Indian Journal of Science and Technology, *Vol 9(17), DOI: 10.17485/ijst/2016/v9i17/93042, May 2016*
ISSN (Print) : 0974-6846
ISSN (Online) : 0974-5645

Incorporating Implied Volatility in Pricing Options using Binomial Tree

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Abstract

Background/Objectives: The main objective of this paper is to present an n-step binomial model which can be used to price an option under any exotic conditions. **Methods/Statistical Analysis:** Mathematical models have been presented using which an n-step binomial model can be developed. The model can be used for estimating price of options under n of number exotic conditions that influence the option price. **Findings:** Pricing of exotic options like Asian, American etc., undertaken through Binomial Trees using only one-step considering maximum and minimum values that can be taken by the underling at the maturity leads to a rough approximation of the option price. The approximation is possible by assuming stock price movements to be in one or two binomial steps during the life of the option. A binomial tree extended to an N-step Model can be used to price various exotic options. A study of the convergence in European option price with respect to Number of steps (N) and variation in price of Asian and American options with respect to confidence factor (k) (proxy for implied volatility) using the maximum and minimum boundaries on the value of k gives the investors the ability to change the value of k so that they can have their own opinions concerning the risk-neutral probability distribution.

Keywords: Binomial Tree, Implied Volatility, Pricing Options

1. Introduction

An option holder can use Trees for taking decisions prior to the maturity of the American Options and other derivatives. No analytical valuations exist for American options and therefore one has to use Binomial tree for valuation. It is generally assumed that the movement of stock prices can be represented in one or two binomial steps during the life of the option. Using binomial trees one can only get rough estimation of the option price. About 30 or more steps can be considered in estimation of the option price and as more steps are increase, the option price generally converges to a particular value.

A method was presented to price European Options at a confidence using one-step binomial model by considering only the maximum and minimum values that the underling takes at the maturity.

A rough estimation of the option price can be made by assuming that one two binomial steps during the period of the existence of the option. In this work, the model discussed is extended to build a binomial tree that can be used to price various exotic options. A study on, 1) The convergence in European option price with respect to number of steps (N). 2) The variation in price of Asian and American options with respect to confidence factor (k) which is a proxy for implied volatility. 3) The maximum and minimum boundaries on the value of k are performed. The proposed binomial tree gives the investors the ability to change the value of k so that they can have their own opinions concerning the risk-neutral probability distribution.

The rest of the paper is organized as follows; Section 2 presents the literature survey on various tree-based models for option pricing. Section 3 describes the model used to arrive at the price of the option in a single step; Section 4 describes how to calculate the percentage

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upward movement (u) and the percentage downward movement (d) at a Probability (P) using[1] and describes the derivation of the boundaries for the confidence factor (k). Section 5 explains the variation in option price with the increase in Number of steps (N) and confidence factor (k), and section 6 discusses the results and concludes.

Lattice methods can be used for computing the derivative prices. Discrete and stochastics approximations are used in the lattice methods. The lattice methods are quite easy for implementation and therefore are used quite frequently. Lattice methods are generally used as the models are simple and quite suitable for computation of derivatives. Various types of Lattice methods have been proposed in literature that includes CRR[2-5]. Using these models binomial and trinomial lattices can be constructed. In the case of binomial models consider that one node in a step-1 will lead to two nodes in step-2 and similarly in the case of trinomial models one node in step-1 leads to three nodes in step-2. Further models proposed by[6,7] considered four calculations at each of the node. The overall convergence has been witnessed when binomial lattices are considered at the cost of more computational time. It has been shown by Broadie and Detemple[8] that stock prices takes lognormal distributions which lead to more accuracy in estimating the option prices especially the American options. The additional accuracy achieved is however leads to extra computations, meaning leading to additional cost of computation. In the real world option prices do not follow the lognormal distribution.

A model has been proposed to price European options irrespective of the underlying distribution. However, the model cannot be used as is to price American options. In this work, we extended the model proposed to price American and other path dependent options. The boundary on the model parameter "k" is also determined. The model gives the ability to change the parameter "k" so that the investors can have their own opinions concerning the risk-neutral probability distribution of the underlying. The convergence of the option price with the increase in the number of steps in the proposed method of constructing the binomial tree is also studied. Influence of stock options, trading behaviors of the foreign investors without looking into issue of collateral amount[9,10].

1.1 The Model

Consider a random process X (t) with mean μ and standard deviation σ. The Chebyshev's inequality given in Equation 1 gives the confidence level about the deviation of the value taken by X (t) from the mean.

$$P\left(|X(t) - \mu| \geq k\sigma\right) \leq 1/k^2 \quad (1)$$

Where $k > 1$

The price movement of a stock is a random process (random walk). Let R (τ) be the returns on the stock with mean μ_r and standard deviation σ_r then as per Equation 1.

$$P\left(|R(\tau) - \mu_r| \geq k\sigma_r\right) \leq 1/k^2 \quad (2)$$

Where

$|R(\tau)| = |S(t_2) - S(t_1)|\ t_2 = t_1 + \tau,$

S (t) is daily close of underlying.

The Chebyshev's inequality means that if we take k = 10 then 99% of R (τ) will fall in the range $[-k\sigma_r + \mu_r, k\sigma_r + \mu_r]$ no matter what the probability distribution of R (τ) may be. It is observed that the volatility for t months is effectively $\sigma_r\sqrt{t}$ and therefore the boundaries become $[-k\,\sigma_r\sqrt{t} + \mu_r, k\,\sigma_r\sqrt{t} + \mu_r]$.

1.2 Determining P, u, d, k

Let the current value of the underlying be S_0
Let the strike price of the option be S_k
Let the risk free interest rate be r
Let the number of steps in the binomial tree be N
Let the time to maturity of the option be T in months
Let the time for each step be t = T / N in months
Let the standard deviation of the returns be σ_r
Let the expected return from the stock be μ_r

Let the probability that the underlying instrument price takes the upper boundary be P.
From Equation 2 we have

$$P\left[S_0 - k\,\sigma_r\sqrt{t} + \mu_r < S(t) < S_0 + k\,\sigma_r\sqrt{t} + \mu_r\right] \geq 1-(1/k^2) \quad (3)$$

The Equation 3 is shown in Figure 1 as a general one step tree. In risk neutral world where all individuals are indifferent to risk, the investors require no compensation for the risk and the expected return on all the securities is the risk free interest rate. The relation shown in Equation 4 should hold for non-dividend paying stocks.

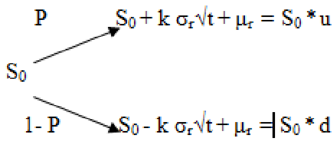

Figure 1. Stock price in a general one step tree.

$$P * (S_0 * u) + (1- P) * (S_0 * d) = S_0 * e^{rt} \quad (4)$$

This gives the value of P, u and d as

$$P = \frac{e^{rt} - d}{u - d} \quad (5)$$

$$u = 1 + \frac{\left(k\sigma_r \sqrt{t} + \mu_r\right)}{S_0} \quad (6)$$

$$d = 1 - \frac{\left(k\sigma_r \sqrt{t} - \mu_r\right)}{S_0} \quad (7)$$

The probability obtained by using Equation 5 is the risk neutral probability of reaching the node S_0 * u by the underlying at time t. The proof for it is shown in Appendix-1. This method of finding the risk neutral probability is much easier than the methods proposed in[11-13] as there is no need for solving any optimization problems to obtain the risk neutral probability of the nodes at each step. Neglecting higher order terms u and d can be approximated as given by Equations 8 and 9.

$$u = \exp \frac{\left\{\left(\sigma_r \sqrt{t} + \mu_r\right)\right\}}{S_0} \quad (8)$$

$$d = \exp \frac{\left\{-\left(\sigma_r \sqrt{t} - \mu_r\right)\right\}}{S_0} \quad (9)$$

It may be noted that t should be expressed in months. Since P is probability, it must satisfy the condition (10)

$$0 \le P \le 1$$

We have

$$\frac{e^{rt} - d \le 1}{u - d} \quad (10)$$

Hence, the minimum value of k is given by the equation

$$k > \max \frac{\left(1, S_0^* \left(e^{rt} - 1\right) - \mu_r\right)}{\sigma_r \sqrt{}} \quad (11)$$

Although there is no limit on the maximum value of k the fact that the value of the underlying should be greater than zero forces the value of the lower boundary to be greater than zero giving rise to inequality (12) that gives the maximum value for k.

Lower boundary = S_0 - k $\sigma_r \sqrt{t} + \mu_r \ge 0$

$$k \le \frac{S_0 + \mu_r}{\sigma_r \sqrt{t}} \quad (12)$$

From equations (11) and (12) we have the below relation

$$Max\left(1^+, \frac{S_0^* \left(e^{rt} - 1\right) - \mu_r}{\sigma_r \sqrt{t}}\right) < k \le \frac{S_0 + \mu_r}{\sigma_r \sqrt{t}}$$

2. Variations in Option Price with Respect to N, K

It has been verified again and again that one or two binomial steps only will help estimating the option prices to an approximations. It is necessary to increase number of steps of processing such that the option price converges to a specific cost and there is no need to approximate the option price. In the case of European options, the calculation of the call option is calculated considering the cost to be in between (0, S_t - S_k) where S_t is the value of underlying at maturity. The calculation is effected for the nodes at maturity.

When period is t, and discount rate is r, the expected cost of the option is calculated at each of the node considering the time to be T – t. Even the expected value of the option at each of the node at time T – 2*t can also be computed considering the expected value at time T – t which is discounted at rate r for a period t. It is mathematically expressed in Equations (13) and (14) for call and put options respectively. It is necessary to check at each of the node whether an early exercise is needed to find whether it is preferable to hold the option for further period t in respect of American options.

$$Call = e^{(-Nrt)} \sum_{j=0}^{N} [N!/ j!(N - j)!] P^j (1 - P)^{N-j} Max(0, u^j d^{N-j} S_0 - S_k) \quad (13)$$

$$Put = e^{(-Nrt)} \sum_{j=0}^{N} [N!/ j!(N - j)!] P^j (1 - P)^{N-j} Max(0, S_k - u^j d^{N-j} S_0) \quad (14)$$

In case of European Asian options the payoff at maturity for a call option is given by arithmetic average of discounted value of the maximum (0, S_{avg}-S_k) or maximum (0, S_t-S_{avg}) across all the paths for average price option (Equations (15), (16)) and average strike option (Equations (17), (18)) respectively. S_{avg} is the arithmetic average value of the underlying in a particular path in the tree, with first and last underlying price being included in the average Table 2, S_t is the value of the underlying at maturity in that path and S_k is the strike price of average price Asian option.

Table 1. Values for various Parameters considered in pricing the Option

μ_r	σ_r	k	R	S_0	T
6.277273	53.96829	5	10%p.a	4076.45	2 months

$$Call_{avg.price} = \frac{e^{(-Nrt)}}{N}\sum \ Max\left(0, S_{avg} - S_k\right) \quad (15)$$

$$Put_{avg.price} = \frac{e^{(-Nrt)}}{N}\sum \ Max\left(0, S_k - S_{avg}\right) \quad (16)$$

$$Call_{avg.stike} = \frac{e^{(-Nrt)}}{N}\sum \ Max\left(0, S_t - S_{avg}\right) \quad (17)$$

$$Put_{avg.stike} = \frac{e^{(-Nrt)}}{N}\sum \ Max\left(0, S_{avg} - S_k\right) \quad (18)$$

In case of American style options, at each node the intrinsic value (difference between underlying price and strike in case of call option and difference between strike and underlying price in case of put option) is calculated. At a node, if the intrinsic value is greater than the discounted pay-off, the intrinsic value is considered for calculating the option price instead of discounted payoff, which is not the case for European option. Hence, pricing of an American requires a lattice based method for pricing as the option cannot be expressed analytically.

For a given volatility (σ_r), confidence factor (k), maturity (t), mean return (μ_r), risk-free rate (r) and current value of underlying (S_0), the price of the option is calculated using trees with different steps. Table 1 gives these parameters used in this work. It can be observed from Table 2 that the option price is converging as the number of steps is increasing. There is not much change in option price after 25 steps. In addition, the value of k has no effect on the number of steps required for the convergence of option price.

Table 2. European (American) option price with varying number of steps for different values of K

Steps	k=3	k=4	k=5	k=6	k=7
1	194.7683	229.4601	265.2295	301.5182	338.0831
4	182.637	208.7608	235.6404	262.8993	290.3701
10	176.9416	205.7935	234.7556	263.7646	292.7917
15	177.4326	205.6631	233.2189	260.4346	287.4507
20	177.2588	205.17	231.5033	257.4101	288.1307
25	177.4472	204.7238	230.0573	259.993	289.833
30	177.8302	204.3942	230.6786	260.5725	289.2627
40	177.9671	204.0078	231.3943	259.6377	287.0224
60	177.1968	203.1763	231.0027	258.8152	287.9873

Once the factors that affect the price of an option are fixed, it can be varied by changing the value of k. As the value of k increases, the price of the option also increases as shown in Tables 3, 4 and 5 for American (European) option, Average Price and Average Strike Asian options

Table 3. Average Price of underlying for each path in a 4 step Tree for different values of k

Path	k=4	k=5	k=6	k=7	k=8	k=9	k=10
Uuuu	4413.716	4500.807	4590.418	4682.631	4777.528	4875.197	4975.726
Uuud	4344.961	4412.403	4481.294	4551.669	4623.561	4697.005	4772.04
Uudu	4278.834	4328.172	4378.29	4429.202	4480.923	4533.464	4586.842
Uudd	4215.04	4247.668	4280.761	4314.327	4348.373	4382.906	4417.934
Uduu	4215.236	4247.916	4281.061	4314.68	4348.78	4383.369	4418.453
Udud	4151.441	4167.412	4183.533	4199.805	4216.23	4232.81	4249.546
Uddu	4090.086	4090.708	4091.473	4092.381	4093.434	4094.63	4095.97
Uddd	4030.895	4017.398	4004.307	3991.617	3979.322	3967.419	3955.902
Duuu	4154.069	4171.447	4189.285	4207.587	4226.361	4245.613	4265.349
Duud	4028.92	4014.24	3999.696	3985.288	3971.015	3956.874	3942.866
Dudu	4028.92	4014.24	3999.696	3985.288	3971.015	3956.874	3942.866
Dudd	3969.728	3940.93	3912.531	3884.524	3856.903	3829.663	3802.798
Dduu	3969.91	3941.156	3912.799	3884.834	3857.254	3830.055	3803.229
Ddud	3910.719	3867.846	3825.633	3784.069	3743.143	3702.844	3663.162
Dddu	3853.791	3797.997	3743.356	3689.841	3637.428	3586.091	3535.808
Dddd	3798.871	3731.239	3665.452	3601.454	3539.19	3478.608	3419.657

Table 4. Payoff of average price Asian option for strike price = 4000 at maturity

Path	k=4	k=5	k=6	k=7	k=8	k=9	k=10
Uuuu	406.8779	492.5294	580.6592	671.3481	764.6766	860.7313	959.5987
Uuud	339.2593	405.5866	473.3389	542.5507	613.2544	685.4845	759.2793
Uudu	274.2253	322.7478	372.0374	422.1079	472.974	524.6466	577.1424
Uudd	211.4857	243.5744	276.1204	309.1316	342.6149	376.5771	411.0262
Uduu	211.6785	243.8183	276.4155	309.4788	343.0152	377.0325	411.5366
Udud	148.9379	164.6449	180.4995	196.5025	212.656	228.962	245.4214
Uddu	88.59701	89.20873	89.96108	90.85408	91.88967	93.0659	94.38376
Uddd	30.38435	17.11044	4.235812	0	0	0	0
Duuu	151.5225	168.6132	186.1564	204.1559	222.6196	241.5534	260.9632
Duud	28.44199	14.00463	0	0	0	0	0
Dudu	28.44199	14.00463	0	0	0	0	0
Dudd	0	0	0	0	0	0	0
Dduu	0	0	0	0	0	0	0
Ddud	0	0	0	0	0	0	0
Dddu	0	0	0	0	0	0	0
Dddd	0	0	0	0	0	0	0
Average Payoff	119.9908	135.9902	152.464	171.6331	191.4813	211.7533	232.4595

Table 5. Payoff of average strike Asian option at maturity

Path	k=4	k=5	K=6	k=7	k=8	k=9	k=10
Uuuu	344.8739347	438.005731	535.4805	637.4783	744.1889	855.806	972.5333
Uuud	74.39764854	90.23547283	106.2011	122.2888	138.4974	154.822	171.2558
Uudu	139.4316654	173.0742569	207.5026	242.7316	278.7778	315.6599	353.3928
Uudd	0	0	0	0	0	0	0
Uduu	201.9784829	252.0037419	303.1246	355.3607	408.7366	463.2741	518.9985
Udud	0	0	0	0	0	0	0
Uddu	11.36007876	10.74835952	9.996004	9.103012	8.067416	6.891184	5.573333
Uddd	0	0	0	0	0	0	0
Duuu	262.1344813	327.2088205	393.3837	460.6836	529.1322	598.7532	669.572
Duud	71.51509371	85.95245465	100.2561	114.4259	128.463	142.3703	156.1467
Dudu	71.51509371	85.95245465	100.2561	114.4259	128.463	142.3703	156.1467
Dudd	0	0	0	0	0	0	0
Dduu	129.5497442	157.8284824	185.7168	213.2196	240.3437	267.0931	293.4757
Ddud	0	0	0	0	0	0	0
Dddu	0	0	0	0	0	0	0
Dddd	0	0	0	0	0	0	0
Average Payoff	81.67226395	101.3131109	121.3698	141.8573	162.7919	184.19	206.0684

respectively. It supports the statement that, increase in value of k increases the confidence that the option writer will not lose because charging more premium decreases the probability of loss.

The path probabilities for a 4-step tree for different values of k using the parameters shown in Table 4 have been calculated. The expected value of the underlying at maturity using the risk free rate of 10% for maturity of 2 months is 4144.960166 (assuming continuous compounding) as shown in Table 6 and Table 7.

The expected value of the underlying at maturity using N step binomial tree is given by

$$\text{Expected value at maturity} = \sum_{\text{Nodes at maturity}}^{\text{Node value * nodal probability}} \quad (19)$$

$$\text{Nodal Probability} = \sum \text{probability of the paths ending at the node} \quad (20)$$

Table 6. Path probabilities for a 4-step tree at step 4 for different values of k

Path	K = 5	K = 6	K = 10
Uuuu	0.071104971	0.067314663	0.057950396
Uuud	0.066592186	0.064839929	0.060161137
Uudu	0.066592186	0.064839929	0.060161137
Uudd	0.062365811	0.062456175	0.062456215
uduu	0.066592186	0.064839929	0.060161137
udud	0.062365811	0.062456175	0.062456215
uddu	0.062365811	0.062456175	0.062456215
uddd	0.05840767	0.060160056	0.064838848
duuu	0.066592186	0.064839929	0.060161137
duud	0.062365811	0.062456175	0.062456215
dudu	0.062365811	0.062456175	0.062456215
dudd	0.05840767	0.060160056	0.064838848
dduu	0.062365811	0.062456175	0.062456215
ddud	0.05840767	0.060160056	0.064838848
dddu	0.05840767	0.060160056	0.064838848
dddd	0.054700738	0.057948351	0.067312375
Sum of path probabilities	1	1	1

Table 7. Path probabilities for a 3-step tree at step 3 for different values of k

Path	K = 5	K = 6	K = 10
uuu	0.137697	0.132155	0.118112
uud	0.128958	0.127296	0.122617
udu	0.128958	0.127296	0.122617
udd	0.120773	0.122616	0.127295
duu	0.128958	0.127296	0.122617
dud	0.120773	0.122616	0.127295
ddu	0.120773	0.122616	0.127295
ddd	0.113108	0.118108	0.132151
Sum of path probabilities	1	1	1

The 4 step binomial trees and the corresponding nodal probabilities at maturity for k = 5, 10 are shown in Table 8 and Table 9.

It can be observed from the results that irrespective of the value of k the expected value of the underlying at maturity calculated using the nodal values and the corresponding nodal probabilities at each step in the

Table 8. Path probabilities for a 4-step tree at step 4 for k=5

K = 5				node	nodal probability
				4946.174	0.071104971
			4712.73		
		4490.304		4504.155	0.266368743
	4278.376		4291.573		
4076.45		4089.024		4101.637	0.374194867
	3896.035		3908.052		
		3723.604		3735.09	0.23363068
			3558.805		
				3401.3	0.054700738
Sum of nodal probabilities					1

Expected value at maturity = 4144.960417

Table 9. Path probabilities for a 4-step tree at step 4 for k=10

K = 10				node	nodal probability
				5964.604	0.057950396
			5423.214		
		4930.965		4946.174	0.240644547
	4483.395		4497.224		
4076.45		4089.024		4101.637	0.37473729
	3717.875		3729.343		
		3390.841		3401.3	0.259355391
			3092.574		
				2820.543	0.067312375
Sum of nodal probabilities					1

Expected value at maturity = 4144.960239 binomial tree is equal to the continuously compounded value using risk-free rate. Therefore, the nodal probabilities are risk-neutral probabilities. Hence the path probabilities from which the nodal probabilities are calculated are risk-neutral. From Table 8 and Table 9, it can be seen that when the value of k is changed from 5 to 10 the risk-neutral probability for the node 4946.174 has increased from 0.071104971 to 0. 240644547 and the risk-neutral probability for the node 3401.3 have increased from 0.054700738 to 0.259355391, which is a parameter dependent on volatility of the underlying. Hence changing the value of k is actually changing the volatility used in pricing the option so that k can be used as a proxy for implied volatility, changing which, the risk-neutral probabilities can be changed as per the view of the investor. Also the Equation 21 is satisfied at each step irrespective of the value of k which proves that the probabilities calculated by using Equation (5) are risk-neutral.

$$1 = \frac{1}{S_0 * (e^{rnt} - 1)} \sum_{Nodes\ at\ n^{th}\ step}^{(node\ value - S_0)*\ nodal\ probability} \quad (21)$$

3. Conclusions

In this research, the model proposed in the literature is extended to price American options along with the inclusion of the confidence parameter k which is a proxy of implied volatility. From the results obtained in this research, we conclude that as the number of steps increases the price of the European (American) option decreases and converges to a particular value. In addition, increase in the value of k increases the option price for both Asian and European (American) options.

The confidence that the option seller will not is determined by the value of k chosen for pricing the option and is given by $(1 - 1/k^2) * 100$ %. Hence, American and Asian options can be priced at certain confidence. Inclusion of the confidence parameter k in the calculation of P, u and d incorporates a proxy for implied volatility in the binomial tree.

The proposed Model can also be used to price various exotic options like barrier, lookback options etc. Changing the value of k changes the nodal probabilities so that investors can have their own opinions concerning the risk-neutral probability distribution at maturity. The value of μ_r can be made equal to zero to make the tree recombining which will result in computational savings because O (n) distinct asset price levels can be computed once and stored.

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