Indian Journal of Science and Technology, *Vol 9(17), DOI: 10.17485/ijst/2016/v9i17/93044, May 2016*
ISSN (Print) : 0974-6846
ISSN (Online) : 0974-5645

Pricing Options Considering Bankruptcy of Underlying Issuer

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Abstract

Background/Objectives: The main objective of this paper is to present a model that takes bankruptcy of the seller while calculating the risk involved in fixing the premium at which the underlying options are sold. **Methods/Statistical Analysis:** A mathematical model which is based on probability distribution have been presented in the paper that extends the binary decision tree model to include bankruptcy for assessing the risk in selling underlying option at premium. **Findings:** A novel method has been presented that extends the binomial tree for taking into account default probability of the seller. The model can be used by the option seller for computing the risks involved in fixing the premium at which the options can be sold. It has been observed that the option prices considering the bankruptcy are higher compared to the option prices without bankruptcy.

Keywords: Bankruptcy, Pricing Option, Underlying Issuer

1. Introduction

Options transfer the risk of the underlying from the option buyer to the option seller. Options on underlying like stock market indices do not suffer from the risk of bankruptcy, as the probability that the index becomes zero is almost negligible. However while trading options on individual firms; the probability of the firm being bankrupt plays a role in pricing the options being traded on the firm's stock.

On bankruptcy of a company, the payoff on the put option is equal to the strike price of the option, which is its maximum. Hence, it is important for the option seller to consider this state while pricing options. In[1,2] do not consider the state of bankruptcy while pricing options. In[3] developed a model to price European options under bankruptcy. In[4] A binomial tree has been developed that introduces a new state of underlying becoming zero from each node in the binomial tree. However, it assumes that the probability of being bankrupty follows a Poisson process and the tree becomes less volatile when the square of volatility approaches default intensity.

In the current study, we address the issue of considering the bankruptcy of a firm while pricing options on its stock. We extend the tree construction methodology proposed in[5] by introducing a new state where the firms stock can become zero. The maximum probability of being bankrupt is calculated using the Chebyshev inequality [6]and is multiplied with a factor α which lies between [0, 1]. The methodology does not assume any distribution of the bankruptcy process and takes into account the current underlying price to measure the probability of being bankrupt.

Infinite life time constant volatility diffusion famously known as GBM has been considered to be built-in Black–Scholes[7] model. In GBM the stock price is expected to follow Brownian motion which geometric. This model does not consider the issue of bankruptcy which is one of the major limitations of the model. The models built for corporate bands consider extensively the issue of bankruptcy and credit spread. The models that are related to stock options, credit spread and bankruptcy have evolved over the time almost simultaneously and independently. However in the recent times both aspects are being con-

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sidered to be built into a single model. The models that combine both the aspects are being used for pricing the convertible bands which are basically the corporate bonds which also embedded in it the stock options.

A new framework has been presented called the reduced-form framework until quite recently. The framework considers hazard rate of bankruptcy $h = h(S)$, a function that reduces the price of the option generally known as decreasing function. The bankruptcy is modelled at the time of first jump that can be witnessed form a stochastic Poisson process that is built with intensity h. The term structure of the credit spreads is determined based on the intensity factor and various other parameters that determine the underling price of the options, negative power intensity will have adverse effect on the price of the option.

$$h(S) = \alpha S^{-p} \qquad (1)$$

for some values of $\alpha > 0$ and $p > 0$.

[8-11]and Buffum have employed the model in the process of converting the bonds.

In these models, lattice methods or finite-difference are used to determine numerically, the price of convertible bonds.

The negative power intensity model is being used for pricing convertible bonds. The models exhibited volatility skews which are implied while computing the equity option prices. The hazard rate function is primarily responsible for skews in the volatility of the options. Thus the hazard rate helps in establishing the link between implied volatility and credit skews.

[12]Have shown the linkage between credit spreads and implied volatility skews. Vadim Linetsky has proposed a model using which one can build a binomial tree. The model presented by Vadim Linetsky considered both aspects of credit spread and bankruptcy[13,14]. Poisson distribution for bankruptcy process has been assumed in the model. The model however fails when default intensity is the square root of the volatility is equal to default intensity.

The main contribution of this work is as follows:

- Consider the approach to build binomial tree following the methodology discussed in K. V. N. M. Ramesh et al. Extend this methodology to consider bankruptcy.
- Calculate an upper bound on probability of default that depends on stock price, stock volatility, time and use a fraction of it in pricing options under bankruptcy.
- Make the probability of default dependent on the underlying price and volatility as opposed to considering only underlying price as in equation (1).
- Make the probability of default and stock price process independent of any distribution.

In[15,16] studied influence of stock options, trading behaviors of the foreign investors without looking into issue of collateral amount.

2. Building the Defaultable Binomial Tree

In the event of default a stock receives recovery rate or zero. The price of the stock jumps to zero upon default. Therefore while building the binomial tree for stock price with default with S_t as price at time t, the stock price S_{t+h} at the nextperiod may take one of three potential values:

$$S_{t+h} = \begin{cases} u\,S_t \text{ (an up move) with probability } q(1-\lambda_t) \\ d\,S_t \text{ (a down move) with probability } (1-q) \\ (1-\lambda_t)0 \text{ (default) with probability } \lambda_t \end{cases} \qquad (2)$$

Where

u is a multiplier for upward movement
d is a multiplier for downward movement
h is the length of the time step
λ_t is the probability of default

Figure 1 shows the binomial tree corresponding to total default where the stock price becomes zero on

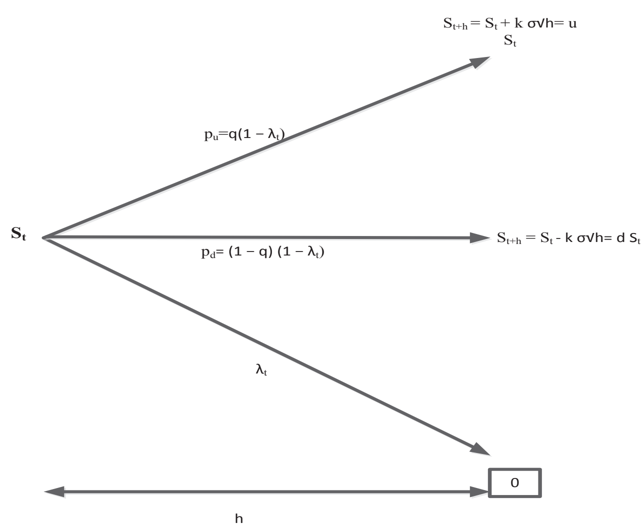

Figure 1. Defaultable Binomial Tree for one-step with stock price becoming zero on default.

default. The tree is in line with the one used in K. V. N. M.[10] for which a new state of default is added.

In case of total default, the stock at time t, becomes zero at time t + h when the return on the stock is equal to $-S_t$. Considering the volatility of returns to be σ applying Chebyshev inequality [14] we have equation (3) from which equation (4) is derived.

$$\text{Prob}(R \le - (S_t / \sigma\sqrt{h})^* \sigma\sqrt{h}) \le 1/(1+(-S_t/\sigma\sqrt{h})^2) \quad (3)$$

Where

R is the return on the underlying

$$\text{Prob}(R \le -S_t) \le 1/(1 + (-S_t/\sigma\sqrt{h})^2) \quad (4)$$

From equation (4), it can be inferred that during default, the probability that the stock price at time t becomes zero within the interval t + h corresponds to the maximum value as expressed in equation (5).

$$\lambda_t=\alpha^*\{1/(1 + (-S_t/\sigma\sqrt{h})^2)\} \quad (5)$$

where $0 \le \alpha \le 1$

From equation (5), it can be observed that the default probability is inversely proportional to S_t^2 and directly proportional to σ^2. It means that the default probability decreases as the stock price increases and increases with the increase in volatility of the underlying. The multipliers for upper and lower bounds are derived from the equation (6) and equation (7).

$$S_{t+h} = S_t + k\, \sigma\sqrt{h} = S_t (1 + k\, \sigma\sqrt{h}/ S_t) \quad (6)$$

$$S_{t+h} = S_t - k\, \sigma\sqrt{h} = S_t (1 - k\, \sigma\sqrt{h}/ S_t) \quad (7)$$

From equation (6) and (7), we have u and d as given in equations (8) and (9). In order to make the tree a recombining binomial tree, the values of "u" and "d" are made dependent on the initial value of the stock (S_0). Therefore, equations (8) and (9) can be re-written as equations (10) and (11).

$$u = (1 + (k\, \sigma\sqrt{h})/S_t) \quad (8)$$

$$d = (1 - (k\, \sigma\sqrt{h})/S_t) \quad (9)$$

$$u = (1 + (k\, \sigma\sqrt{h})/S_0) \quad (10)$$

$$d = (1 - (k\, \sigma\sqrt{h})/S_0) \quad (11)$$

Applying arbitrage free pricing, the value of "q" is derived from equations (12) for the case shown in Figure 1.

$$q(1 - \lambda_t)\, u^*\, S_t + (1 - q)(1 - \lambda_t)^* d^*\, S_t = S_t{}^*\, e^{rh} \quad (12)$$

From equation (11) the value of q considering bankruptcy is given by equation (13).

$$q = ((e^{rh}/(1- \lambda_t)) - d)/(u - d) \quad (13)$$

The tree parameters considering bankruptcy are given in Table 1.

Bounds on k, λ_t

The probability p_u should lie in the interval [0,1]. Hence, the condition below should hold. This gives an upper bound on the default as given in equation (14).

$$0 < (e^{rh} - d(1 - \lambda_t))/(u - d) < 1\lambda_t < (u - e^{rh})/d \quad (14)$$

The value of d, which is the multiplier for the lower jump, should be greater than zero. Hence, the value of k should be selected such that the downward jump multiplier should be greater than zero. This gives an upper bound on the value of k as expressed in equation (15) which is used to build the binomial tree given in equation (1).

$$k < S_0/\sigma\sqrt{h} \quad (15)$$

3. Comparison with Other Tree Based Models

Comparing the proposed model with the methodology proposed in Hull[8] where the model parameters are given in Table 2, the methodology proposed in[8] has the following disadvantages:

- The value of "u" approaches as the variance approaches the default intensity, which means that the tree becomes less volatile irrespective of the actual volatility of the underlying.

Table 1. Tree Parameters of a default table stock that drops to zero

Parameter	Definition
U	$(1 + (k\, \sigma\sqrt{h})/S_0)$
D	$(1 - (k\, \sigma\sqrt{h})/S_0)$
p_u	$(e^{rh} - d(1 - \lambda_t))/(u - d)$
p_d	$-(e^{rh} - u(1 - \lambda_t))/(u - d)$
λ_t	$\alpha^*(1/(1 + (-S_t/\sigma\sqrt{h})^2))$
Length of tree step	h

- The default intensity should be determined using the data of bond prices or credit default swap spreads of the issuer. However in the proposed model it requires only equity data.
- The default probability is assumed to be constant. However, in the proposed model the probability of default varies with the value of the underlying stock at each step and with its volatility.
- The model assumes Poisson process for the probability of default and geometric Brownian motion for the underlying stock process.

From Table 3, it can be observed that the price of an option without considering bankruptcy is less compared to the price of the option considering bankruptcy. From Table 4 it can be observed that as the parameter **α** which is used as a multiplier while calculating default probability, approaches zero, the option prices approaches to the prices calculated without considering bankruptcy.

Table 2. Tree Parameters of a defaultable stock that drops to zero using methodology in[8].

Parameter	Definition
U	$e^{(\sqrt{(\sigma 2 - \lambda)h})}$
D	1/u
p_u	$(e^{rh} - de^{\lambda h}))/(u - d)$
p_d	$(ue^{\lambda h} - e^{rh})/(u - d)$

Table 3. Price of call and put options with and without considering bankruptcy.

α	Call Price with Bankruptcy	PutPrice with Bankruptcy
0.1	1021.69	545.94
0.2	1021.83	546.17
0.3	1022.05	546.41
0.4	1022.23	546.64
0.5	1022.40	546.87
0.6	1022.58	547.11
0.7	1022.76	547.34
0.8	1022.93	547.58
0.9	1023.11	547.81

Table 4. Option price with bankruptcy for different values of

Strike	CallPrice with Bankruptcy(α = 1)	CallPrice without Bankruptcy	PutPrice with Bankruptcy(α = 1)	PutPrice without Bankruptcy
3500	1909.49	1906.18	76.82	73.11
4000	1614.09	1611.29	233.89	230.64
4500	1318.69	1316.40	390.97	388.17
5000	1023.29	1021.52	548.05	545.71
5500	727.89	726.63	705.12	703.23
6000	432.49	431.74	862.20	860.77
6500	137.10	136.86	1019.28	1018.30

4. Conclusions

In this work, we have presented a model to price an option without assuming any distribution of the underlying stock process and for default probability. The model depends on, issuer Stock price and does not need any bond price or credit default swap related information for calculating default probability. The formula for probability of default proposed in this work depends on the issuer stock price and its volatility apart from the length of the time step. It is observed that the price of an option considering bankruptcy of the underlying issuer is more than the price of the option which doesn't consider the same

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