF, the situation in underwater, at the time of testing it is assumed to be with favorable conditions and hence the measurements are obtained for every second[8–14].

The algorithm is implemented using MATLAB in a PC environment. The noise in the bearing measurement is assumed to be following white Gaussian with Standard Deviation (SD) of 0.5°. So, each bearing measurement available at every 1 sec is added with white Gaussian of SD, 0.5o.

It is assumed that the observer makes S-maneuver at a turning rate of 0.5°/s on the line of sight as shown in Figure1. The observer travels for 2 minutes in the first leg at 90° course and then turns towards 270⁰ course. In the second leg, it travels for 4 minutes and then turns towards 90° course.

The third and fourth legs are just like second leg except in the third leg, observer course is 270⁰ and in fourth leg it is 90°. The length of each run is 42 minutes (2520 samples) covering 5 legs with 4 maneuvers. The algorithms are evaluated using scenarios given in Table 1 Table 2 and Table 3. The targets are assumed to be submarine, ship and torpedo in Table 1, Table 2 and Table 3 scenarios respectively. The target range and speeds are chosen which are close to realistic values.

Based on the intercept target weapon guidance algorithm the acceptance of the solution of this algorithm is assumed to be as follows[18–24].

Error in the range estimate<=8% of the actual range
Error in the course estimate<=3°.

Table 1. Scenarios chosen for low ATB

Scenario	Initial Range in meters	Initial bearing in degrees	Target Speed in meter per second	Observer Speed in meter per second	Target Course in degrees
1. Submarine to Submarine	4800	0	6	5	168
2. Submarine to ship	9000	0	9	5	165
3. Submarine to Torpedo	18000	0	12	5	162

Table 2. Scenarios chosen for medium ATB

Scenario	Initial Range in meters	Initial bearing in degrees	Target Speed in meter per second	Observer Speed in meter per second	Target Course in degrees
1. Submarine to Submarine	5500	0	5	5	132
2. Submarine to ship	9300	0	8	5	137
3. Submarine to Torpedo	18500	0	10	5	130

Table 3. Scenarios chosen for high ATB

Scenario	Initial Range in meters	Initial bearing in degrees	Target Speed in meter per second	Observer Speed in meter per second	Target Course in degrees
1. Submarine to Submarine	5300	0	6	5	100
2. Submarine to ship	9600	0	8	5	120
3. Submarine to Torpedo	19000	0	10	5	110

Error in the speed estimate<=1m/s.

The algorithm is said to be converged in the range, course and speed, once the solution (target location is tracked) is accepted. The convergence time in seconds of CKF algorithms are shown in the Tables 4 to 6 with single run mode. The errors in estimate of range (R-error), course (C-error) and speed (S-error) in case of Low, medium and high ATB's are shown in Figures 2 to 4, Figures 5 to 7 and Figures 8 to 10 for scenarios in Tables 1 to 3 respectively.

Table 4. Convergence time in seconds for Low ATB Scenarios with single run

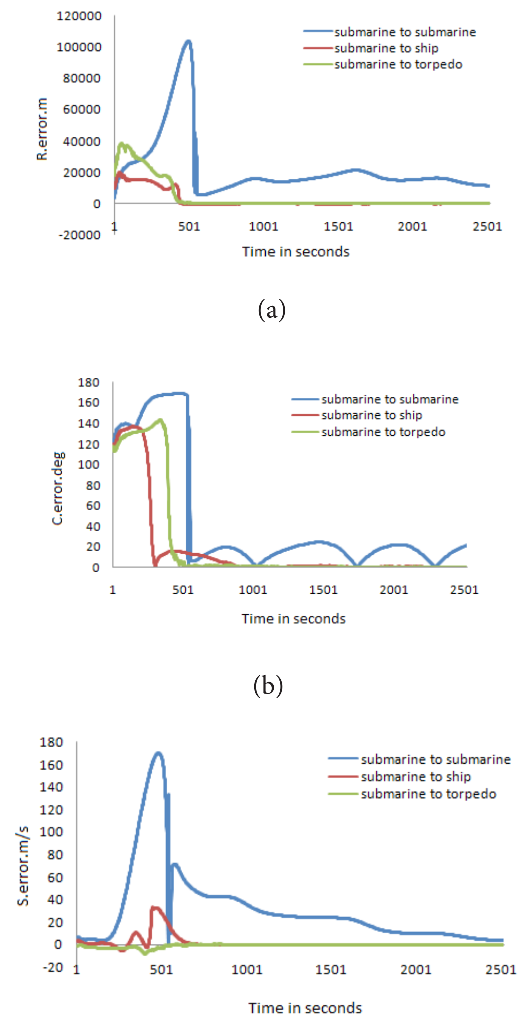
Scenario	CKF			
	R	C	S	Total solution
1. Submarine to Submarine	NC	1008	NC	NC
2. Submarine to ship	452	854	668	854
3. Submarine to Torpedo	453	512	872	872

Table 5. Convergence time in seconds for Medium ATB Scenarios with single run

Scenario	CKF			
	R	C	S	Total solution
1. Submarine to Submarine	563	872	628	628
2. Submarine to ship	317	508	234	508
3. Submarine to Torpedo	343	793	280	793

Table 6. Convergence time in seconds for High ATB Scenarios with single run

Scenario	CKF			
	R	C	S	Total solution
1. Submarine to Submarine	362	640	214	640
2. Submarine to ship	312	661	218	661
3. Submarine to Torpedo	340	805	308	805

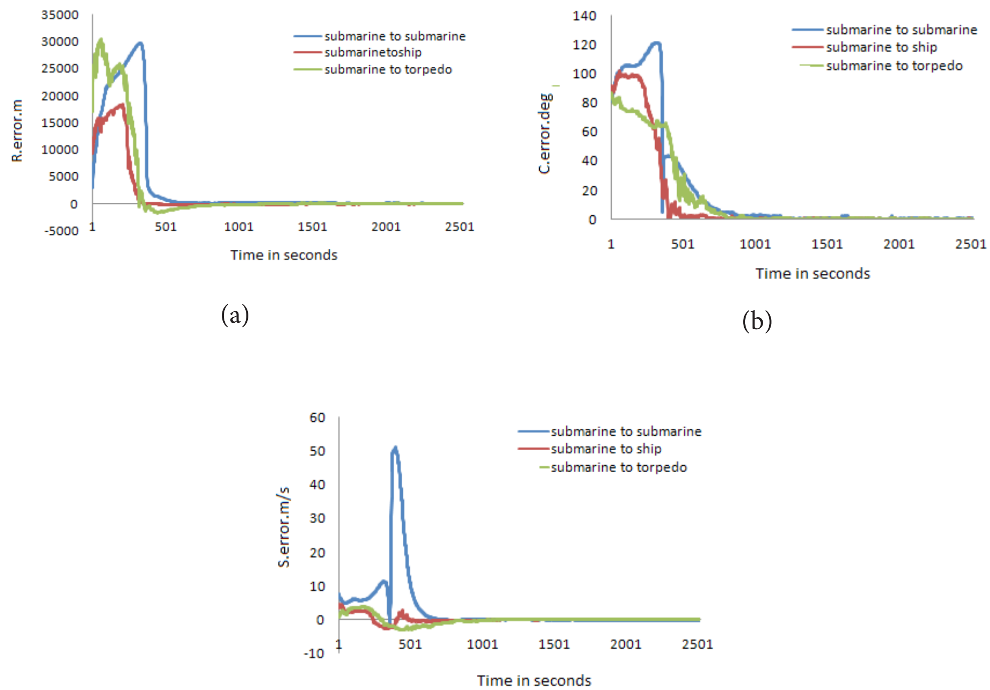


(a) (b) (c)

Figure 2. Errors in estimates for low ATB (single run). (a) Error in Range estimate. (b) Error in Course estimate. **(c)** Error in Speed estimate.

For example, in Table 1, the scenario describes a low ATB submarine target moving at 6 m/s at the course of 168°. Initial range between the target and submarine is 4800m and the target moves at an initial bearing of 0°. The observer moves at a speed of 5 m/s in S-maneuver as shown in Figure 1.

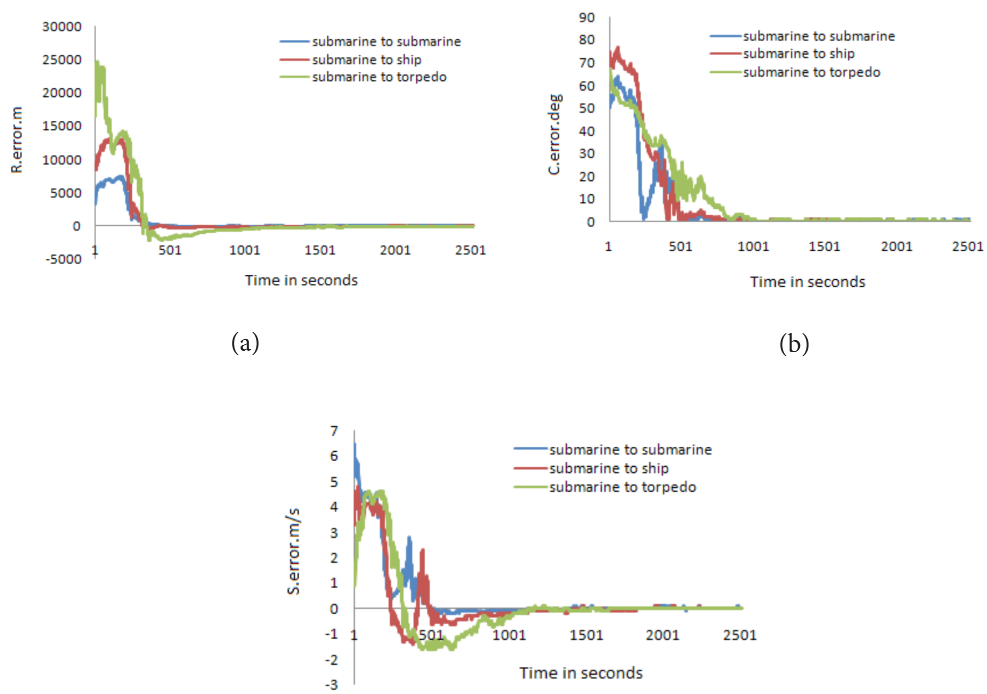
With CKF algorithm, the estimated range, course and speed of the target are converged at 452,854,668 seconds and hence the total solution is said to be converged at 854 seconds, which is tabulated in Table 4. The errors in estimated target motion parameters with respect to time obtained with CKF are shown in Figure 2. Similarly, for all other scenarios, same procedure is followed.



(a) (b)

(c)

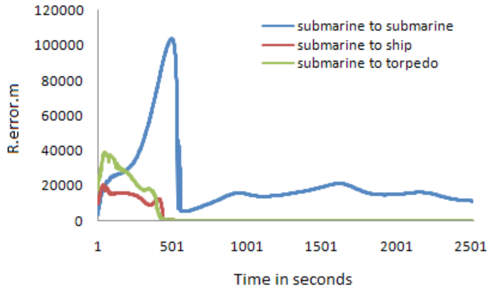
Figure 3. Errors in estimates for Medium ATB (single run). **(a)** Error in Range estimate. **(b)** Error in Course estimate. **(c)** Error in Speed estimate.



(a) (b)

(c)

Figure 4. Errors in estimates for high ATB (single run). **(a)** Error in Range estimate. **(b)** Error in Course estimate. **(c)** Error in Speed estimate.



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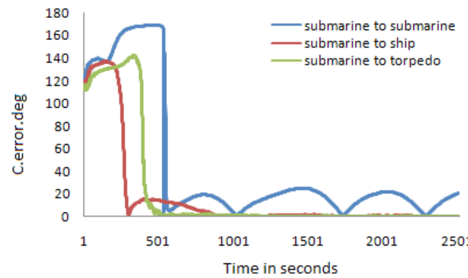

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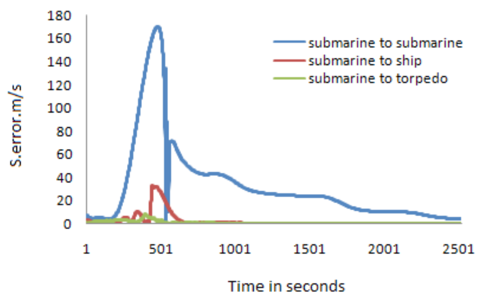
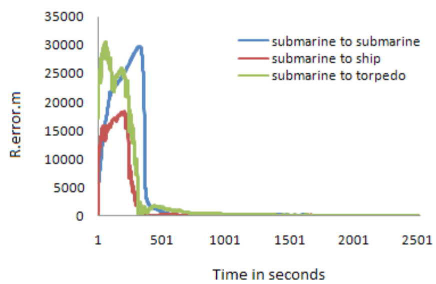
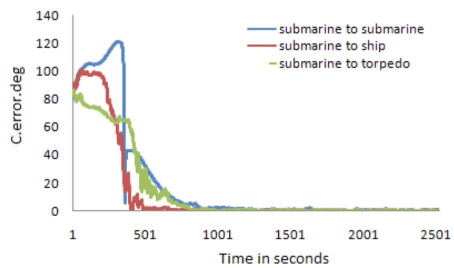


(c)

Figure 5. RMS errors of estimates for low ATB (MonteCarlo runs).**(a)** Error in Range estimate. **(b)** Error in Course estimate. **(c)** Error in Speed estimate.



(a)



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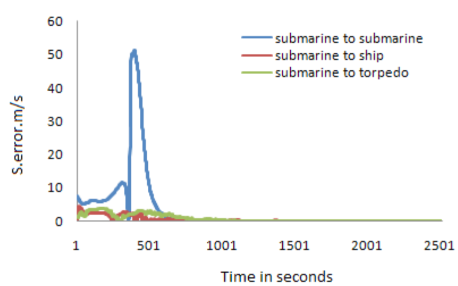
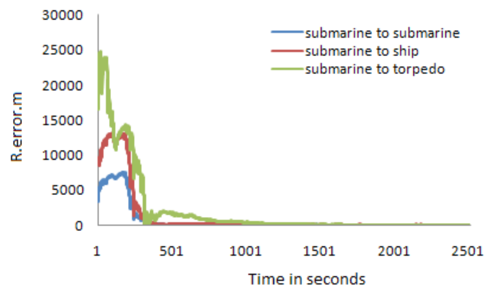
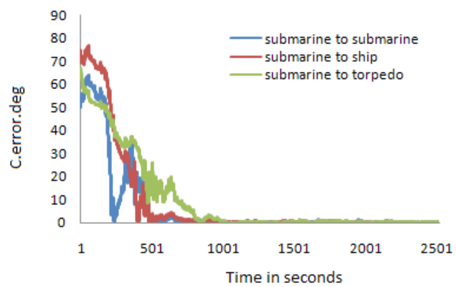


(c)

Figure 6. RMS errors in estimates for medium ATB scenario (Monte Carlo runs). **(a)** Error in Range estimate. **(b)** Error in Course estimate. **(c)** Error in Speed estimate.



(a)



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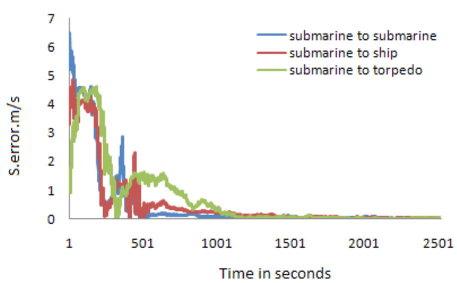

(c)

Figure 7. RMS errors in estimates for high ATB scenarios (Monte Carlo runs). **(a)** Error in Range estimate. **(b)** Error in Course estimate. **(c)** Error in Speed estimate.

Table 7. Convergence time in seconds for Low ATB Scenarios with 100 Monte-Carlo runs

Scenario	CKF			
	R	C	S	Total solution
1. Submarine to Submarine	NC	NC	NC	NC
2. Submarine to ship	453	876	692	876
3. Submarine to Torpedo	453	872	872	872

Table 8. Convergence time in seconds for Medium ATB Scenarios with 100 Monte-Carlo runs

Scenario	CKF			
	R	C	S	Total solution
1. Submarine to Submarine	563	896	692	896
2. Submarine to ship	340	680	672	680
3. Submarine to Torpedo	550	960	980	980

Table 9. Convergence time in seconds for High ATB Scenarios with 100 Monte-Carlo runs

Scenario	CKF			
	R	C	S	Total solution
1. Submarine to Submarine	362	1957	448	1957
2. Submarine to ship	312	923	771	923
3. Submarine to Torpedo	641	1010	1019	1019

5. Summary and Conclusion

In underwater, the observer can be submarine and the target will be submarine, ship or torpedo. Accordingly, scenario is considered covering low ATB. CKF algorithm is considered for the performance evaluation with respect to convergence of the solution. Simulation was carried out and the solution for CKF obtained.

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